

Número de posets con n elementos

Luiz F. Monteiro ^(*), Sonia Savini ^(**) e Ignacio Viglizzo ^(***)

^(*) Universidad Nacional del Sur

^(**) Departamento de Matemática-Universidad Nacional del Sur

^(***) INMABB-CONICET-Universidad Nacional del Sur

luiz.monteiro@speedy.com.ar, ssavini@criba.edu.ar, viglizzo@gmail.com

Resumen

Si $n \in \mathbb{N}$, notaremos con $P(n)$ el conjunto de todos los posets con n elementos, con $PC(n)$ ($PNC(n)$) el conjunto de todos los posets conexos (no conexos) con n elementos, con $PNI(n)$ el conjunto de todos los posets, no isomorfos, con n elementos, con $PCNI(n)$ ($PNCNI(n)$) el conjunto de los posets conexos (no conexos), no isomorfos, con n elementos. La determinación del número de elementos de estos conjuntos es un problema abierto desde hace mucho tiempo. No se conocen fórmulas, en función de n , para $|P(n)|$, $|PNI(n)|$, $|PCNI(n)|$ y $|PNCNI(n)|$.

Hasta el presente solo se conocen los valores de $|PNI(n)|$, $|PCNI(n)|$, para $1 \leq n \leq 16$, y de $|P(n)|$, $|PC(n)|$ para $1 \leq n \leq 18$. Los mismos se indican en la sección **1**, así como la historia de la determinación de $|P(n)|$. Para $n \geq 10$ los resultados obtenidos por diferentes autores son mediante el uso de computadoras.

En la sección **2** fijamos la nomenclatura a utilizar, en la sección **3** indicamos importantes resultados de M. Ern , en la sección **4** determinamos $|P(7)|$ y utilizando algunos resultados de Ern  determinamos $|P(10)|$ sin uso de computadoras, en la sección **5** interesantes resultados de R.P. Stanley y algunos resultados originales, en las secciones **6**, **7**, **8** y **10** resultados originales, y una determinaci n, diferente de las conocidas, de $|P(n)|$ para $1 \leq n \leq 8$, sin uso de computadoras. En la secci n **9** indicamos los diagramas de Hasse de $PNI(n)$ para $1 \leq n \leq 7$, as  como la cantidad de cada uno de ellos. Finalmente indicamos las referencias, que consideramos m s importantes, relacionadas con este tema. Otras referencias se pueden encontrar en la bibliograf a citada.

1. Resultados conocidos

n	$ P(n) $	$ PNI(n) $
1	1	1
2	3	2
3	19	5
4	219	16
5	4.231	63
6	130.023	318
7	6.129.859	2.045
8	431.723.379	16.999
9	44.511.042.511	183.231
10	6.611.065.248.783	2.567.284
11	1.396.281.677.105.899	46.749.427
12	414.864.951.055.853.499	1.104.891.746
13	171.850.728.381.587.059.351	33.823.827.452
14	98.484.324.257.128.207.032.183	1.338.193.159.771
15	77.567.171.020.440.688.353.049.939	68.275.077.901.156
16	83.480.529.785.490.157.813.844.256.579	44.831.306.651.195.087
17	122.152.541.250.295.322.862.941.281.269.151	
18	241.939.392.597.201.176.602.897.820.148.085.023	

n	$ PC(n) $	$ PCNI(n) $
1	1	1
2	2	1
3	12	3
4	146	10
5	6.060	44
6	101.642	238
7	5.106.612	1.650
8	377.403.266	14.512
9	40.299.722.580	163.341
10	6.138.497.261.882	2.360.719
11	1.320.327.172.853.172	43.944.974
12	397.571.105.288.091.506	1.055.019.099
13	166.330.355.795.371.103.700	32.664.984.238
14	96.036.130.723.851.671.469.482	1.303.143.553.205
15	76.070.282.980.382.554.147.600.692	66.900.392.672.168
16	82.226.869.197.428.315.925.408.327.266	4.413.439.778.321.689
17	120.722.306.604.121.583.767.045.993.825.620	
18	239.727.397.782.668.638.856.762.574.296.226.842	

Determinación de $ P(n) $				
Referencias Bibliográficas			A	B
[2]	1948	$1 \leq n \leq 4$	*	
[12]	1966	$1 \leq n \leq 6$	*	
[9]	1966	$1 \leq n \leq 6$	*	
[25]	1966	$1 \leq n \leq 6$	*	
[34]	1966	$1 \leq n \leq 6$	*	
[35]	1966	$1 \leq n \leq 6$	*	
[23]	1967	$1 \leq n \leq 7$		*
[17]	1970	$1 \leq n \leq 8$		*
[18, 19]	1972	$1 \leq n \leq 9$	*	
[15]	1977	$1 \leq n \leq 11$		*
[4]	1978	$1 \leq n \leq 9$		*
[5]	1979	$1 \leq n \leq 10$		*
[31]	1979	$1 \leq n \leq 11$		*
[32]	1982	$1 \leq n \leq 12$		*
[33]	1985	$1 \leq n \leq 6$	*	
[13]	1991	$1 \leq n \leq 11$		*
[21]	1991	$1 \leq n \leq 14$		*
[22]	1993	$n = 15$		*
[24]	2000	$1 \leq n \leq 14$		*
[7]	2002	$1 \leq n \leq 18$		*

Columna A, sin uso de computadoras, Columna B, con uso de computadoras.

2. Preliminares

Sea $\mathbf{n} = \{1, 2, \dots, n\}$. Si $X \in P(n)$ sea $m(X)$ el conjunto de los elementos minimales de X . Si $m \leq n$ sean

- $P(n, m) = \{X \in P(n) : |m(X)| = m\}$,
- $PC(n, m) = \{X \in PC(n) : |m(X)| = m\}$,
- $PNC(n, m) = \{X \in PNC(n) : |m(X)| = m\}$,

Para $1 \leq m \leq n$, notaremos con $PCNI(n, m)$, el conjunto de los elementos de $PC(n, m)$ que no son isomorfos, con $PNCNI(n, m)$ el conjunto de los elementos de $PNC(n, m)$ que no son isomorfos, con $PNI(n, m)$ el conjunto de los elementos de $P(n, m)$ que no son isomorfos.

Es claro que:

$$|P(n)| = \sum_{m=1}^n |P(n, m)|, \quad |PC(n)| = \sum_{m=1}^{n-1} |PC(n, m)|, \quad |PNC(n)| = \sum_{m=1}^n |PNC(n, m)|,$$

$$|PCNI(n, m)| + |PNCNI(n, m)| = |PNI(n, m)|,$$

$$|PCNI(n)| = \sum_{m=1}^{n-1} |PCNI(n, m)|, \quad |PNCNI(n)| = \sum_{m=1}^n |PNCNI(n, m)|,$$

$$|PNI(n)| = |PCNI(n)| + |PNCNI(n)|.$$

Brinkmann y McKay [7], en 2002 determinaron $|PC(n)|$ y $|P(n)|$ para $1 \leq n \leq 18$.

Dado $X \in P(n, n-1)$ existen $\binom{n}{n-1} = n$ formas de elegir el conjunto $m(X)$. Luego $|X \setminus m(X)| = 1$. Sea $X \setminus m(X) = \{a\}$ y $P_a = \{x \in m(X) : x < a\}$. Entonces $X \in PC(n, n-1)$ si y solo si $P_a = m(X)$, luego $|PC(n, n-1)| = \binom{n}{n-1} = n$. Además $X \in PNC(n, n-1)$ si y solo si $1 \leq |P_a| \leq n-2$ y por lo tanto $|PNC(n, n-1)| = n \sum_{i=1}^{n-2} \binom{n-1}{i} = n(2^{n-1} - 2)$.

Por lo tanto, si $n \geq 2$

$$|PC(n, n-1)| = n. \quad (2.1)$$

$$|PNC(n, n-1)| = \binom{n}{n-1} (2^{n-1} - 2). \quad (2.2)$$

Luego

$$|P(n, n-1)| = \binom{n}{n-1} + \binom{n}{n-1} (2^{n-1} - 2) = \binom{n}{n-1} (2^{n-1} - 1).$$

Es claro que:

$$|PC(n, n)| = 0 \quad \text{y} \quad |P(n, n)| = |PNC(n, n)| = 1. \quad (2.3)$$

Es bien conocido ([1], [2]) que existe una correspondencia biunívoca entre el conjunto $P(n)$ y el conjunto $T_0(n)$ de todas las topologías T_0 que se pueden definir sobre $\mathbf{n}$. A los efectos de hacer lo más autocontenido posible, los resultados de estas notas, vamos a indicar una demostración de este resultado.

Dado un poset $(\mathbf{n}, \leq)$ y $x \in \mathbf{n}$ sea $C(x) = \{y \in \mathbf{n} : y \leq x\}$ luego $x \in C(x)$ y si $Y \subseteq \mathbf{n}$ sea $C(Y) = \bigcup_{y \in Y} C(y)$. Entonces C es un operador de clausura en $\mathbf{n}$.

En efecto,

C1) $C(\emptyset) = \emptyset$.

C2) $Y \subseteq C(Y)$. En efecto, si $y \in Y$ entonces $y \in C(y) \subseteq C(Y)$.

C3) $C(C(Y)) = C(Y)$. Por C2) $C(Y) \subseteq C(C(Y))$. Sea $z \in C(C(Y))$ luego $z \in C(w)$ para algún $w \in C(Y)$, esto es (1) $z \leq w$ con $w \in C(y)$ para algún $y \in Y$, esto es (2) $w \leq y$. De (1) y (2) resulta que $z \leq y$ con $y \in Y$. Luego $z \in C(Y)$.

C4) $C(Y \cup Z) = C(Y) \cup C(Z)$.

Probemos en primer lugar que: si (1) $Y \subseteq Z$ entonces $C(Y) \subseteq C(Z)$. En efecto, si $a \in C(Y)$ entonces $a \in C(y)$ para algún $y \in Y$, luego por (1) $a \in C(y)$ para algún $y \in Z$, luego $a \in C(Z)$.

Como $Y \subseteq Y \cup Z$ y $Z \subseteq Y \cup Z$, entonces $C(Y) \subseteq C(Y \cup Z)$ y $C(Z) \subseteq C(Y \cup Z)$, luego $C(Y) \cup C(Z) \subseteq C(Y \cup Z)$.

Si $w \in C(Y \cup Z)$ entonces $w \in C(b)$ para algún $b \in Y \cup Z$. Si $b \in Y$ entonces $w \in C(Y)$ y si $b \in Z$ entonces $w \in C(Z)$, luego $w \in C(Y) \cup C(Z)$.

Luego $(\mathbf{n}, C)$ es un espacio topológico, diremos que es el espacio topológico generado por el poset $(\mathbf{n}, \leq)$. Es bien conocido que el conjunto de los cerrados de un espacio topológico, es un reticulado distributivo acotado. Notaremos con $RB(\mathbf{n})$ al conjunto de los cerrados de $(\mathbf{n}, C)$.

Veamos que $(\mathbf{n}, C)$ es T_0 , para ello basta ver que si $C(y) = C(z)$ entonces $y = z$. Como $y \in C(y) = C(z)$ entonces $y \leq z$ y como $z \in C(z) = C(y)$ entonces $z \leq y$ luego $z = y$.

Si $(\mathbf{n}, \leq) \in P(n)$ pongamos $\alpha((\mathbf{n}, \leq)) = (\mathbf{n}, C)$, luego $\alpha : P(n) \rightarrow T_0(n)$.

Si $(\mathbf{n}, \leq_1), (\mathbf{n}, \leq_2) \in P(n)$ son diferentes, entonces existe por lo menos un par $x, y \in \mathbf{n}$ tal que por ejemplo $x \leq_1 y$ y $x \not\leq_2 y$. Luego $x \in C_1(y)$ y $x \notin C_2(y)$, por lo tanto las topologías son diferentes, esto es $\alpha((\mathbf{n}, \leq_1)) \neq \alpha((\mathbf{n}, \leq_2))$

Sea $(\mathbf{n}, C) \in T_0(n)$, definamos $y \sqsubseteq z$ si y solo si $y \in C(z)$.

O1) $y \sqsubseteq y$, dado que $y \in C(y)$.

O2) Si $y \sqsubseteq z$ y $z \sqsubseteq y$, entonces $y = z$. Por hipótesis $y \in C(z)$ y $z \in C(y)$, luego $C(y) \subseteq C(C(z)) = C(z)$ y $C(z) \subseteq C(C(y)) = C(y)$ luego $C(y) = C(z)$ y como el espacio topológico es T_0 , $y = z$.

O3) Si $y \sqsubseteq z$ y $z \sqsubseteq w$ entonces $y \sqsubseteq w$. Por hipótesis (1) $y \in C(z)$ y (2) $z \in C(w)$. De (2) resulta (3) $C(z) \subseteq C(C(w)) = C(w)$. De (1) y (3) $y \in C(w)$ luego $y \sqsubseteq w$.

Luego $(\mathbf{n}, \sqsubseteq)$ es un poset. Diremos que $(\mathbf{n}, \sqsubseteq)$ es el poset generado por el espacio topológico $(\mathbf{n}, C)$. Notemos con $(\mathbf{n}, C^{(\sqsubseteq)})$ al espacio topológico que se obtiene a partir de $\sqsubseteq$ y probemos que $\alpha((\mathbf{n}, \sqsubseteq)) = (\mathbf{n}, C)$, esto es que $(\mathbf{n}, C^{(\sqsubseteq)}) = (\mathbf{n}, C)$.

Sea Y un cerrado de $(\mathbf{n}, C^{(\sqsubseteq)})$, esto es, $C^{(\sqsubseteq)}(Y) = Y$. Sabemos que $Y \subseteq C(Y)$. Sea $z \in C(Y) = \bigcup_{y \in Y} C(y)$, luego $z \in C(y)$ para algún $y \in Y$, por lo tanto $z \sqsubseteq y$ y en consecuencia $z \in C^{(\sqsubseteq)}(Y) = Y$.

Recíprocamente sea Y es un cerrado de $(\mathbf{n}, C)$, esto es, $C(Y) = Y$. Sabemos que $Y \subseteq C^{(\sqsubseteq)}(Y)$. Sea $z \in C^{(\sqsubseteq)}(Y)$, luego $z \in C^{(\sqsubseteq)}(y)$ para algún $y \in Y$ esto es $z \sqsubseteq y$ con $y \in Y$, entonces por definición $z \in C(y) \subseteq C(Y) = Y$.

Un espacio topológico E se dice conexo si los los únicos conjuntos abiertos (cerrados) son $\emptyset$ y E .

Si $(\mathbf{n}, \leq)$ es no conexo, esto es (1) $\mathbf{n} = X_1 + X_2$ con $X_1 \cap X_2 = \emptyset$, $X_1 \cup X_2 = \mathbf{n}$, $X_1, X_2 \neq \emptyset, \mathbf{n}$. Sea $y \in C(X_1)$ luego $y \in C(x)$ con $x \in X_1$ por lo tanto $y \leq x$, con $x \in X_1$, luego por (1) $y \in X_1$ y por lo tanto $C(X_1) = X_1$, luego $(\mathbf{n}, C)$ es no conexo.

Supongamos que $(\mathbf{n}, C)$ es no conexo, luego existe un abierto y cerrado $X_1 \subset \mathbf{n}$, $X_1 \neq \emptyset, \mathbf{n}$, luego $X_2 = \mathbf{n} \setminus X_1$ es abierto y cerrado. Por lo tanto $X_1 \cap X_2 = \emptyset$, $X_1 \cup X_2 = \mathbf{n}$, $C(X_1) = X_1$ y $C(X_2) = X_2$. Sean $x_1 \in X_1$ y $x_2 \in X_2$, si $x_1 \leq x_2$ entonces $x_1 \in C(x_2) \subseteq C(X_2) = X_2$, absurdo y si $x_2 \leq x_1$ entonces $x_2 \in C(x_1) \subseteq C(X_1) = X_1$, absurdo. Luego $\mathbf{n} = X_1 + X_2$ y $(\mathbf{n}, \leq)$ es no conexo. Como α es una biyección entonces $(\mathbf{n}, \leq)$ es no conexo (conexo) si y solo si $(\mathbf{n}, C)$ es no conexo (conexo).

Si $(X, \leq)$ es un poset y $f \in X$ notaremos $[f] = \{x \in X : x \leq f\}$ y $\lceil f \rceil = \{x \in X : f \leq x\}$. L. Monteiro en 1990, obtuvo el siguiente resultado, que recién fue publicado en 2001, [29]. Si X es un poset finito y $f \in X$ entonces:

$$|RB(X)| = |RB(X \setminus (f))| + |RB(X \setminus \lceil f \rceil)|.$$

Observemos que si X tiene primer elemento entonces $|RB(X)| = |RB(X \setminus (f))| + 1$ y que si X tiene último elemento entonces $|RB(X)| = 1 + |RB(X \setminus [f])|$. Es bien conocido que $|RB(X)|$ es igual al número de anticadenas del poset X .

Si $1 \leq j \leq n$, notaremos con $P^{(j)}(n)$, el conjunto de todos los elementos $X \in P(n)$ tales que $|RB(X)| = j$. Si conocemos $|P^{(j)}(k)|$ para $1 \leq j \leq 2^k$, $1 \leq k \leq n-1$, entonces dado el diagrama de Hasse de $X \in P(n)$, por el resultado precedente, sabemos determinar a que $P^{(j)}(n)$ pertenece X . Es claro que si X^* es el poset dual de X entonces $|RB(X)| = |RB(X^*)|$. Notaremos con $PC^{(j)}(n)$ ($PNC^{(j)}(n)$), el conjunto de todos los elementos $X \in PC(n)$ ($X \in PNC(n)$) tales que $|RB(X)| = j$. Si $j < 2^n$ es primo entonces $|PNC^{(j)}(n)| = 0$. En efecto, si existe $X \in PNC^{(j)}(n)$ entonces $X = X_1 + X_2$ y por lo tanto $j = |RB(X)| = |RB(X_1)| \times |RB(X_2)|$, absurdo.

Si X es un poset finito, notaremos con $Aut(X)$ el conjunto de todos los automorfismos de orden de X . Es bien conocido, ver por ejemplo ([24], página 337) que si X es un poset y $|X| = n$ entonces el número de posets isomorfos a X es igual a $\frac{n!}{|Aut(X)|}$.

3. Los resultados de M. Ern 

En 1981, M. Ern  [20] mediante una sofisticada demostraci n prueba que, si $n \geq 2$:

$$|P(n, 1)| = n|P(n-1)| = |PC(n, 1)|. \quad (3.1)$$

$$|P(n, 2)| = \binom{n}{2}|P(n-1)|. \quad (3.2)$$

La f rmula 3.1 se puede probar de un modo mucho m s sencillo que la indicada por Ern , como el mismo lo reconoce en [21]. En efecto, para $1 \leq i \leq n$, sea $P_{(i)}(n) = \{X \in P(n, 1) : m(X) = \{i\}\}$. Como $\{P_{(i)}(n)\}_{i=1}^n$ es una n -partici n de $P(n, 1)$ entonces $|P(n, 1)| = \sum_{i=1}^n |P_{(i)}(n)|$.

Es claro que $|P(n, 1)| = |PC(n, 1)|$ y que $|P_{(i)}(n)| = |P(n-1)|$ para $1 \leq i \leq n$, luego:

$$|P(n, 1)| = \sum_{i=1}^n |P_{(i)}(n)| = \sum_{i=1}^n |P(n-1)| = n|P(n-1)|.$$

Ern  denota con $T_0(n, j)$ el n mero de topolog as T_0 con j abiertos, o lo que lo mismo con j cerrados, que se pueden definir sobre $\mathbf{n}$ y con $T_0(n)$ el n mero de topolog as T_0 que se pueden definir sobre $\mathbf{n}$. Es claro que $T_0(n, j) = |P^{(j)}(n)|$. Es bien conocido que el n mero de conjuntos cerrados de un espacio topol gico T_0 con n puntos var a entre $n+1$ y 2^n , ([20], p gina 128). Luego

$$|P^{(j)}(n)| = 0, \text{ para } 1 \leq j \leq n, \text{ y } j \geq 2^n + 1. \quad (3.3)$$

Por lo tanto

$$|P(n)| = \sum_{j=n+1}^{2^n} |P^{(j)}(n)|.$$

Erné [20] determina los números $T_0(n, j) = |P^{(j)}(n)|$ para $1 \leq n \leq 6$, $1 \leq j \leq 2^n$, cuyas tablas se indican más adelante y por lo tanto $|P(n)|$ para $1 \leq n \leq 6$. Es claro que $|P^{(2^n)}(n)| = 1 = |PNI(n, n)|$. Para determinar $|P(n)|$ para $2 \leq n \leq 9$, Erné [20] define, para $n \in \mathbb{N}$:

$$\tilde{P}^{(j)}(n) = \sum_{r=0}^n \binom{n}{r} |P^{(j)}(n-r)| (-j)^r, \quad 1 \leq j \leq 2^n. \quad (3.4)$$

Es claro que

$$|P^{(1)}(0)| = 1 \quad \text{y} \quad |P^{(1)}(n)| = 0, \quad \text{para } n \geq 1, \quad (3.5)$$

luego

$$\tilde{P}^{(1)}(n) = \sum_{r=0}^n \binom{n}{r} |P^{(1)}(n-r)| (-1)^r$$

luego por (3.5) tenemos que:

$$\tilde{P}^{(1)}(n) = \binom{n}{n} |P^{(1)}(0)| (-1)^n$$

por lo tanto $\tilde{P}^{(1)}(n) = 1$ si n es par y $\tilde{P}^{(1)}(n) = -1$ si n es impar.

Por (3.3) $|P^{(j)}(n-1)| = 0$ para $j \geq 2^{n-1} + 1$ entonces como $2^{n-1} + 1 > 2^{n-2} + 1 > \dots > 2^1 + 1 > 2^0 + 1$ tenemos $|P^{(j)}(n-r)| = 0$ para $1 \leq r \leq n$ y $j \geq 2^{n-1} + 1$ luego

$$\tilde{P}^{(j)}(n) = |P^{(j)}(n)| \quad \text{para } j \geq 2^{n-1} + 1.$$

Por el Lema 5.1 de la sección 5: $\tilde{P}^{(2^{n-1})}(n) = |P^{(2^{n-1})}(n)| - n|P^{(2^{n-1})}(n-1)|2^{n-1} = (n-2)V_n^3 - n(2^{n-1}) = n((n-2)^2(n-1) - 2^{n-1})$.

Observemos que si $n \geq 4$ y $|P^{(j)}(n-1)| = 0$, para $2^{n-2} < j \leq 2^{n-1} - 1$ entonces $|P^{(j)}(k)| = 0$ para $k \leq n-1$ luego por (3.4) $\tilde{P}^{(j)}(n) = |P^{(j)}(n)|$.

Erné prueba que:

$$|P(n)| = \sum_{m=1}^n \binom{n}{m} E(n, m), \quad \text{donde } E(n, m) = \sum_{j=1}^{2^{n-m}} \tilde{P}^{(j)}(n-m) j^m. \quad (3.6)$$

$$E(n, 1) = E(n, 2) = |P(n-1)|. \quad (3.7)$$

Observemos que

$$|P(n, m)| = \binom{n}{m} E(n, m), \quad E(n, n-1) = 2^{n-1} - 1, \quad E(n, n) = 1.$$

Erné obtiene las siguientes tablas:

Tabla 3.1										
	$ P^{(j)}(n) $					$\tilde{P}^{(j)}(n)$				
	n					n				
j	1	2	3	4	5	1	2	3	4	5
1	0	0	0	0	0	-1	1	-1	1	-1
2	1	0	0	0	0	1	-4	12	-32	80
3		2	0	0	0		2	-18	108	-540
4		1	6	0	0		1	-6	0	320
5			6	24	0			6	-96	900
6			6	36	120			6	-108	1.200
7			0	54	240			0	54	-1.650
8			1	48	450			1	16	-830
9				20	600				20	-300
10				24	660				24	-540
11				0	500				0	500
12				12	540				12	-180
13				0	240				0	240
14				0	390				0	390
15				0	120				0	120
16				1	180				1	100
17					10					10
18					100					100
19					0					0
20					60					60
21 a 23					0					0
24					20					20
25 a 31					0					0
32					1					1

Tabla 3.2					
$n = 6$					
j	$ P^{(j)}(n) $	$\tilde{P}^{(j)}(n)$	j	$ P^{(j)}(n) $	$\tilde{P}^{(j)}(n)$
1	0	1	22	4.800	4.800
2	0	-192	23	720	720
3	0	2.430	24	3.600	720
4	0	-3.840	25	600	600
5	0	-6.000	26	1.680	1.680
6	0	-10.800	27	360	360
7	720	30.330	28	1.530	1.530
8	1.800	16.040	29	0	0
9	3.960	-4.140	30	720	720
10	6.570	2.970	31	0	0
11	9.480	-23.520	32	480	288
12	11.520	-1.440	33	12	12
13	11.760	-6.960	34	60	60
14	12.960	-19.800	35	0	0
15	10.820	20	36	300	300
16	12.240	-1.200	37 a 39	0	0
17	8.280	7.260	40	120	120
18	10.290	-510	41 a 47	0	0
19	4.110	4.110	48	30	30
20	7.080	-120	49 a 63	0	0
21	3.420	3.420	64	1	1

A partir de las tablas precedentes Ern  obtiene:

$E(n, j)$									
	n								
j	1	2	3	4	5	6	7	8	9
1	1	1	3	19	219	4.231	130.023	6.129.859	431.723.379
2		1	3	19	219	4.231	130.023	6.129.859	431.723.379
3			1	7	87	1.783	57.459	2.816.707	204.959.247
4				1	15	355	12.819	689.047	54.097.935
5					1	31	1.383	88.039	7.932.819
6						1	63	5.299	598.899
7							1	127	20.247
8								1	255
9									1

y por lo tanto, ver (3.6) determina $|P(n)|$, para $1 \leq n \leq 9$.

A partir de los diagramas de Hasse de $P(n)$ para $1 \leq n \leq 6$ que se encuentran en la secci n 9, podemos indicar m s detalles sobre las tablas de Ern :

Tabla 3.1(A)						
j	$ PC^{(j)}(1) $	$ PNC^{(j)}(1) $	$ PC^{(j)}(2) $	$ PNC^{(j)}(2) $	$ PC^{(j)}(3) $	$ PNC^{(j)}(3) $
1	0	0	0	0	0	0
2	1	0	0	0	0	0
3			2	0	0	0
4			0	1	6	0
5					6	0
6					0	6
7					0	0
8					0	1

Luego $|PC(1)| = 1$, $|PNC(1)| = 0$, $|PC(2)| = 2$, $|PNC(2)| = 1$, $|PC(3)| = 12$, $|PNC(3)| = 7$.

Tabla 3.1(B)				
j	$ PC^{(j)}(4) $	$ PNC^{(j)}(4) $	$ PC^{(j)}(5) $	$ PNC^{(j)}(5) $
1 a 4	0	0	0	0
5	24	0	0	0
6	36	0	120	0
7	54	0	240	0
8	24	24	450	0
9	8	12	600	0
10	0	24	540	120
11	0	0	500	0
12	0	12	240	300
13	0	0	240	0
14	0	0	120	270
15	0	0	0	120
16	0	1	0	180
17			10	0
18			0	100
19			0	0
20			0	60
21 a 23			0	0
24			0	20
25 a 31			0	0
32			0	1

Luego $|PC(4)| = 146$, $|PNC(4)| = 73$, $|PC(5)| = 3.060$, $|PNC(5)| = 1.171$.

Tabla 3.2(A)					
j	$ PC^{(j)}(6) $	$ PNC^{(j)}(6) $	j	$ PC^{(j)}(6) $	$ PNC^{(j)}(6) $
1	0	0	22	1.800	3.000
2	0	0	23	720	0
3	0	0	24	180	3.420
4	0	0	25	240	360
5	0	0	26	240	1.440
6	0	0	27	0	360
7	720	0	28	0	1.530
8	1.800	0	29	0	0
9	3.960	0	30	0	720
10	6.570	0	31	0	0
11	9.480	0	32	0	480
12	10.800	720	33	12	0
13	11.760	0	34	0	60
14	11.520	1.440	35	0	0
15	10.100	720	36	0	300
16	9.180	3.060	37 a 39	0	0
17	8.280	0	40	0	120
18	5.610	4.680	41 a 47	0	0
19	4.110	0	48	0	30
20	2.760	4.320	49 a 63	0	0
21	1.800	1.620	64	0	1

Luego $|PC(6)| = 101.642$, $|PNC(6)| = 28.381$.

4. $|PC^{(j)}(7)|$, $|PNC^{(j)}(7)|$, para $1 \leq j \leq 128$.

Los valores que se indican a continuación se obtienen a partir de los diagramas de Hasse de $P(7)$ que se encuentran en la sección 9.

	$ P(7) $			
j	$ PC^{(j)}(7) $	$ PNC^{(j)}(7) $	$ P^{(j)}(7) $	$\tilde{P}^{(j)}(7)$
1	0	0	0	-1
2	0	0	0	448
3	0	0	0	-10.206
4	0	0	0	32.256
5	0	0	0	26.250
6	0	0	0	90.720
7	0	0	0	-436.590
8	5.040	0	5.040	-207.760
9	15.120	0	15.120	275.940
10	37.800	0	37.800	123.900
11	73.080	0	73.080	613.620
12	123.900	0	123.900	63.420
13	183.960	0	183.960	-34.440
14	240.030	5.040	245.070	580.230
15	288.120	0	288.120	-280.980
16	337.680	12.600	350.280	-196.280
17	355.740	0	355.740	-568.890
18	381.780	32.760	414.540	-201.600
19	381.360	0	381.360	-165.270
20	379.890	51.030	430.920	-56.280
21	317.940	10.080	328.020	-174.720
22	335.580	66.360	401.940	-337.260
23	290.710	0	290.710	174.790
24	241.920	104.580	346.500	-16.380
25	204.120	5.040	209.160	104.160
26	210.420	82.320	292.740	-13.020
27	125.160	25.200	150.360	82.320
28	135.240	97.020	232.260	-67.620
29	95.760	0	95.760	95.760
30	86.520	105.980	192.500	41.300
31	67.200	0	67.200	67.200
32	56.700	81.270	137.970	51.954
33	29.400	21.000	50.400	47.628
34	42.126	57.960	100.086	85.806
35	9.282	11.340	20.622	20.622
36	16.380	73.710	90.090	14.490
37	10.080	0	10.080	10.080
PARCIAL	5.078.038	843.290	5.921.328	***

	$ P(7) $			
j	$ PC^{(j)}(7) $	$ PNC^{(j)}(7) $	$ P^{(j)}(7) $	$\tilde{P}^{(j)}(7)$
38	13.440	28.770	42.210	42.210
39	2.520	10.080	12.600	12.600
40	3.780	41.580	45.360	11.760
41	2.940	0	2.940	2.940
42	2.520	28.980	31.500	31.500
43	1.680	0	1.680	1.680
44	840	23.100	23.940	23.940
45	0	4.200	4.200	4.200
46	0	5.040	5.040	5.040
47	0	0	0	0
48	0	15.120	15.120	5.040
49	420	0	420	420
50	420	4.200	4.620	4.620
51	0	420	420	420
52	0	6.720	6.720	6.720
53	0	0	0	0
54	0	2.520	2.520	2.520
55	0	0	0	0
56	0	4.410	4.410	4.410
57 a 59	0	0	0	0
60	0	2.520	2.520	2.520
61 a 63	0	0	0	0
64	0	1.050	1.050	602
65	14	0	14	14
66	0	84	84	84
67	0	0	0	0
68	0	210	210	210
69 a 71	0	0	0	0
72	0	700	700	700
73 a 79	0	0	0	0
80	0	210	210	210
81 a 95	0	0	0	0
96	0	42	42	42
97 a 127	0	0	0	0
128	0	1	1	1
PARCIAL	28.574	179.957	208.531	***
TOTAL	5.106.612	1.023.247	6.129.859	***

$E(10, 1) = |P(9)| = E(10, 2)$, $E(10, 10) = 1$. Por las tablas precedentes y la fórmula (3.6) de Ern , obtenemos $E(10, m)$ para $3 \leq m \leq 9$.

$E(10, 1) =$	44.511.042.511	$E(10, 2) =$	44.511.042.511
$E(10, 3) =$	21.724.257.583	$E(10, 4) =$	6.110.970.115
$E(10, 5) =$	1.004.241.331	$E(10, 6) =$	91.323.031
$E(10, 7) =$	4.109.383	$E(10, 8) =$	77.635
$E(10, 9) =$	511	$E(10, 10) =$	1

Luego:

$$|P(10)| = \sum_{j=1}^{10} \binom{10}{j} E(10, j) = 6.611.065.248.783$$

que coincide con los resultados, obtenidos v a computadoras, en [5], [31, 32], [33], [13], [21], [24], [7].

5. Los resultados de R.P. Stanley

Lema 5.1 ([38], p gina 78). *Si $n \geq 5$:*

- | | |
|--|--|
| (a) $ P^{(7 \cdot 2^{n-4})}(n) = \frac{9}{4}V_n^4 + V_n^5,$ | (b) $ P^{(15 \cdot 2^{n-5})}(n) = V_n^5,$ |
| (c) $ P^{(2^{n-1})}(n) = V_n^3 + V_n^4 = (n-2)V_n^3,$ | (d) $ P^{(17 \cdot 2^{n-5})}(n) = \frac{1}{12}V_n^5,$ |
| (e) $ P^{(9 \cdot 2^{n-4})}(n) = \frac{5}{6}V_n^4,$ | (f) $ P^{(5 \cdot 2^{n-3})}(n) = V_n^3,$ |
| (g) $ P^{(3 \cdot 2^{n-2})}(n) = V_n^2 = n(n-1),$ | (h) $ P^{(2^n)}(n) = 1.$ |

Lema 5.2 ([38], p gina 78). *Si $n \geq 6$*

$$|P^{(2^{n-1}+2^{n-m})}(n)| = \frac{2 V_n^m}{(m-1)!}, \text{ para } 6 \leq m \leq n.$$

Corolario 5.1 *Si $n \geq 6$ entonces $|P^{(2^{n-1}+1)}(n)| = 2n$ y $|P^{(2^{n-1}+2)}(n)| = 2(n-1)n$.*

Dem. Por el Lema 5.2, si $m = n$ entonces $|P^{(2^{n-1}+1)}(n)| = \frac{2 V_n^n}{(n-1)!} = \frac{2n!}{(n-1)!} = 2n$ y

si $m = n-1$ entonces $|P^{(2^{n-1}+2)}(n)| = \frac{2 V_n^{n-1}}{(n-2)!} = 2(n-1)n$. $\square$

Stanley indica adem s que si $j \in \mathbb{N}$ y $j < 7 \cdot 2^{n-4}$ pueden existir elementos en $P^{(j)}(n)$ y

Lema 5.3 *Si $n \geq 5$, $j \in \mathbb{N}$, $j > 7 \cdot 2^{n-4}$ entonces:*

- (a) $P^{(j)}(n) = \emptyset$, para $j \in [7 \cdot 2^{n-4} + 1, 15 \cdot 2^{n-5} - 1]$,

- (b) $P^{(j)}(n) = \emptyset$, para $j \in [15 \cdot 2^{n-5} + 1, 2^{n-1} - 1]$,
- (c) $P^{(j)}(n) = \emptyset$, para $j \in [17 \cdot 2^{n-5} + 1, 9 \cdot 2^{n-4} - 1]$,
- (d) $P^{(j)}(n) = \emptyset$, para $j \in [9 \cdot 2^{n-4} + 1, 5 \cdot 2^{n-3} - 1]$,
- (e) $P^{(j)}(n) = \emptyset$, para $j \in [5 \cdot 2^{n-3} + 1, 3 \cdot 2^{n-2} - 1]$,
- (f) $P^{(j)}(n) = \emptyset$, para $j \in [3 \cdot 2^{n-2} + 1, 2^n - 1]$. (ver también [36], [39])

Lema 5.4 ([38], página 79), ([8], página 287), ([21], página 256). Si $n \geq 2$:

$$|P^{(n+1)}(n)| = n!, \quad |P^{(n+2)}(n)| = \frac{(n-1)n!}{2}, \quad |P^{(n+3)}(n)| = \frac{(n-2)(n+5)n!}{8}.$$

Lema 5.5 ([21], página 256). Si $n \geq 3$, $|P^{(n+4)}(n)| = \frac{(n-3)(n^2 + 15n + 20)n!}{48}$.

Vamos a indicar un nuevo resultado. Por las tablas indicadas en la sección 3 sabemos que si $1 \leq n \leq 5$ y $j \geq 2^{n-1} + 2$ entonces $|PC^{(j)}(n)| = 0$.

Lema 5.6 Si $n \geq 6$ y $j \geq 2^{n-1} + 2$ entonces $|PC^{(j)}(n)| = 0$.

Dem. Supongamos que existe $X \in PC^{(j)}(n)$ con $j \geq 2^{n-1} + 2$, esto es, X es conexo y $2^{n-1} + 2 \leq |RB(X)|$.

Sea $m \in m(X)$, como $n \geq 6$ entonces $|X \setminus (m)| = n - 1 \geq 5$. Luego por el lema 5.3 (f), $|RB(X \setminus (m))| = 2^{n-1}$ ó $|RB(X \setminus (m))| \leq 3 \cdot 2^{n-3}$.

Si $|RB(X \setminus (m))| = 2^{n-1}$, como por el Lema 5.1 (h), $|P^{(2^{n-1})}(n-1)| = 1$, entonces $X \setminus (m)$ es una anticadena con $n - 1$ elementos y por ser X conexo resulta que m es primer elemento, de donde

$$2^{n-1} + 2 \leq |RB(X)| = |RB(X \setminus (m))| + |RB(X \setminus [m])| \leq 2^{n-1} + 1$$

absurdo.

Luego $|RB(X \setminus (m))| \leq 3 \cdot 2^{n-3}$ y por lo tanto

$$2^{n-1} + 2 \leq |RB(X)| = |RB(X \setminus (m))| + |RB(X \setminus [m])| \leq 3 \cdot 2^{n-3} + |RB(X \setminus [m])|$$

$$2^{n-3} + 2 = 2^{n-1} + 2 - 3 \cdot 2^{n-3} \leq |RB(X \setminus [m])|$$

Como X es conexo entonces $|[m]| \neq 1$.

Si $|[m]| \geq 3$ entonces $|X \setminus [m]| \leq n - 3$, luego

$$2^{n-3} + 2 \leq |RB(X \setminus [m])| \leq 2^{n-3}$$

absurdo.

Luego $|[m]| = 2$, esto es, (1) $[m] = \{m, u\}$, con $m < u$, u elemento maximal de X .

Como $|X \setminus [u]| = n - 1 \geq 5$, si $|RB(X \setminus [u])| = 2^{n-1}$ entonces $X \setminus [u]$ es una anticadena con $n - 1$ elementos y como X es conexo resulta que u es último elemento de X , de donde

$$2^{n-1} + 2 \leq |RB(X)| = |RB(X \setminus [u])| + |RB(X \setminus (u))| \leq 2^{n-1} + 1$$

absurdo.

Por lo tanto $|RB(X \setminus [u])| \leq 3 \cdot 2^{n-3}$, de donde

$$2^{n-1} + 2 \leq |RB(X)| = |RB(X \setminus (u))| + |RB(X \setminus [u])| \leq |RB(X \setminus (u))| + 3 \cdot 2^{n-3}$$

entonces

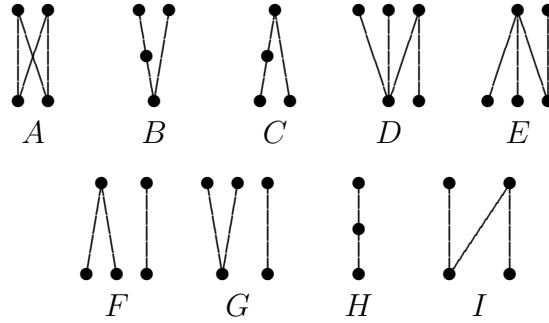
$$2^{n-3} + 2 \leq |RB(X \setminus (u))|$$

Por (1), $|[u]| \neq 1$ y como X es conexo $|[u]| \neq 2$. Luego $|[u]| \geq 3$, de donde resulta que $|RB(X \setminus (u))| \leq 2^{n-3}$ y por lo tanto

$$2^{n-3} + 2 \leq |RB(X \setminus (u))| \leq 2^{n-3}$$

absurdo. □

Sean $A, B, C, D, E, F, G, H, I$ los posets cuyos diagramas se indican a continuación:



Por lo indicado en la sección 9, sabemos que:

Poset X	$ RB(X) $	Número de posets isomorfos a X
A	7	6
B	7	24
C	7	24
D	14	60
E	14	60
F	15	60
G	15	60
H	4	6
I	8	24

Lema 5.7 Si $n \geq 6$, $|P^{(7 \cdot 2^{n-4})}(n)| = |PNC^{(7 \cdot 2^{n-4})}(n)|$.

Dem. Si $X = Y + Z \in P(n)$ donde Y es isomorfo a A y Z es una anticadena con $n - 4$ elementos $|RB(X)| = |RB(Y)||RB(Z)| = 7 \cdot 2^{n-4}$ y existen 6 C_n^4 posets isomorfos a él.

Si $X = Y + Z \in P(n)$ donde Y es isomorfo a B ó C y Z es una anticadena con $n - 4$ elementos $|RB(X)| = |RB(Y)||RB(Z)| = 7 \cdot 2^{n-4}$ y existen 24 $C_n^4 = V_n^4$ posets isomorfos a cada uno de ellos.

Si $X = Y + Z \in P(n)$ donde Y es isomorfo a D ó E y Z es una anticadena con $n - 5$ elementos $|RB(X)| = |RB(Y)||RB(Z)| = 14 \cdot 2^{n-5} = 7 \cdot 2^{n-4}$ y existen $60 C_n^5$ posets isomorfos a cada uno de ellos.

Luego $\frac{9}{4}V_n^4 + V_n^5 = 6 C_n^4 + 2 V_n^4 + 2 C_n^5 \cdot 60 \leq |PNC^{(7 \cdot 2^{n-4})}(n)|$.

Como $|PNC^{(7 \cdot 2^{n-4})}(n)| \leq |P^{(7 \cdot 2^{n-4})}(n)|$ y por el Lema 5.1 (a), $|P^{(7 \cdot 2^{n-4})}(n)| = \frac{9}{4}V_n^4 + V_n^5$, entonces

$$|PNC^{(7 \cdot 2^{n-4})}(n)| = |P^{(7 \cdot 2^{n-4})}(n)| = \frac{9}{4}V_n^4 + V_n^5 \text{ y } |PC^{(7 \cdot 2^{n-4})}(n)| = 0.$$

□

Lema 5.8 Si $n \geq 6$, $|P^{(15 \cdot 2^{n-5})}(n)| = |PNC^{(15 \cdot 2^{n-5})}(n)|$.

Dem. Si $X = Y + Z \in P(n)$ donde Y es isomorfo a F ó G y Z es una anticadena con $n - 5$ elementos $|RB(X)| = |RB(Y)||RB(Z)| = 15 \cdot 2^{n-5}$ y existen $60 C_n^5$ posets isomorfos a cada uno de ellos.

Luego $V_n^5 = 2(60 C_n^5) \leq |PNC^{(15 \cdot 2^{n-5})}(n)|$. Como $|PNC^{(15 \cdot 2^{n-5})}(n)| \leq |P^{(15 \cdot 2^{n-5})}(n)|$ y por el Lema 5.1 (b), $|P^{(15 \cdot 2^{n-5})}(n)| = V_n^5$, entonces

$$|PNC^{(15 \cdot 2^{n-5})}(n)| = |P^{(15 \cdot 2^{n-5})}(n)| = V_n^5 \text{ y } |PC^{(15 \cdot 2^{n-5})}(n)| = 0.$$

□

Lema 5.9 Si $n \geq 5$, $|P^{(2^{n-1})}(n)| = |PNC^{(2^{n-1})}(n)|$.

Dem. Si $X = Y + Z \in P(n)$ donde Y es isomorfo a H y Z es una anticadena con $n - 3$ elementos $|RB(X)| = |RB(Y)||RB(Z)| = 4 \cdot 2^{n-3} = 2^{n-1}$ y existen $6 C_n^3 = V_n^3$ posets isomorfos a él.

Si $X = Y + Z \in P(n)$ donde Y es isomorfo a I y Z es una anticadena con $n - 4$ elementos $|RB(X)| = |RB(Y)||RB(Z)| = 8 \cdot 2^{n-4} = 2^{n-1}$ y existen $24 C_n^4 = V_n^4$ posets isomorfos a él.

Luego $V_n^3 + V_n^4 \leq |PNC^{(2^{n-1})}(n)|$. Como $|PNC^{(2^{n-1})}(n)| \leq |P^{(2^{n-1})}(n)|$ y por el Lema 5.1 (c), $|P^{(2^{n-1})}(n)| = V_n^3 + V_n^4$, entonces

$$|PNC^{(2^{n-1})}(n)| = |P^{(2^{n-1})}(n)| = V_n^3 + V_n^4 \text{ y } |PC^{(2^{n-1})}(n)| = 0.$$

□

En el Corolario 5.1 vimos que: Si $n \geq 6$ entonces $|P^{(2^{n-1}+1)}(n)| = 2n$ y $|P^{(2^{n-1}+2)}(n)| = 2(n-1)n$.

Lema 5.10 Si $n \geq 6$ entonces $|P^{(2^{n-1}+1)}(n)| = |PC^{(2^{n-1}+1)}(n)|$.

Dem. Sea $X \in PC(n)$ un poset con primer elemento p , tal que si $x \in X$ $x \neq p$ entonces x cubre a p , luego $|RB(X)| = |RB(X \setminus \{p\})| + |RB(X \setminus [p])| = 2^{n-1} + 1$. Es claro que existen n conjuntos isomorfos a él. Análogamente si $X \in PC(n)$ un poset con último elemento u , tal que si $x \in X$ $x \neq u$ entonces u cubre a x , luego $|RB(X)| = |RB(X \setminus \{u\})| + |RB(X \setminus [u])| = 1 + 2^{n-1}$. Es claro que existen n conjuntos isomorfos a él. Como $2n \leq |PC^{(2^{n-1}+1)}(n)| \leq |P^{(2^{n-1}+1)}(n)| = 2n$, entonces

$$|PC^{(2^{n-1}+1)}(n)| = |P^{(2^{n-1}+1)}(n)| = 2n \text{ y } |PNC^{(2^{n-1}+1)}(n)| = 0.$$

□

Por las tablas indicadas en la sección 8 el Lema también es válido para $n = 3, 4, 5$.

Lema 5.11 Si $n \geq 6$ entonces $|PNC^{(2^{n-1}+2)}(n)| = 2(n-1)n$ y $|PC^{(2^{n-1}+2)}(n)| = 0$.

Dem. Por el Lema 5.6 $|PC^{(2^{n-1}+2)}(n)| = 0$, y por el Corolario 5.1 $|P^{(2^{n-1}+2)}(n)| = 2(n-1)n$. $\square$

Corolario 5.2 $|PC^{(7 \cdot 2^{n-4})}(n)| = |PC^{(15 \cdot 2^{n-5})}(n)| = |PC^{(2^{n-1})}(n)| = |PNC^{(2^{n-1}+1)}(n)| = 0$.

Luego si $n \geq 6$, todas las fórmulas indicadas por Stanley en el Lema 5.1, para $n \geq 6$, $j \geq 7 \cdot 2^{n-4}$ y $j \neq 2^{n-1} + 1$ corresponden a posets no conexos.

6. Fórmulas de S. Savini e I. Viglizzo

Si $X \in P(n)$ sea $M(X)$ el conjunto de los elementos maximales de X . Si $X \in PC(n)$ y $n \geq 2$ entonces $m(X) \cap M(X) = \emptyset$ y si $X \in PNC(n)$ sea $M'(X)$ el conjunto de los elementos maximales que no son minimales.

Si $m, M \leq n$ sean

- $PC(n, m, M) = \{X \in PC(n, m) : |M(X)| = M\},$
- $PNC^{(')} (n, m, M) = \{X \in PNC(n, m) : |M'(X)| = M\}.$

Es claro que

$$|PC(n, m, 1)| = n|P(n-1, m)|. \quad (6.1)$$

Los detalles de los siguientes resultados se encuentran en la sección 10.

Si $n \geq 6$:

Lema 6.1 $|PC(n, n-5, 1)| = \binom{n}{n-5} (5 \cdot 16^{n-5} + 60 \cdot 12^{n-5} + 120 \cdot 10^{n-5} + 100 \cdot 9^{n-5} + 80 \cdot 8^{n-5} + 270 \cdot 7^{n-5} - 540 \cdot 6^{n-5} - 480 \cdot 5^{n-5} + 540 \cdot 3^{n-5} - 160 \cdot 2^{n-5} + 5).$

También por (6.1) $|PC(n, n-5, 1)| = n|P(n-1, n-5)|$.

Lema 6.2 $|PC(n, n-5, 2)| = \binom{n}{n-5} (20 \cdot 17^{n-5} + 60 \cdot 14^{n-5} + 180 \cdot 13^{n-5} + 60 \cdot 12^{n-5} + 420 \cdot 11^{n-5} + 310 \cdot 10^{n-5} - 20 \cdot 9^{n-5} - 120 \cdot 8^{n-5} - 1.620 \cdot 7^{n-5} - 1.020 \cdot 6^{n-5} + 1.260 \cdot 5^{n-5} + 1.200 \cdot 4^{n-5} - 810 \cdot 3^{n-5} + 80 \cdot 2^{n-5}).$

Lema 6.3 $|PC(n, n-5, 3)| = \binom{n}{n-5} (30 \cdot 19^{n-5} + 60 \cdot 17^{n-5} + 180 \cdot 15^{n-5} + 60 \cdot 14^{n-5} + 210 \cdot 13^{n-5} + 120 \cdot 12^{n-5} - 540 \cdot 11^{n-5} + 10 \cdot 10^{n-5} - 580 \cdot 9^{n-5} - 400 \cdot 8^{n-5} + 190 \cdot 7^{n-5} + 600 \cdot 6^{n-5} + 840 \cdot 5^{n-5} - 960 \cdot 4^{n-5} + 180 \cdot 3^{n-5}).$

Lema 6.4 $|PC(n, n-5, 4)| = \binom{n}{n-5} (20 \cdot 23^{n-5} + 30 \cdot 19^{n-5} + 20 \cdot 17^{n-5} + 5 \cdot 16^{n-5} - 75 \cdot 15^{n-5} - 60 \cdot 12^{n-5} - 60 \cdot 10^{n-5} - 40 \cdot 9^{n-5} + 100 \cdot 8^{n-5} + 90 \cdot 7^{n-5} + 270 \cdot 6^{n-5} - 420 \cdot 5^{n-5} + 120 \cdot 4^{n-5}).$

Lema 6.5 $|PC(n, n-5, 5)| = \binom{n}{n-5} (31^{n-5} - 5 \cdot 16^{n-5} - 10 \cdot 10^{n-5} + 20 \cdot 9^{n-5} + 30 \cdot 7^{n-5} - 60 \cdot 6^{n-5} + 24 \cdot 5^{n-5}).$

Corolario 6.1 $|PC(n, n-5)| = \binom{n}{n-5} (31^{n-5} + 20 \cdot 23^{n-5} + 60 \cdot 19^{n-5} + 100 \cdot 17^{n-5} + 5 \cdot 16^{n-5} + 105 \cdot 15^{n-5} + 120 \cdot 14^{n-5} + 390 \cdot 13^{n-5} + 180 \cdot 12^{n-5} - 120 \cdot 11^{n-5} + 370 \cdot 10^{n-5} - 520 \cdot 9^{n-5} - 340 \cdot 8^{n-5} - 1.040 \cdot 7^{n-5} - 750 \cdot 6^{n-5} + 1.224 \cdot 5^{n-5} + 360 \cdot 4^{n-5} - 90 \cdot 3^{n-5} - 80 \cdot 2^{n-5} + 5).$

Si $n \geq 5$:

Lema 6.6 $|PC(n, n-4, 1)| = \binom{n}{n-4} (4 \cdot 8^{n-4} + 24 \cdot 6^{n-4} + 24 \cdot 5^{n-4} - 24 \cdot 4^{n-4} - 72 \cdot 3^{n-4} + 48 \cdot 2^{n-4} - 4)$.

También por (6.1) $|PC(n, n-4, 1)| = n|P(n-1, n-4)|$.

Lema 6.7 $|PC(n, n-4, 2)| = \binom{n}{n-4} (12 \cdot 9^{n-4} + 12 \cdot 8^{n-4} + 48 \cdot 7^{n-4} + 30 \cdot 6^{n-4} - 120 \cdot 5^{n-4} - 84 \cdot 4^{n-4} + 126 \cdot 3^{n-4} - 24 \cdot 2^{n-4})$.

Lema 6.8 $|PC(n, n-4, 3)| = \binom{n}{n-4} (12 \cdot 11^{n-4} + 12 \cdot 9^{n-4} + 4 \cdot 8^{n-4} - 28 \cdot 7^{n-4} - 24 \cdot 6^{n-4} - 24 \cdot 5^{n-4} + 72 \cdot 4^{n-4} - 24 \cdot 3^{n-4})$.

Lema 6.9 $|PC(n, n-4, 4)| = \binom{n}{n-4} (15^{n-4} - 4 \cdot 8^{n-4} - 3 \cdot 6^{n-4} + 12 \cdot 5^{n-4} - 6 \cdot 4^{n-4})$.

Corolario 6.2 $|PC(n, n-4)| = \binom{n}{n-4} (15^{n-4} + 12 \cdot 11^{n-4} + 24 \cdot 9^{n-4} + 16 \cdot 8^{n-4} + 20 \cdot 7^{n-4} + 27 \cdot 6^{n-4} - 108 \cdot 5^{n-4} - 42 \cdot 4^{n-4} + 30 \cdot 3^{n-4} + 24 \cdot 2^{n-4} - 4)$.

Si $n \geq 4$:

Lema 6.10 $|PC(n, n-3, 1)| = \binom{n}{n-3} (3 \cdot 4^{n-3} + 6 \cdot 3^{n-3} - 12 \cdot 2^{n-3} + 3)$.

También por (6.1) $|PC(n, n-3, 1)| = n|P(n-1, n-3)|$.

Lema 6.11 $|PC(n, n-3, 2)| = \binom{n}{n-3} (6 \cdot 5^{n-3} + 3 \cdot 4^{n-3} - 15 \cdot 3^{n-3} + 6 \cdot 2^{n-3})$.

Lema 6.12 $|PC(n, n-3, 3)| = \binom{n}{n-3} (7^{n-3} - 3 \cdot 4^{n-3} + 2 \cdot 3^{n-3})$.

Corolario 6.3 $|PC(n, n-3)| = \binom{n}{n-3} (7^{n-3} + 6 \cdot 5^{n-3} + 3 \cdot 4^{n-3} - 7 \cdot 3^{n-3} - 6 \cdot 2^{n-3} + 3)$.

Si $n \geq 3$:

Lema 6.13 $|PC(n, n-2, 1)| = \binom{n}{n-2} (2 \cdot 2^{n-2} - 2)$.

También por (6.1) $|PC(n, n-2, 1)| = n|P(n-1, n-2)|$.

Lema 6.14 $|PC(n, n-2, 2)| = \binom{n}{n-2} (3^{n-2} - 2^{n-2})$.

Corolario 6.4 $|PC(n, n-2)| = \binom{n}{n-2} (3^{n-2} + 2^{n-2} - 2)$.

Si $n \geq 5$:

Lema 6.15 $|PNC^{(')} (n, n-4, 1)| = \binom{n}{n-4} (4 \cdot 9^{n-4} - 4 \cdot 8^{n-4} + 24 \cdot 7^{n-4} - 48 \cdot 5^{n-4} - 48 \cdot 4^{n-4} + 120 \cdot 3^{n-4} - 52 \cdot 2^{n-4} + 4)$.

Lema 6.16 $|PNC^{(')} (n, n-4, 2)| = \binom{n}{n-4} (12 \cdot 10^{n-4} + 36 \cdot 8^{n-4} - 18 \cdot 7^{n-4} - 138 \cdot 6^{n-4} + 60 \cdot 5^{n-4} + 114 \cdot 4^{n-4} - 78 \cdot 3^{n-4} + 12 \cdot 2^{n-4})$.

Lema 6.17 $|PNC^{(')} (n, n-4, 3)| = \binom{n}{n-4} (12 \cdot 12^{n-4} - 12 \cdot 11^{n-4} + 12 \cdot 10^{n-4} - 8 \cdot 9^{n-4} - 32 \cdot 8^{n-4} + 28 \cdot 7^{n-4} + 12 \cdot 5^{n-4} - 36 \cdot 4^{n-4} + 36 \cdot 3^{n-4} - 12 \cdot 2^{n-4})$.

Lema 6.18 $|PNC^{(\prime)}(n, n-4, 4)| = \binom{n}{n-4} (16^{n-4} - 15^{n-4} + 3 \cdot 6^{n-4} - 12 \cdot 5^{n-4} + 12 \cdot 4^{n-4} - 4 \cdot 2^{n-4} + 1)$.

Corolario 6.5 $|PNC(n, n-4)| = \binom{n}{n-4} (16^{n-4} - 15^{n-4} + 12 \cdot 12^{n-4} - 12 \cdot 11^{n-4} + 24 \cdot 10^{n-4} - 4 \cdot 9^{n-4} + 34 \cdot 7^{n-4} - 135 \cdot 6^{n-4} + 12 \cdot 5^{n-4} + 42 \cdot 4^{n-4} + 78 \cdot 3^{n-4} - 56 \cdot 2^{n-4} + 5)$.

Si $n \geq 4$:

Lema 6.19 $|PNC^{(\prime)}(n, n-3, 1)| = \binom{n}{n-3} (3 \cdot 5^{n-3} + 3 \cdot 4^{n-3} - 18 \cdot 3^{n-3} + 15 \cdot 2^{n-3} - 3)$.

Lema 6.20 $|PNC^{(\prime)}(n, n-3, 2)| = \binom{n}{n-3} (6 \cdot 6^{n-3} - 3 \cdot 5^{n-3} - 12 \cdot 4^{n-3} + 9 \cdot 3^{n-3})$.

Lema 6.21 $|PNC^{(\prime)}(n, n-3, 3)| = \binom{n}{n-3} (8^{n-3} - 7^{n-3} - 2 \cdot 3^{n-3} + 3 \cdot 2^{n-3} - 1)$.

Corolario 6.6 $|PNC(n, n-3)| = \binom{n}{n-3} (8^{n-3} - 7^{n-3} + 6 \cdot 6^{n-3} - 9 \cdot 4^{n-3} - 11 \cdot 3^{n-3} + 18 \cdot 2^{n-3} - 4)$.

Si $n \geq 3$:

Lema 6.22 $|PNC^{(\prime)}(n, n-2, 1)| = \binom{n}{n-2} (2 \cdot 3^{n-2} - 4 \cdot 2^{n-2} + 2)$.

Lema 6.23 $|PNC^{(\prime)}(n, n-2, 2)| = \binom{n}{n-2} (4^{n-2} - 3^{n-2} - 2^{n-2} + 1)$.

Corolario 6.7 $|PNC(n, n-2)| = \binom{n}{n-2} (4^{n-2} + 3^{n-2} - 5 \cdot 2^{n-2} + 3)$.

Luego por Corolario (6.4)

Corolario 6.8 $|P(n, n-2)| = \binom{n}{n-2} (4^{n-2} + 2 \cdot 3^{n-2} - 4 \cdot 2^{n-2} + 1)$.

7. $|PNC(n, 2)|, n \geq 3$.

Si $X \in P(n)$, entonces $X \in PC(n)$ ó $X = \sum_{i=1}^k K_i$, $k \geq 2$, donde los K_i son conexos.

Si $n_i = |K_i| < n$, entonces $n_1, n_2, \dots, n_k$ es una k -partición de n y si $k > 2$ entonces $|m(X)| > 2$ pues en cada K_i tenemos al menos un elemento minimal.

Si $X \in PNC(n, 2)$ sea $m(X) = \{m_1, m_2\}$ y

$$\emptyset \neq K_1 = \{x \in X : m_1 \leq x\} \subset \mathbf{n}, \quad \emptyset \neq K_2 = \{x \in X : m_2 \leq x\} \subset \mathbf{n}.$$

Luego $K_1 \cap K_2 = \emptyset$ y $|K_1| + |K_2| = n$.

Si (i) $y \in K_1$, (ii) $z \in K_2$ e $y \leq z$ por (i) $m_1 \leq y$ luego $m_1 \leq z$ entonces $z \in K_1$ absurdo. Análogamente no puede ser $z \leq y$. Luego $X = K_1 + K_2$, K_1 y K_2 posets con un elemento minimal cada uno, luego posets conexos, con $1 \leq |K_1| = n_1$ entonces $n_1 < n$ y $1 \leq |K_2| = n_2 = n - n_1$. Podemos suponer sin inconvenientes que $n_1 \leq n_2$.

Sea $1 \leq n_1 \leq \left\lceil \frac{n}{2} \right\rceil$. Si n es par entonces $n_1 \leq n - n_1$. En efecto, si $n - n_1 < n_1$ esto es

$n < 2 n_1$, luego como $2 n_1 \leq 2 \left\lceil \frac{n}{2} \right\rceil = 2 \frac{n}{2}$, entonces $n < n$, absurdo.

Si n es impar, entonces $n_1 < n - n_1$. En efecto, si n es impar y $n - n_1 \leq n_1$ esto es $n \leq 2 n_1$

luego como $2 n_1 \leq 2 \left\lceil \frac{n}{2} \right\rceil = 2 \frac{n-1}{2} = n - 1$ entonces $n \leq n - 1$, absurdo.

Si n es impar y $K_1 \subset \mathbf{n}$, $1 \leq n_1 = |K_1| \leq \left\lfloor \frac{n}{2} \right\rfloor$ y $K_1 \in PC(n_1, 1)$, $K_2 \in PC(n - n_1, 1)$ entonces $|K_1| \neq |\mathbf{n} \setminus K_1|$ y $X = K_1 + K_2 \in PNC(n, 2)$.

Observemos que si $W_1 \subset \mathbf{n}$, $1 \leq n_1 = |W_1| \leq \left\lfloor \frac{n}{2} \right\rfloor$, $W_1 \in PC(n_1, 1)$, y $W_1 \neq K_1$ entonces $W_1 + (\mathbf{n} \setminus W_1) \neq K_1 + (\mathbf{n} \setminus K_1)$.

Como tenemos $\binom{n}{n_1}$ formas de elegir subconjuntos de $\mathbf{n}$ con n_1 elementos $1 \leq n_1 \leq \left\lfloor \frac{n}{2} \right\rfloor$ de este modo obtenemos

$$\sum_{n_1=1}^{\left\lfloor \frac{n}{2} \right\rfloor} \binom{n}{n_1} |PC(n_1, 1)| |PC(n - n_1, 1)|$$

elementos de $PNC(n, 2)$, luego

$$\sum_{n_1=1}^{\left\lfloor \frac{n}{2} \right\rfloor} \binom{n}{n_1} |PC(n_1, 1)| |PC(n - n_1, 1)| \leq |PNC(n, 2)|$$

y por lo visto precedentemente si $X \in PNC(n, 2)$ entonces $X = K_1 + K_2$ donde K_1, K_2 son posets conexos con un elemento minimal cada uno, tales que $1 \leq |K_1| \leq \left\lfloor \frac{n}{2} \right\rfloor$ y $K_2 = \mathbf{n} \setminus K_1$, $K_1 \neq K_2$, luego si n es impar:

$$|PNC(n, 2)| = \sum_{n_1=1}^{\left\lfloor \frac{n}{2} \right\rfloor} \binom{n}{n_1} |PC(n_1, 1)| |PC(n - n_1, 1)|$$

Si n es par, y $K_1 \subset \mathbf{n}$, $1 \leq n_1 = |K_1| \leq \left\lfloor \frac{n}{2} \right\rfloor$ y $K_1 \in PC(n_1, 1)$, $K_2 \in PC(n - n_1, 1)$ entonces $X = K_1 + K_2 \in PNC(n, 2)$, pero puede ser $|K_1| = |\mathbf{n} \setminus K_1| = \frac{n}{2}$.

Supongamos que $K_1 \subset \mathbf{n}$, $1 \leq n_1 = |K_1| \leq \left\lfloor \frac{n}{2} \right\rfloor - 1$ y $K_1 \in PC(n_1, 1)$, $K_2 \in PC(n - n_1, 1)$ entonces $|K_1| \neq |\mathbf{n} \setminus K_1|$ y $X = K_1 + K_2 \in PNC(n, 2)$.

Observemos que si $W_1 \subset \mathbf{n}$, $1 \leq n_1 = |W_1| \leq \left\lfloor \frac{n}{2} \right\rfloor - 1$, $W_1 \in PC(n_1, 1)$, y $W_1 \neq K_1$ entonces $W_1 + (\mathbf{n} \setminus W_1) \neq K_1 + (\mathbf{n} \setminus K_1)$.

Como tenemos $\binom{n}{n_1}$ formas de elegir subconjuntos de $\mathbf{n}$ con n_1 elementos $1 \leq n_1 \leq \left\lfloor \frac{n}{2} \right\rfloor - 1$ de este modo obtenemos

$$\sum_{n_1=1}^{\left\lfloor \frac{n}{2} \right\rfloor - 1} \binom{n}{n_1} |PC(n_1, 1)| |PC(n - n_1, 1)|$$

elementos de $PNC(n, 2)$.

Si $|K_1| = \left\lfloor \frac{n}{2} \right\rfloor$ entonces $|\mathbf{n} \setminus K_1| = \left\lfloor \frac{n}{2} \right\rfloor$. Tenemos $\binom{n}{\left\lfloor \frac{n}{2} \right\rfloor}$ formas de elegir subconjuntos K_1

de $\mathbf{n}$ con $\frac{n}{2}$ elementos luego si $K_2 = \mathbf{n} \setminus K_1$ entonces $|K_2| = \frac{n}{2}$, como $K_1 + K_2 = K_2 + K_1$

entonces tenemos $\binom{n}{\frac{n}{2}} \frac{|PC(\frac{n}{2}, 1)|^2}{2}$ elementos de $PNC(n, 2)$ luego

$$\sum_{n_1=1}^{\lfloor \frac{n}{2} \rfloor - 1} \binom{n}{n_1} |PC(n_1, 1)| |PC(n - n_1, 1)| + \binom{n}{\frac{n}{2}} \frac{|PC(\frac{n}{2}, 1)|^2}{2} \leq |PNC(n, 2)|.$$

Si $X \in PNC(n, 2)$ entonces $X = K_1 + K_2$ con $1 \leq |K_1| \leq \frac{n}{2}$ y $1 \leq |K_2| \leq \frac{n}{2}$, y K_1, K_2 tienen un elemento minimal cada uno. Luego por lo visto precedentemente

$$|PNC(n, 2)| = \sum_{n_1=1}^{\lfloor \frac{n}{2} \rfloor - 1} \binom{n}{n_1} |PC(n_1, 1)| |PC(n - n_1, 1)| + \binom{n}{\frac{n}{2}} \frac{|PC(\frac{n}{2}, 1)|^2}{2}.$$

Luego si $n \geq 3$

$$|PNC(n, 2)| = \begin{cases} \text{Si } n \text{ es par,} \\ \sum_{k=1}^{\frac{n}{2}-1} \binom{n}{k} |PC(k, 1)| |PC(n - k, 1)| + \binom{n}{\frac{n}{2}} \frac{|PC(\frac{n}{2}, 1)|^2}{2}, \\ \text{si } n \text{ es impar,} \\ \sum_{k=1}^{\frac{n-1}{2}} \binom{n}{k} |PC(k, 1)| |PC(n - k, 1)|. \end{cases} \quad (7.1)$$

Es fácil ver que:

$$|PNCNI(n, 2)| = \begin{cases} \text{Si } n \text{ es par, } n \geq 4 \\ \sum_{k=1}^{\frac{n}{2}-1} |PCNI(k, 1)| |PCNI(n - k, 1)| + \\ \frac{|PCNI(\frac{n}{2}, 1)|^2 + |PCNI(\frac{n}{2}, 1)|}{2}, \\ \text{si } n \text{ es impar, } n \geq 3 \\ \sum_{k=1}^{\frac{n-1}{2}} |PCNI(k, 1)| |PCNI(n - k, 1)|. \end{cases} \quad (7.2)$$

8. $|P(n)|$, para $1 \leq n \leq 8$

Si $m, M \leq n$ sean

- $PNC(n, m, M) = \{X \in PNC(n, m) : |M(X)| = M\}$,
- $PCNI(n, m, M)$ el conjunto de los posets de $PC(n, m, M)$ que no son isomorfos,
- $PNCNI(n, m, M)$ el conjunto de los posets de $PNC(n, m, M)$ que no son isomorfos,
- $PNI(n, m, M) = PCNI(n, m, M) \cup PNCNI(n, m, M)$.

Brinkmann y McKay [7] determinaron $|PNI(n, m, M)|$ y

$$|PNI(n, m)| = \sum_{M=1}^n |PNI(n, m, M)|, \text{ para } 1 \leq n, m \leq 16.$$

A partir de los diagramas indicados en la sección 9, obtenemos

$$|PCNI(n, m, M)|, \quad |PNCNI(n, m, M)|, \quad \text{para } 1 \leq n, m, M \leq 6,$$

y los números $|PNI(n, m, M)| = |PCNI(n, m, M)| + |PNCNI(n, m, M)|$ coinciden con los resultados de Brinkmann y McKay.

Para $1 \leq m \leq n \leq 8$, indicamos $|PNCNI(n, m)|$ y utilizando los valores $|PNI(n, m)|$ dados en [7], determinamos $|PCNI(n, m)| = |PNI(n, m)| - |PNCNI(n, m)|$.

Observemos que $|PCNI(n, 1)| = |PNI(n - 1)|$ y $|PNCNI(n, 1)| = 0$. Por (7.2) conocemos $|PNCNI(n, 2)|$. Además $|PCNI(n, n - 1)| = 1$, $|PNCNI(n, n - 1)| = n - 2$, $|PCNI(n, n)| = 1$ si $n > 1$ y $|PNCNI(n, n)| = 0$ si $n = 1$.

Utilizando los valores de las Tablas 2 a 6, que se indican más adelante, obtenemos $|PNCNI(7, m)|$ para $3 \leq m \leq 5$:

$$\begin{array}{rcl}
 |PCNI(6, 2)||PCNI(1, 1)| & = & 110 \\
 |PCNI(5, 2)||PCNI(2, 1)| & = & 20 \\
 |PCNI(4, 2)||PCNI(3, 1)| & = & 8 \\
 |PCNI(4, 1)||PCNI(3, 2)| & = & 5 \\
 |PCNI(5, 1)||PCNI(1, 1)||PCNI(1, 1)| & = & 16 \\
 |PCNI(4, 1)||PCNI(2, 1)||PCNI(1, 1)| & = & 5 \\
 |PCNI(3, 1)||PCNI(2, 1)||PCNI(2, 1)| & = & 2 \\
 \frac{|PCNI(3, 1)||PCNI(3, 1)||PCNI(1, 1)| - 1}{|PNCNI(7, 3)|} & = & \frac{3}{169}
 \end{array}$$

$$\begin{array}{rcl}
|PCNI(6, 3)||PCNI(1, 1)| & = & 54 \\
|PCNI(5, 3)||PCNI(2, 1)| & = & 7 \\
|PCNI(4, 3)||PCNI(3, 1)| & = & 2 \\
|PCNI(4, 2)||PCNI(3, 2)| & = & 4 \\
|PCNI(5, 2)||PCNI(1, 1)||PCNI(1, 1)| & = & 20 \\
|PCNI(4, 2)||PCNI(2, 1)||PCNI(1, 1)| & = & 4 \\
|PCNI(3, 2)||PCNI(3, 1)||PCNI(1, 1)| & = & 2 \\
|PCNI(3, 2)||PCNI(2, 1)||PCNI(2, 1)| & = & 1 \\
|PCNI(4, 1)||PCNI(1, 1)||PCNI(1, 1)||PCNI(1, 1)| & = & 5 \\
|PCNI(3, 1)||PCNI(2, 1)||PCNI(1, 1)||PCNI(1, 1)| & = & 2 \\
|PCNI(2, 1)||PCNI(2, 1)||PCNI(2, 1)||PCNI(1, 1)| & = & 1 \\
\hline
|PNCNI(7, 4)| & = & 102
\end{array}$$

$$\begin{array}{rcl}
|PCNI(6, 4)||PCNI(1, 1)| & = & 10 \\
|PCNI(5, 4)||PCNI(2, 1)| & = & 1 \\
|PCNI(4, 3)||PCNI(3, 2)| & = & 1 \\
|PCNI(5, 3)||PCNI(1, 1)||PCNI(1, 1)| & = & 7 \\
|PCNI(4, 3)||PCNI(2, 1)||PCNI(1, 1)| & = & 1 \\
|PCNI(3, 2)||PCNI(3, 2)||PCNI(1, 1)| & = & 1 \\
|PCNI(4, 2)||PCNI(1, 1)||PCNI(1, 1)||PCNI(1, 1)| & = & 4 \\
|PCNI(3, 2)||PCNI(2, 1)||PCNI(1, 1)||PCNI(1, 1)| & = & 1 \\
|PCNI(3, 1)||PCNI(1, 1)||PCNI(1, 1)||PCNI(1, 1)||PCNI(1, 1)| & = & 2 \\
|PCNI(2, 1)||PCNI(2, 1)||PCNI(1, 1)||PCNI(1, 1)||PCNI(1, 1)| & = & 1 \\
\hline
|PNCNI(7, 5)| & = & 29
\end{array}$$

Observemos que $|PNC(n)| = \sum_{k=2}^n |PNC(n, k)|$. Sabemos determinar:

- $|PNC(n, 2)|$, ver (7.1),
- $|PNC(n, n-4)|$, para $n \geq 5$, (Corolario 6.5),
- $|PNC(n, n-3)|$, para $n \geq 4$, (Corolario 6.6),
- $|PNC(n, n-2)|$, para $n \geq 3$, (Corolario 6.7),
- $|PNC(n, n-1)|$, ver (2.2),
- $|PNC(n, n)|$, ver (2.3).

Es claro que:

Tabla 1						
$ P(1) $						
m	$ PC(1, m) $	$ PNC(1, m) $	$ P(1, m) $	No isomorfos		
				$ PC(1, m) $	$ PNC(1, m) $	$ P(1, m) $
1	1	0	1	1	0	1
$\sum$	1	0	1	1	0	1

Tabla 2						
$ P(2) $						
m	$ PC(2, m) $	$ PNC(2, m) $	$ P(2, m) $	No isomorfos		
				$ PC(2, m) $	$ PNC(2, m) $	$ P(2, m) $
1	2	0	2	1	0	1
2	0	1	1	0	1	1
$\sum$	2	1	3	1	1	2

Por (3.1), (3.2) y Tabla 2 tenemos $|P(3, 1)| = 3|P(2)| = 9$, $|P(3, 2)| = 3|P(2)| = 9$.
 Por (7.1) $|PNC(3, 2)| = \binom{3}{2}|PC(2, 1)||PC(1, 1)| = 6$, luego por (2.1) $|PC(3, 2)| = 3$. Por lo tanto

Tabla 3						
$ P(3) $						
m	$ PC(3, m) $	$ PNC(3, m) $	$ P(3, m) $	No isomorfos		
				$ PC(3, m) $	$ PNC(3, m) $	$ P(3, m) $
1	9	0	9	2	0	2
2	3	6	9	1	1	2
3	0	1	1	0	1	1
$\sum$	12	7	19	3	2	5

$ PNI(3) $								
			$ PCNI(3, m) $			$ PNCNI(3, m) $		
			M			M		
m	1	2	3	$\sum$	1	2	3	$\sum$
1	1	1	0	2	0	0	0	0
2	1	0	0	1	0	1	0	1
3	0	0	0	0	0	0	1	1
$\sum$	2	1	0	3	0	1	1	2

Por (3.1), (3.2) y Tabla 3 tenemos $|P(4, 1)| = 4|P(3)| = 76$, $|P(4, 2)| = \binom{4}{2}|P(3)| = 114$.
 Por Lema 6.14, $|PC(4, 2)| = 66$. Por (7.1), $|PNC(4, 2)| = 48$.
 Por (2.1) y (2.2), $|PC(4, 3)| = 4$, $|PNC(4, 3)| = 24$. Luego

Tabla 4						
$ P(4) $						
m	$ PC(4, m) $	$ PNC(4, m) $	$ P(4, m) $	No isomorfos		
				$ PC(4, m) $	$ PNC(4, m) $	$ P(4, m) $
1	76	0	76	5	0	5
2	66	48	114	4	3	7
3	4	24	28	1	2	3
4	0	1	1	0	1	1
$\sum$	146	73	219	10	6	16

$ PNI(4) $										
$ PCNI(4, m) $					$ PNCNI(4, m) $					
M					M					
m	1	2	3	4	$\sum$	1	2	3	4	$\sum$
1	2	2	1	0	5	0	0	0	0	0
2	2	2	0	0	4	0	2	1	0	3
3	1	0	0	0	1	0	1	1	0	2
4	0	0	0	0	0	0	0	0	1	1
$\sum$	5	4	1	0	10	0	3	2	1	6

Por (3.1), (3.2) y Tabla 4 tenemos:

$$|P(5, 1)| = 5|P(4)| = 1.095, \quad |P(5, 2)| = \binom{5}{2}|P(4)| = 2.190.$$

Por Corolario 6.3 $|PC(5, 2)| = 1.630$. Por (7.1) ó por Corolario 6.6, $|PNC(5, 2)| = 560$.

Por Corolario 6.4, $|PC(5, 3)| = 330$. Por Corolario 6.7, $|PNC(5, 3)| = 540$.

Por (2.1) y (2.2), $|PC(5, 4)| = 5$, $|PNC(5, 4)| = 70$. Luego

Tabla 5						
$ P(5) $						
m	$ PC(5, m) $	$ PNC(5, m) $	$ P(5, m) $	No isomorfos		
				$ PC(5, m) $	$ PNC(5, m) $	$ P(5, m) $
1	1.095	0	1.095	16	0	16
2	1.630	560	2.190	20	7	27
3	330	540	870	7	8	15
4	5	70	75	1	3	4
5	0	1	1	0	1	1
$\sum$	3.060	1.171	4.231	44	19	63

PNI(5)															
		PCNI(5, m)							PNCNI(5, m)						
		M							M						
m	1	2	3	4	5	Σ	1	2	3	4	5	Σ			
1	5	7	3	1	0	16	0	0	0	0	0	0			
2	7	9	4	0	0	20	0	3	3	1	0	7			
3	3	4	0	0	0	7	0	3	4	1	0	8			
4	1	0	0	0	0	1	0	1	1	1	0	3			
5	0	0	0	0	0	0	0	0	0	0	1	1			
Σ	16	20	7	1	0	44	0	7	8	3	1	19			

Por (3.1), (3.2) y Tabla 5 tenemos:

$$|P(6, 1)| = 6|P(5)| = 25.386, \quad |P(6, 2)| = \binom{6}{2}|P(5)| = 63.465.$$

Por Corolario 6.2, $|PC(6, 2)| = 53.805$. Por (7.1), $|PNC(6, 2)| = 9.660$.

Por Corolario 6.3, $|PC(6, 3)| = 21.020$. Por Corolario 6.6, $|PNC(6, 3)| = 14.640$.

Por Corolario 6.4, $|PC(6, 4)| = 1.425$. Por Corolario 6.7, $|PNC(6, 4)| = 3.900$.

Por (2.1) y (2.2), $|PC(6, 5)| = 6$, $|PNC(6, 5)| = 180$. Luego

Tabla 6						
$ P(6) $						
m	$ PC(6, m) $	$ PNC(6, m) $	$ P(6, m) $	No isomorfos		
				$ PC(6, m) $	$ PNC(6, m) $	$ P(6, m) $
1	25.386	0	25.386	63	0	63
2	53.805	9.660	63.465	110	24	134
3	21.020	14.640	35.660	54	34	88
4	1.425	3.900	5.325	10	17	27
5	6	180	186	1	4	5
6	0	1	1	0	1	1
$\sum$	101.642	28.381	130.023	238	80	318

PNI(6)														
	PCNI(6, m)							PNCNI(6, m)						
	M							M						
m	1	2	3	4	5	6	Σ	1	2	3	4	5	6	Σ
1	16	27	15	4	1	0	63	0	0	0	0	0	0	0
2	27	51	26	6	0	0	110	0	8	10	5	1	0	24
3	15	26	13	0	0	0	54	0	10	16	7	1	0	34
4	4	6	0	0	0	0	10	0	5	7	4	1	0	17
5	1	0	0	0	0	0	1	0	1	1	1	1	0	4
6	0	0	0	0	0	0	0	0	0	0	0	0	1	1
Σ	63	110	54	10	1	0	238	0	24	34	17	4	1	80

Por (3.1), (3.2) y Tabla 6 tenemos: $|P(7, 1)| = 7|P(6)| = 910.161$,

y $|P(7, 2)| = 2.730.483$. Por (7.1) $|PNC(7, 2)| = 247.632$

Por Corolario 6.1 $|PC(7, 2)| = 2.482.851$.

Por Corolario 6.2, $|PC(7, 3)| = 1.488.795$.

Por Corolario 6.5, $|PNC(7, 3)| = 522.270$. Por Corolario 6.3, $|PC(7, 4)| = 219.065$.

Por Corolario 6.6, $|PNC(7, 4)| = 229.600$. Por Corolario 6.4, $|PC(7, 5)| = 5.733$.

Por Corolario 6.7, $|PNC(7, 5)| = 23.310$.

Por (2.1) y (2.2) $|PC(7, 6)| = 7$, $|PNC(7, 6)| = 434$. Luego

Tabla 7						
$ P(7) $						
m	$ PC(7, m) $	$ PNC(7, m) $	$ P(7, m) $	No isomorfos		
				$ PC(7, m) $	$ PNC(7, m) $	$ P(7, m) $
1	910.161	0	910.161	318	0	318
2	2.482.851	247.632	2.730.483	725	89	814
3	1.488.795	522.270	2.011.065	473	169	642
4	219.065	229.600	448.665	119	102	221
5	5.733	23.310	29.043	14	29	43
6	7	434	441	1	5	6
7	0	1	1	0	1	1
$\sum$	5.106.612	1.023.247	6.129.859	1.650	395	2.045

Para los no isomorfos vamos a mejorar la información:

$ PNI(7) $																
$ PCNI(7, m) $									$ PNCNI(7, m) $							
M									M							
m	1	2	3	4	5	6	7	$\sum$	1	2	3	4	5	6	7	$\sum$
1	63	134	88	27	5	1	0	318	0	0	0	0	0	0	0	0
2	134	311	213	58	9	0	0	725	0	23	38	21	6	1	0	89
3	88	213	138	34	0	0	0	473	0	38	75	44	11	1	0	169
4	27	58	34	0	0	0	0	119	0	21	44	29	7	1	0	102
5	5	9	0	0	0	0	0	14	0	6	11	7	4	1	0	29
6	1	0	0	0	0	0	0	1	0	0	1	1	1	1	0	5
7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
$\sum$	318	725	473	119	14	1	0	1.650	0	89	169	102	29	5	1	395

Por (3.1), (3.2) y Tabla 7 tenemos: $|P(8, 1)| = 8|P(7)| = 49.038.872$ y

$|P(8, 2)| = \binom{8}{2}|P(7)| = 171.636.052$. Por (7.1) $|PNC(8, 2)| = 9.456.944$ luego

$|PC(8, 2)| = |P(8, 2)| - |PNC(8, 2)| = 162.179.108$.

$$\begin{array}{rcl}
\binom{8}{7}|PC(7, 2)||PC(1, 1)| & = & 19.862.808 \\
\binom{8}{6}|PC(6, 2)||PC(2, 1)| & = & 3.013.080 \\
\binom{8}{5}|PC(5, 2)||PC(3, 1)| & = & 821.520 \\
\binom{8}{5}|PC(5, 1)||PC(3, 2)| & = & 183.960 \\
\binom{8}{4}|PC(4, 2)||PC(4, 1)| & = & 351.120 \\
\binom{8}{6}|PC(6, 1)|\binom{2}{1}|PC(1, 1)||PC(1, 1)|/2! & = & 710.808 \\
\binom{8}{5}|PC(5, 1)|\binom{3}{2}|PC(2, 1)||PC(1, 1)| & = & 367.920 \\
\binom{8}{4}|PC(4, 1)|\binom{4}{3}|PC(3, 1)||PC(1, 1)| & = & 191.520 \\
\binom{8}{4}|PC(4, 1)|\binom{4}{2}|PC(2, 1)||PC(2, 1)|/2! & = & 63.840 \\
\binom{8}{3}|PC(3, 1)|\binom{5}{3}|PC(3, 1)||PC(2, 1)|/2! & = & 45.360 \\
\hline
|PNC(8, 3)| & = & 25.611.936
\end{array}$$

Por Corolario 6.1, $|PC(8, 3)| = 132.123.656$,
 por Corolario 6.2, $|PC(8, 4)| = 31.982.090$,
 por Corolario 6.5, $|PNC(8, 4)| = 16.251.200$,
 por Corolario 6.3, $|PC(8, 5)| = 2.057.384$,
 por Corolario 6.6, $|PNC(8, 5)| = 2.872.800$,
 por Corolario 6.4, $|PC(8, 6)| = 22.148$,
 por Corolario 6.7, $|PNC(8, 6)| = 126.224$,
 por (2.1) y (2.2), $|PC(8, 7)| = 8$, $|PNC(8, 7)| = 1.008$.

Tabla 8			
$ P(8) $			
m	$ PC(8, m) $	$ PNC(8, m) $	$ P(8, m) $
1	49.038.872	0	49.038.872
2	162.179.108	9.456.944	171.636.052
3	132.123.656	25.611.936	157.735.592
4	31.982.090	16.251.200	48.233.290
5	2.057.384	2.872.800	4.930.184
6	22.148	126.224	148.372
7	8	1.008	1.016
8	0	1	1
$\sum$	377.403.266	54.320.113	431.723.379

$ P(8) $			
m	No isomorfos		
	$ PC(8, m) $	$ PNC(8, m) $	$ P(8, m) $
1	2.045	0	2.045
2	5.830	428	6.258
3	4.820	1.008	5.828
4	1.567	752	2.319
5	231	246	477
6	18	46	64
7	1	6	7
8	0	1	1
Σ	14.512	2.487	16.999

Para los no isomorfos vamos a mejorar la información:

$ PCNI(8, m) $									
	M								
m	1	2	3	4	5	6	7	8	Σ
1	318	814	642	221	43	6	1	0	2.045
2	814	2.317	1.916	659	112	12	0	0	5.830
3	642	1.916	1.649	537	76	0	0	0	4.820
4	221	659	537	150	0	0	0	0	1.567
5	43	112	76	0	0	0	0	0	231
6	6	12	0	0	0	0	0	0	18
7	1	0	0	0	0	0	0	0	1
8	0	0	0	0	0	0	0	0	0
Σ	2.045	5.830	4.820	1.567	231	18	1	0	14.512

$ PNCNI(8, m) $									
	M								
m	1	2	3	4	5	6	7	8	Σ
1	0	0	0	0	0	0	0	0	0
2	0	87	177	118	37	8	1	0	428
3	0	177	419	302	94	15	1	0	1.008
4	0	118	302	242	78	11	1	0	752
5	0	37	94	78	29	7	1	0	246
6	0	8	15	11	7	4	1	0	46
7	0	1	1	1	1	1	1	0	6
8	0	0	0	0	0	0	0	1	1
Σ	0	428	1.008	752	246	46	6	1	2.487

Con $S(n, k)$ representamos los números de Stirling de segunda especie, esto es:

$$S(n, k) = \frac{1}{k!} \sum_{i=0}^{k-1} (-1)^i \binom{k}{i} (k-i)^n,$$

Sea $PO(n)$ el conjunto de los órdenes parciales que se pueden definir sobre un conjunto finito con n elementos y $POC(n)$ el conjunto de los órdenes parciales conexos que se pueden definir sobre un conjunto finito con n elementos. Es bien conocido, ver por ejemplo [21], que:

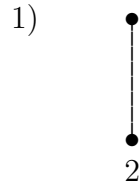
- 1) $|PO(n)|$ es igual al número de topologías que se pueden definir sobre un conjunto finito con n elementos,
- 2) $|POC(n)|$ es igual al número de topologías conexas que se pueden definir sobre un conjunto finito con n ,
- 3) $|POC(n)| = \sum_{k=1}^n S(n, k) |PC(k)|,$
- 4) $|PO(n)| = \sum_{k=1}^n S(n, k) |P(k)|.$

9. Diagramas de Hasse de $PNI(\mathbf{n})$, $2 \leq n \leq 7$.

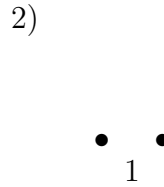
Debajo de los diagramas de cada poset indicamos la cantidad de posets isomorfos a él.

$PNI(\mathbf{2})$

$$|RB(\mathbf{2})| = 3$$

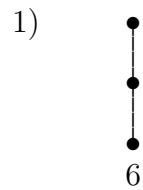


$$|RB(\mathbf{2})| = 4$$

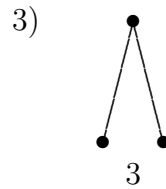
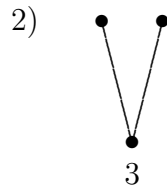


$PNI(\mathbf{3})$

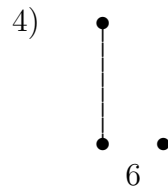
$$|RB(\mathbf{3})| = 4$$



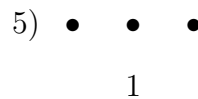
$$|RB(\mathbf{3})| = 5$$



$$|RB(\mathbf{3})| = 6$$

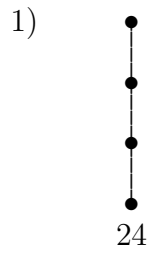


$$|RB(\mathbf{3})| = 8$$

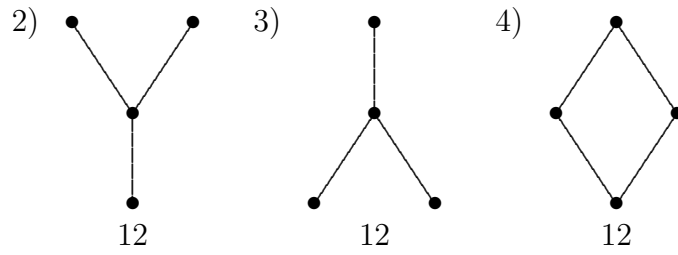


$PNI(4)$

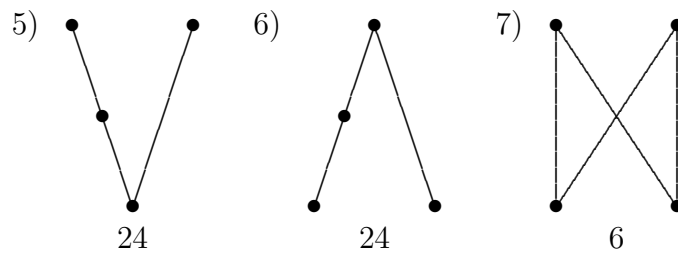
$|RB(4)| = 5$



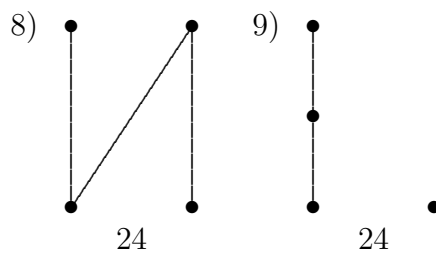
$|RB(4)| = 6$



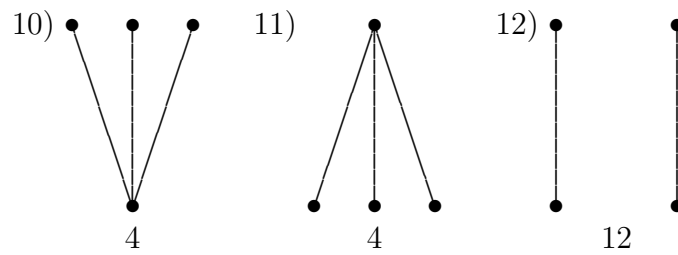
$|RB(4)| = 7$



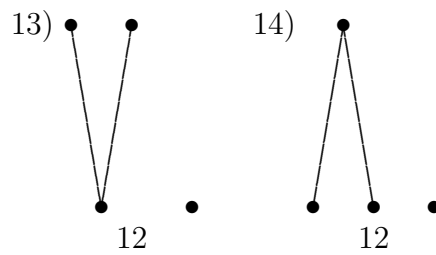
$|RB(4)| = 8$



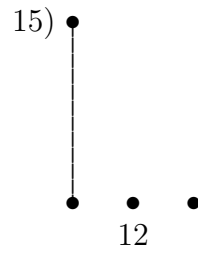
$|RB(4)| = 9$



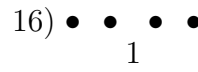
$|RB(4)| = 10$



$$|RB(4)| = 12$$

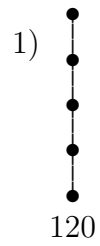


$$|RB(4)| = 16$$

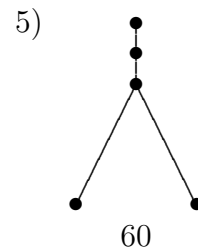
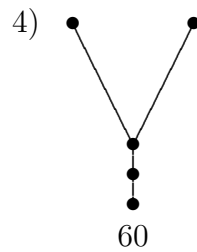
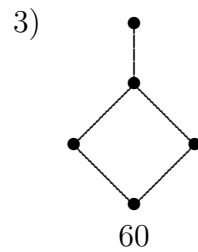
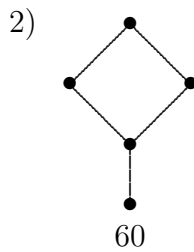


$$PNI(5)$$

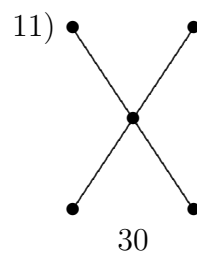
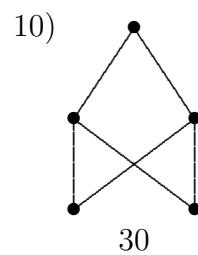
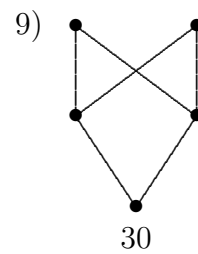
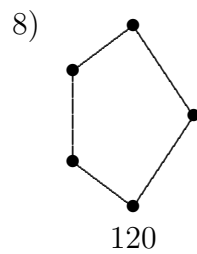
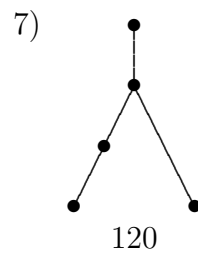
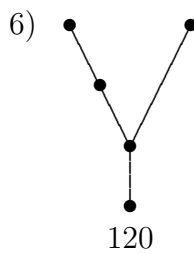
$$|RB(5)| = 6$$



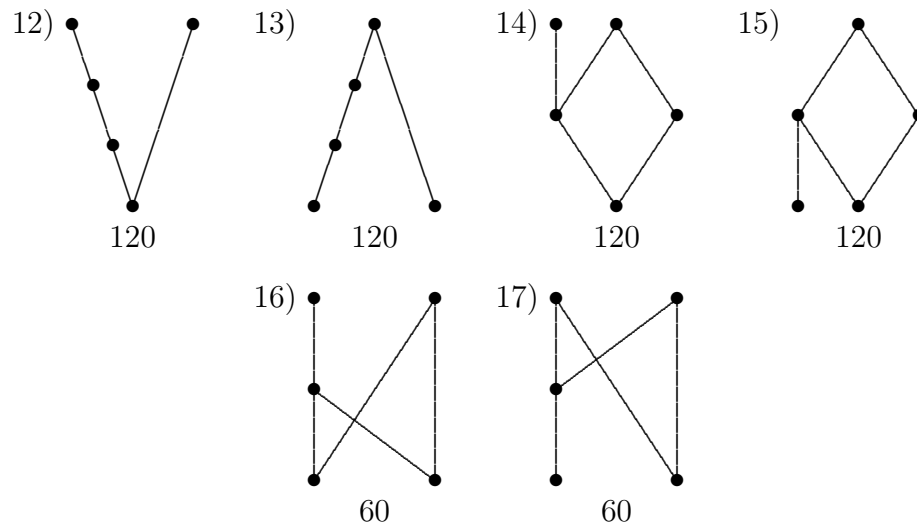
$$|RB(5)| = 7$$



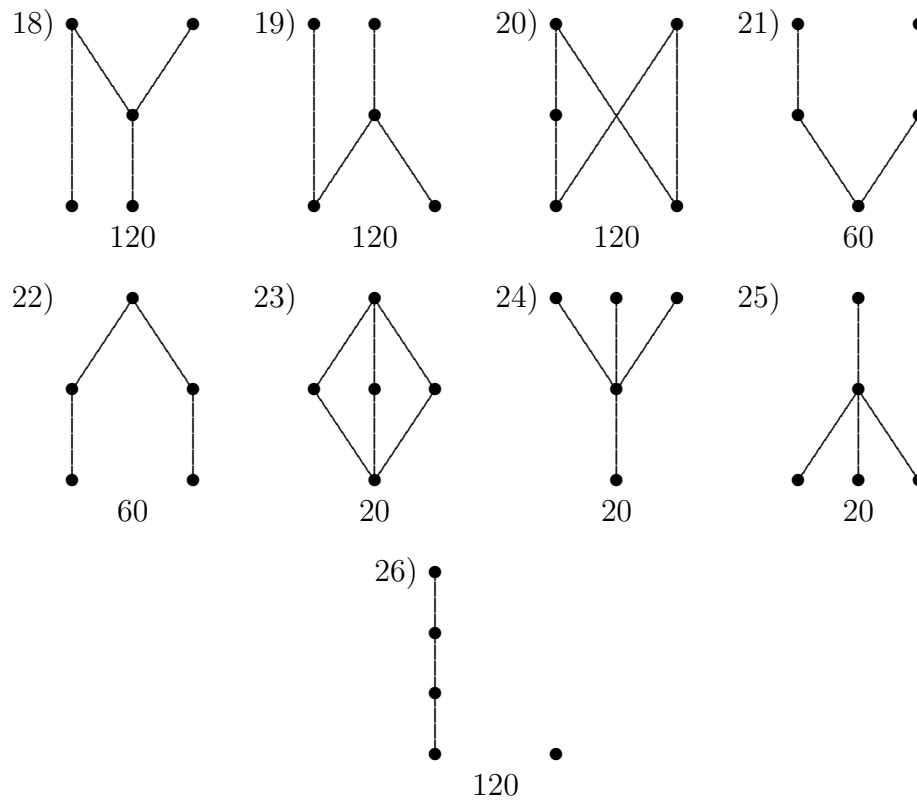
$$|RB(5)| = 8$$



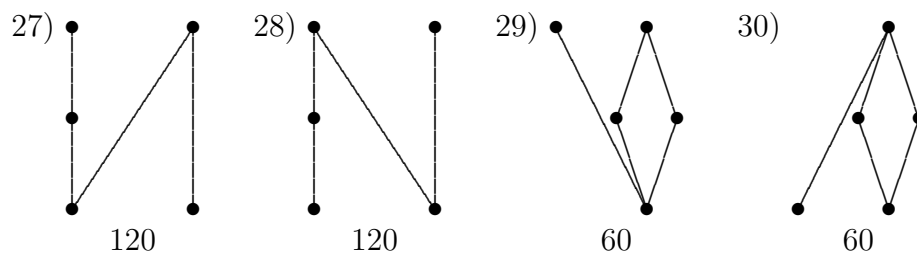
$$|RB(\mathbf{5})| = 9$$

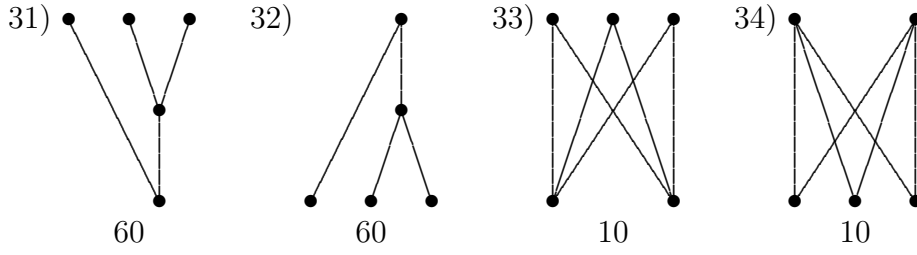


$$|RB(\mathbf{5})| = 10$$

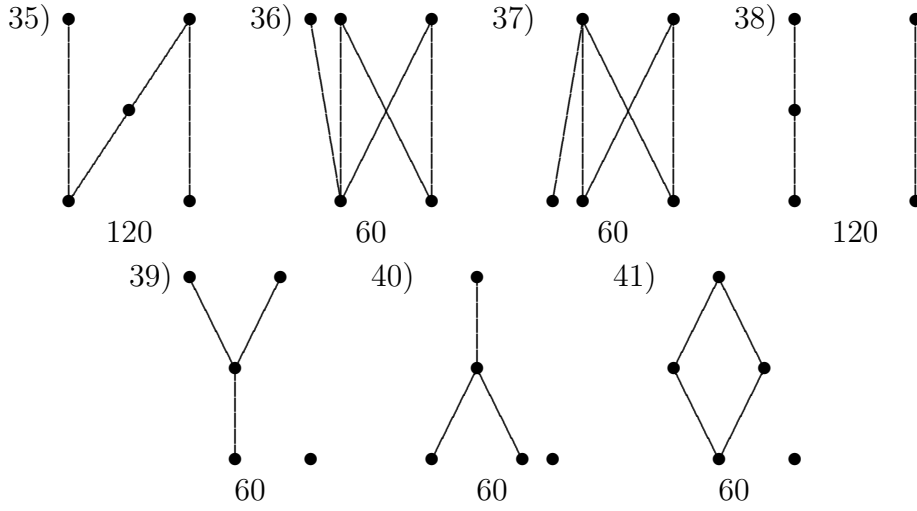


$$|RB(\mathbf{5})| = 11$$

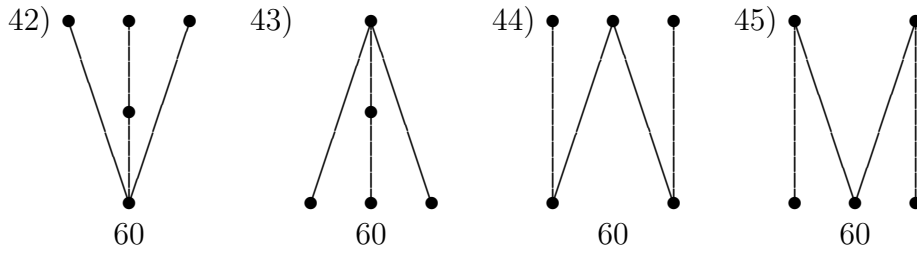




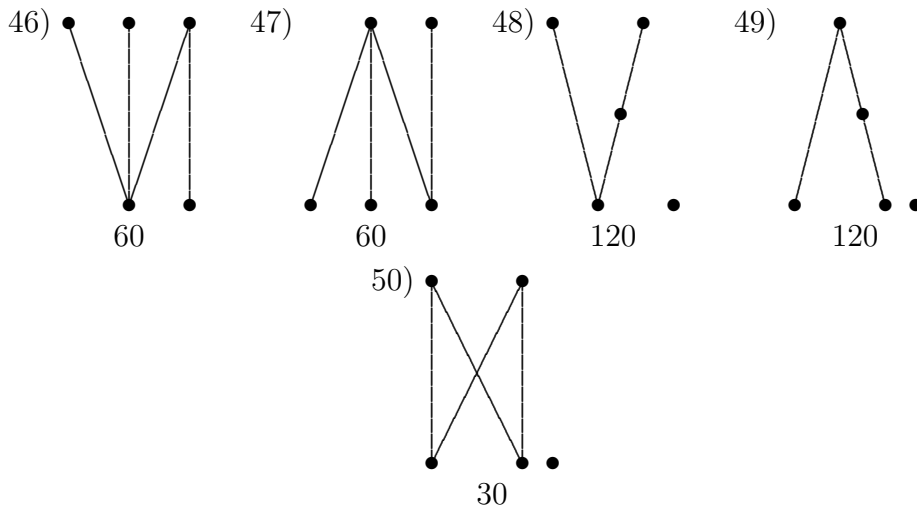
$$|RB(\mathbf{5})| = 12$$



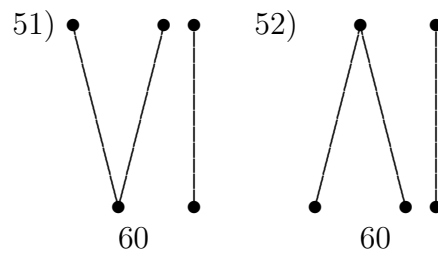
$$|RB(\mathbf{5})| = 13$$



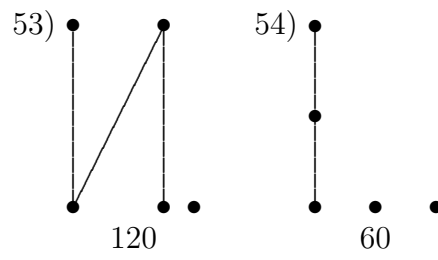
$$|RB(\mathbf{5})| = 14$$



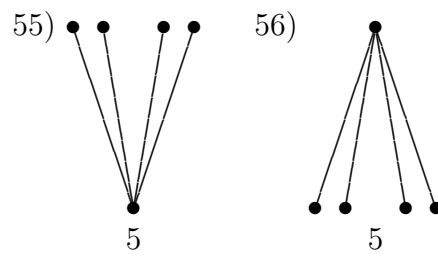
$$|RB(\mathbf{5})| = 15$$



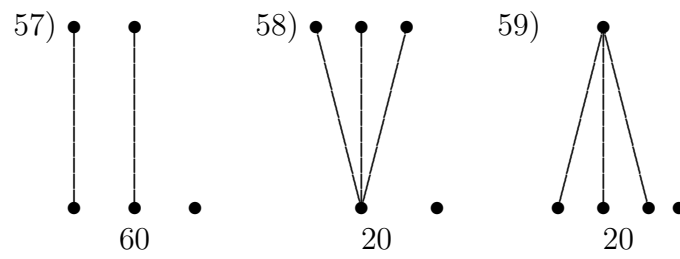
$$|RB(\mathbf{5})| = 16$$



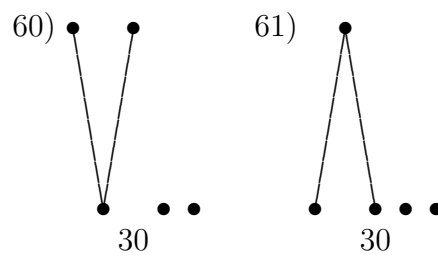
$$|RB(\mathbf{5})| = 17$$



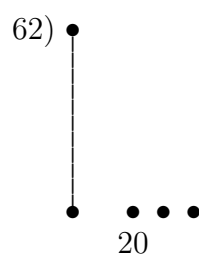
$$|RB(\mathbf{5})| = 18$$



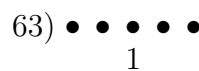
$$|RB(\mathbf{5})| = 20$$



$$|RB(\mathbf{5})| = 24$$

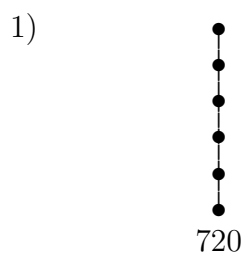


$$|RB(\mathbf{5})| = 32$$



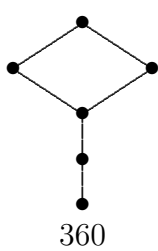
$PNI(\mathbf{6})$

$$|RB(\mathbf{6})| = 7$$

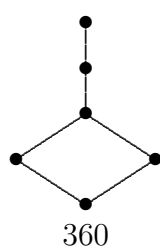


$$|RB(\mathbf{6})| = 8$$

2)



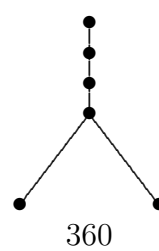
3)



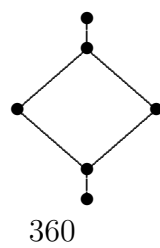
4)



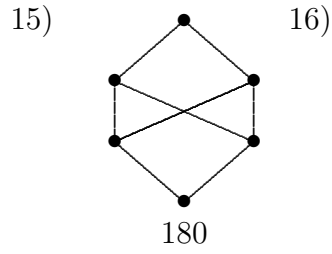
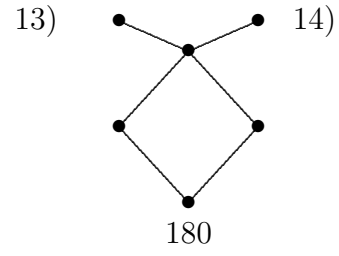
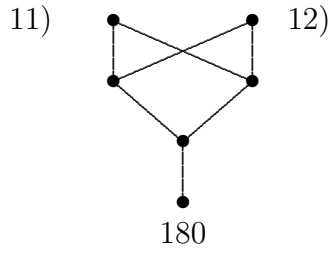
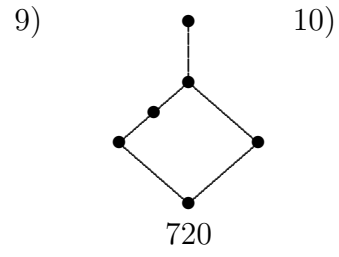
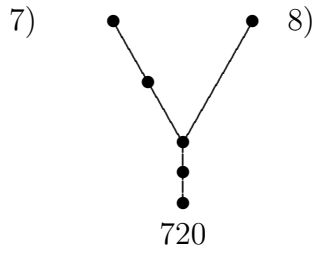
5)



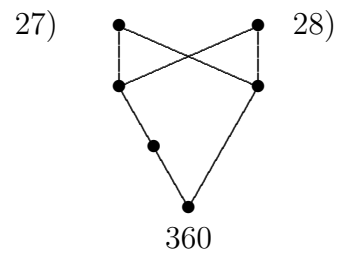
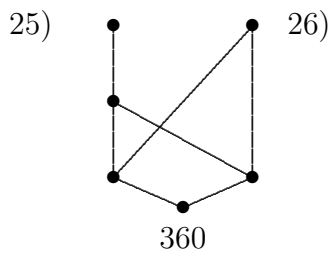
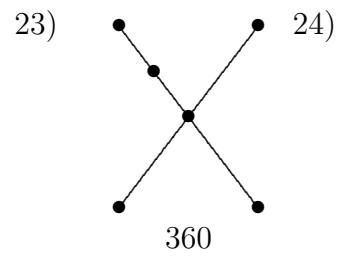
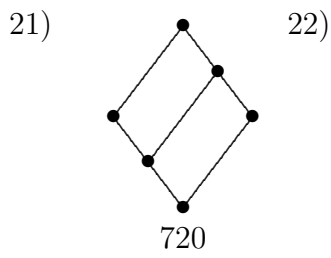
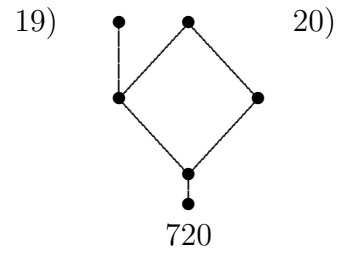
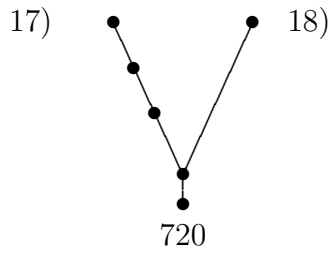
6)

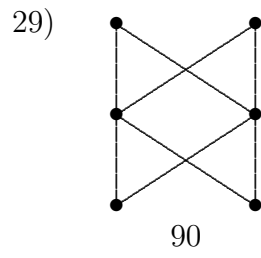


$$|RB(\mathbf{6})| = 9$$

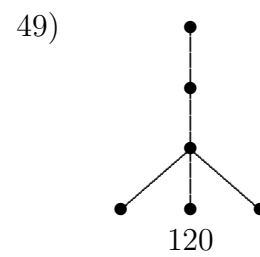
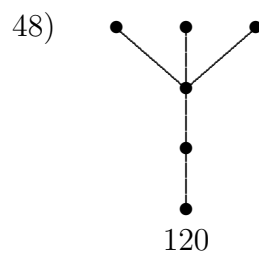
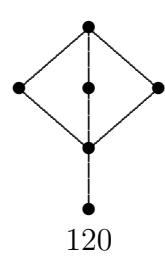
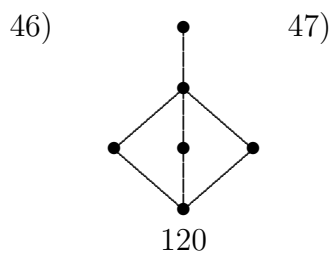
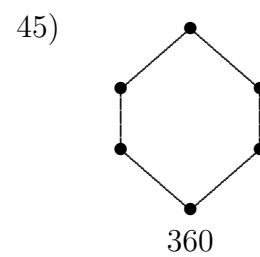
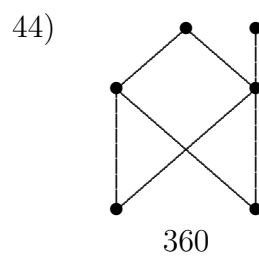
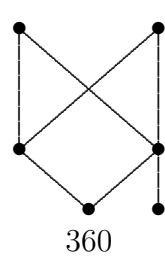
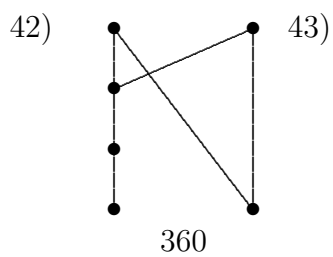
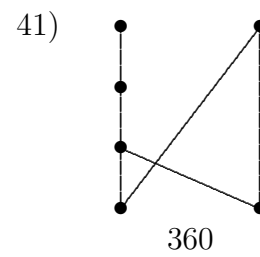
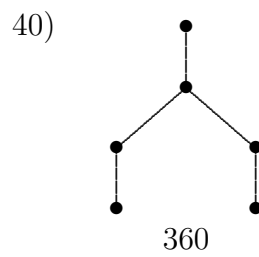
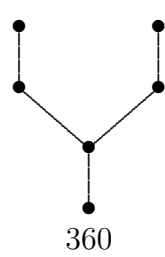
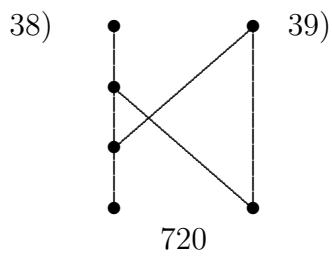
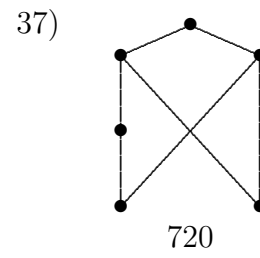
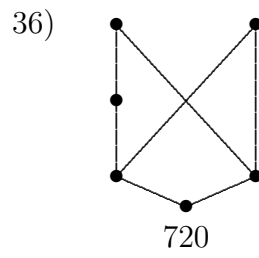
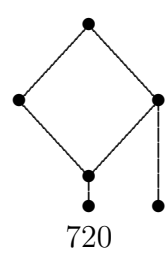
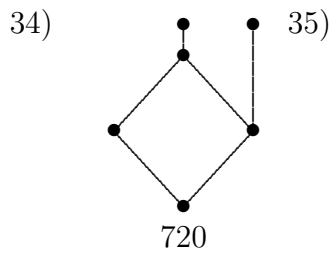
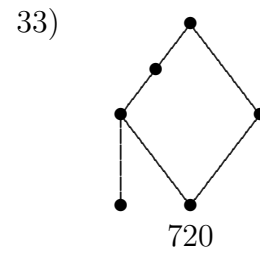
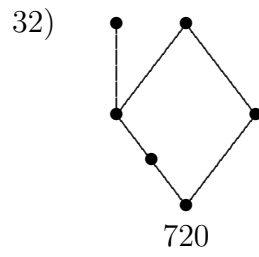
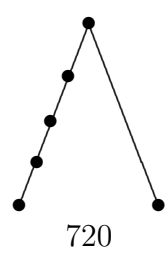
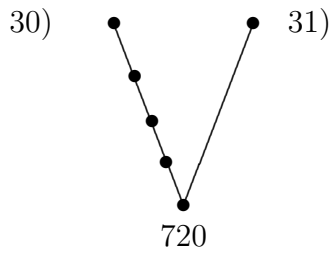


$$|RB(\mathbf{6})| = 10$$



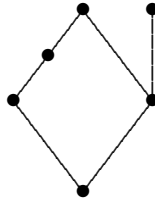


$$|RB(\mathbf{6})| = 11$$



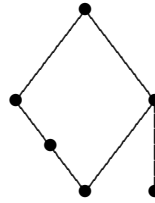
$$|RB(6)| = 12$$

50)



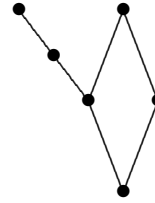
720

51)



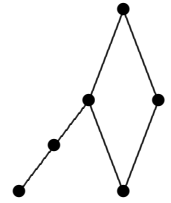
720

52)



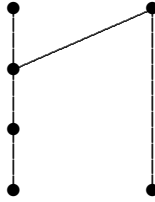
720

53)



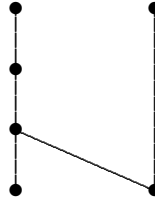
720

54)



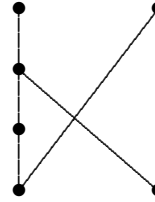
720

55)



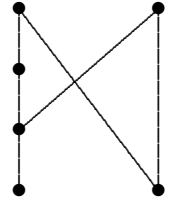
720

56)



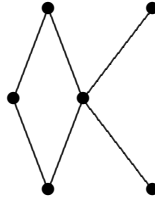
720

57)



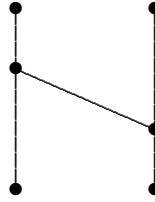
720

58)



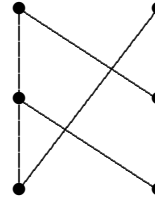
720

59)



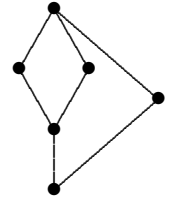
720

60)



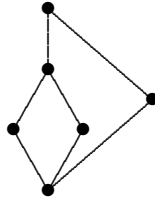
720

61)



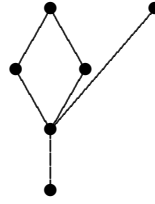
360

62)



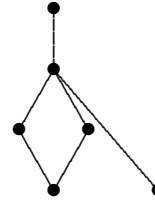
360

63)



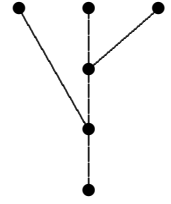
360

64)



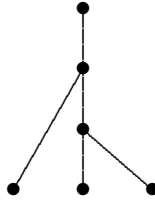
360

65)



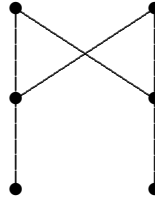
360

66)



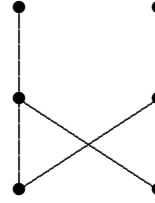
360

67)



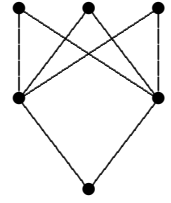
180

68)



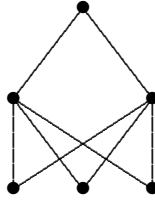
180

69)



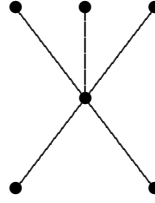
60

70)



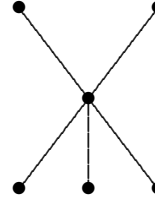
60

71)



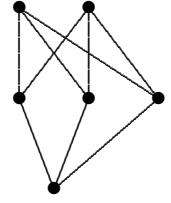
60

72)



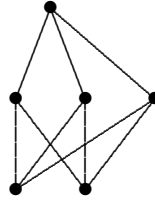
60

73)



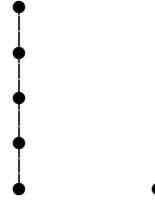
60

74)



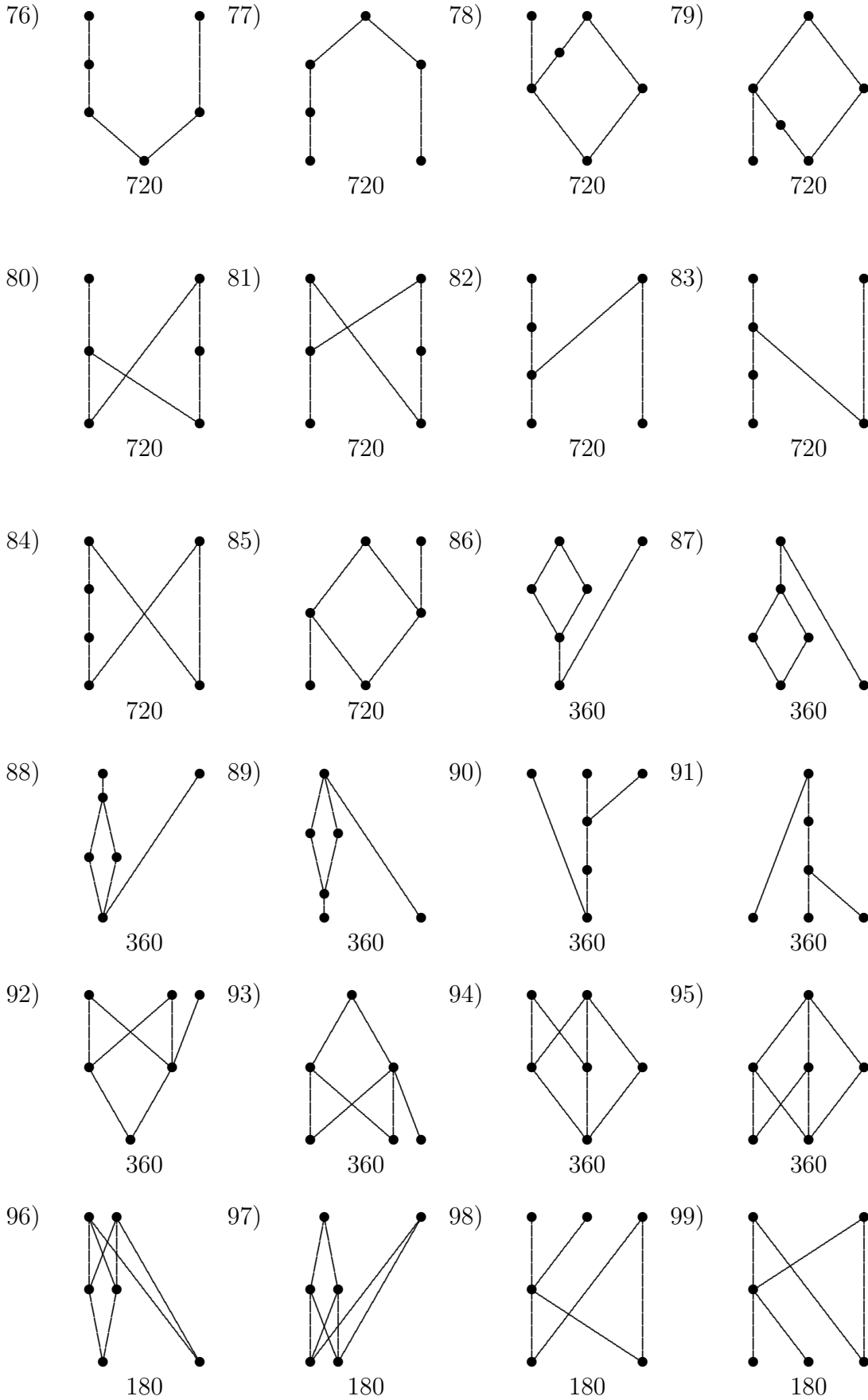
60

75)



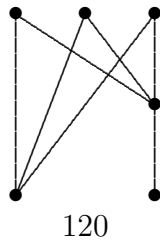
720

$$|RB(\mathbf{6})| = 13$$



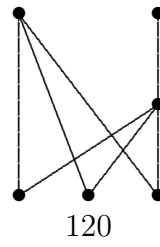
$$|RB(6)| = 14$$

100)



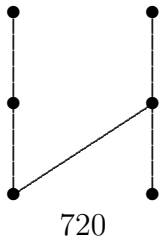
120

101)



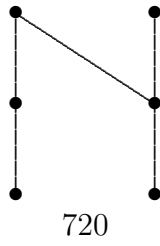
120

102)



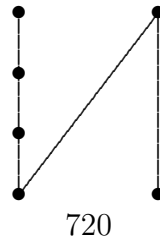
720

103)



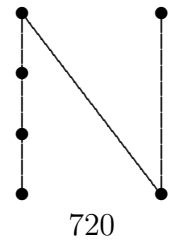
720

104)



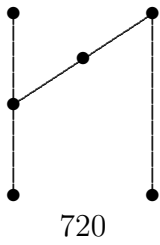
720

105)



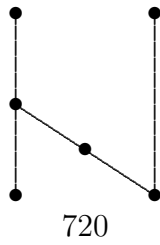
720

106)



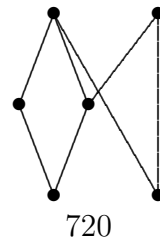
720

107)



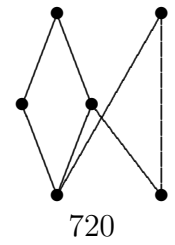
720

108)



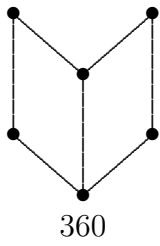
720

109)



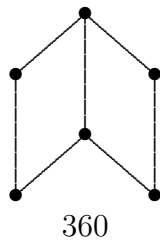
720

110)



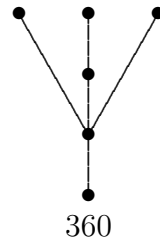
360

111)



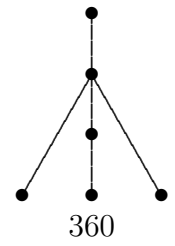
360

112)



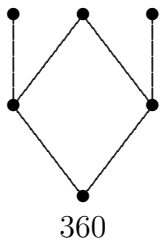
360

113)



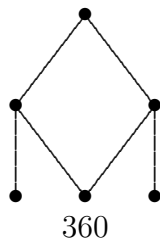
360

114)



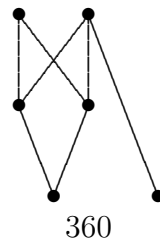
360

115)



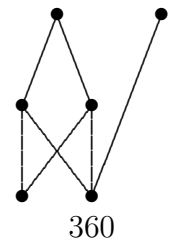
360

116)



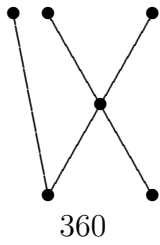
360

117)



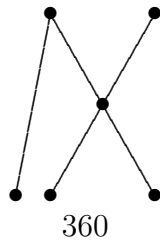
360

118)



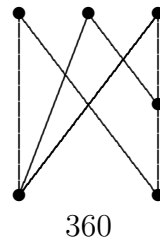
360

119)



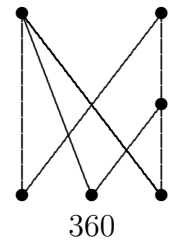
360

120)



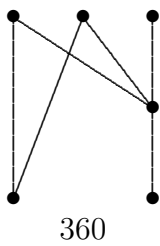
360

121)



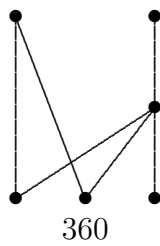
360

122)



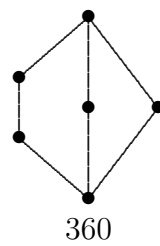
360

123)



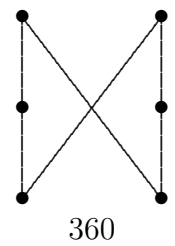
360

124)



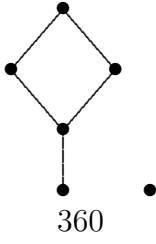
360

125)

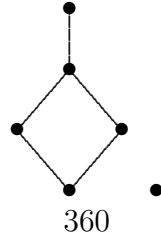


360

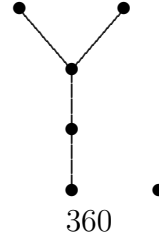
126)



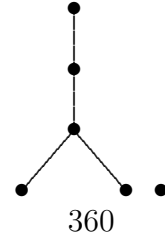
127)



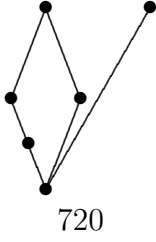
128)



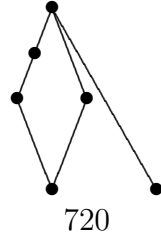
129)

 $|RB(\mathbf{6})| = 15$

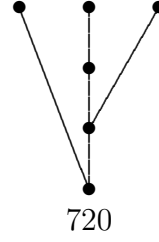
130)



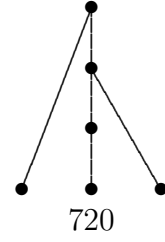
131)



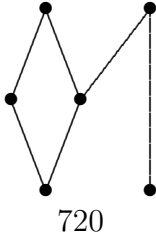
132)



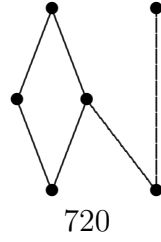
133)



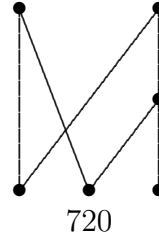
134)



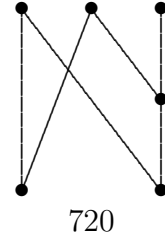
135)



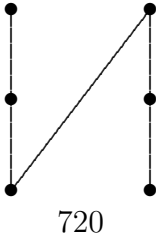
136)



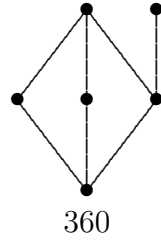
137)



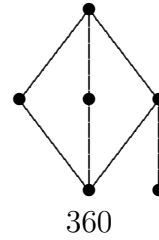
138)



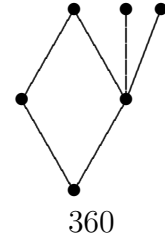
139)



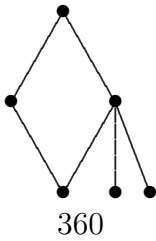
140)



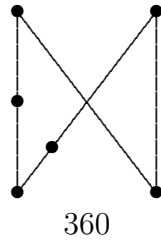
141)



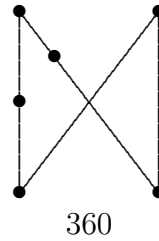
142)



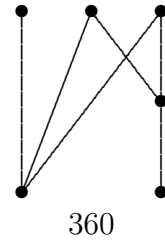
143)



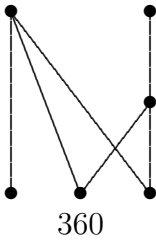
144)



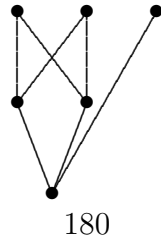
145)



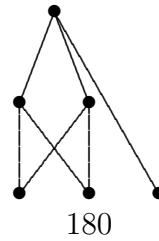
146)



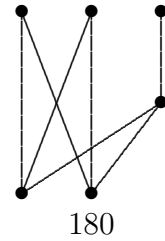
147)

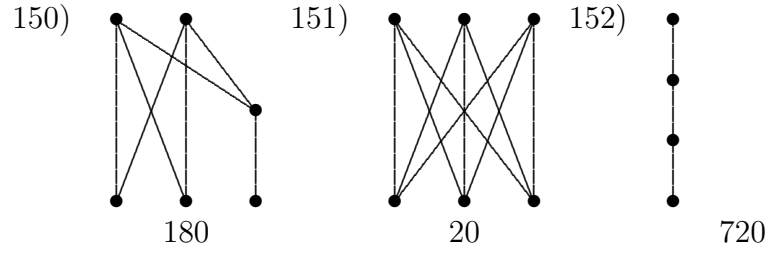


148)

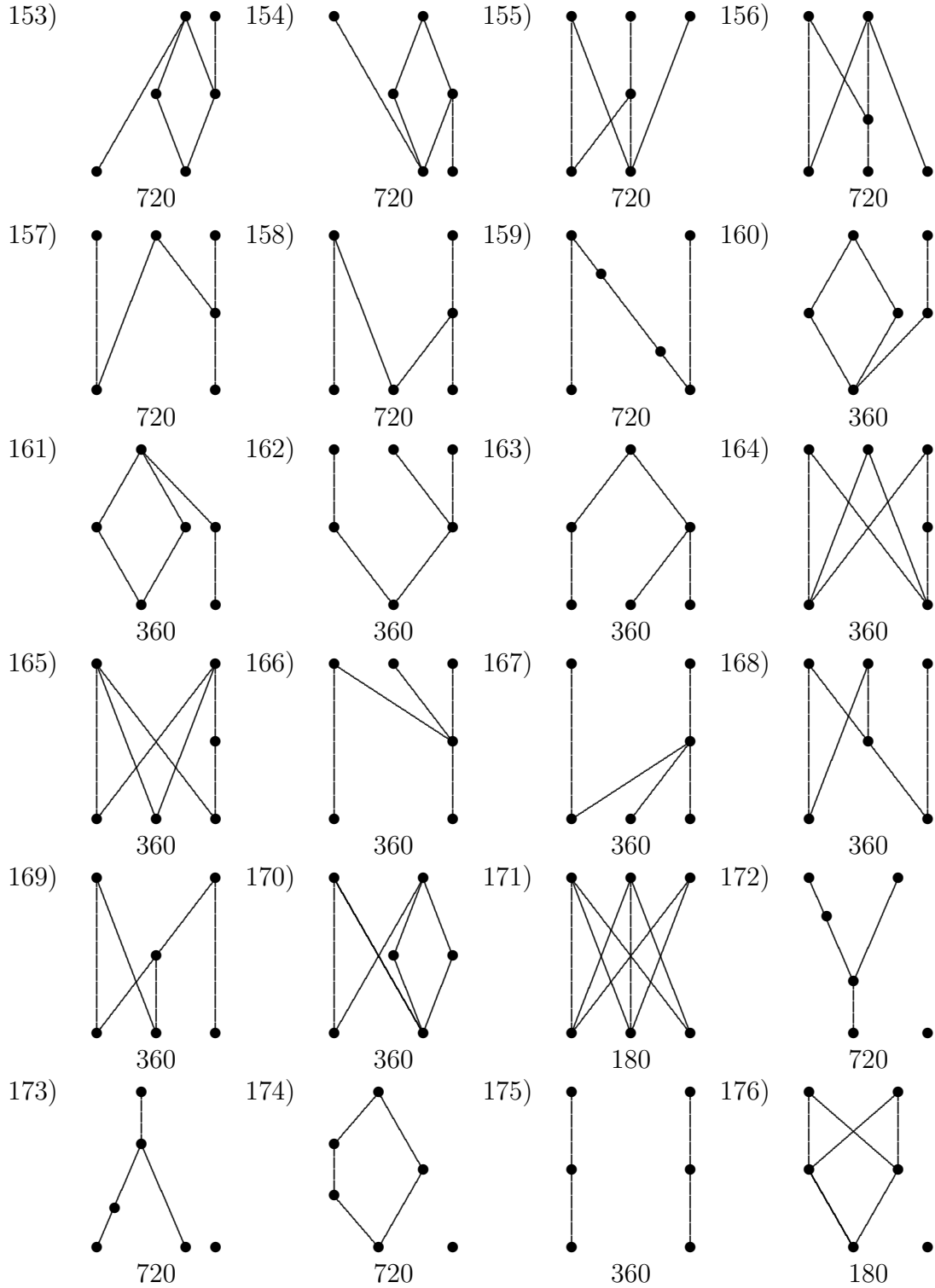


149)

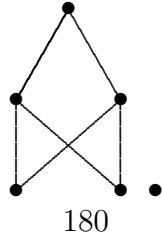




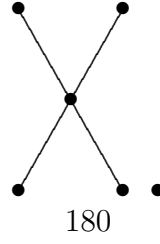
$|RB(6)| = 16$



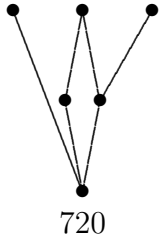
177)



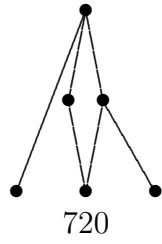
178)

 $|RB(\mathbf{6})| = 17$

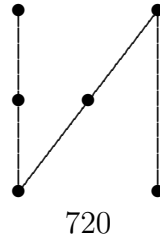
179)



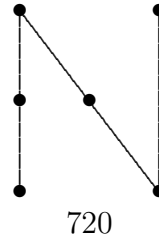
180)



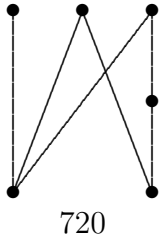
181)



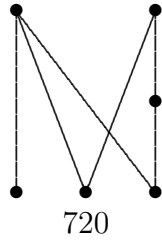
182)



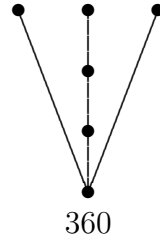
183)



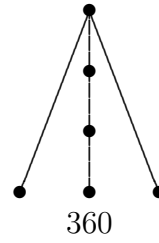
184)



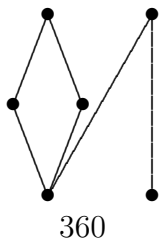
185)



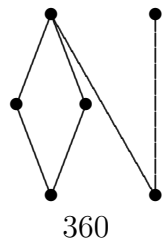
186)



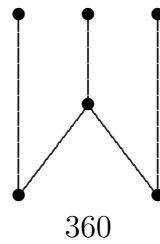
187)



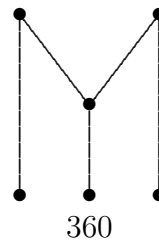
188)



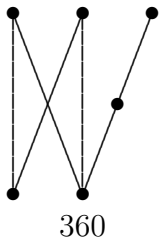
189)



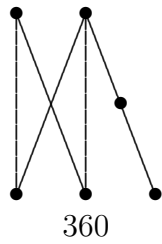
190)



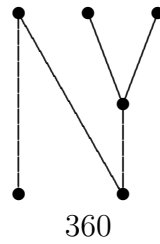
191)



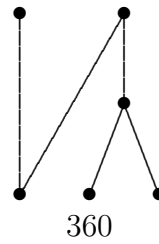
192)



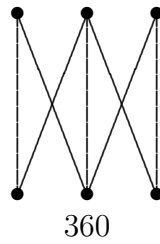
193)



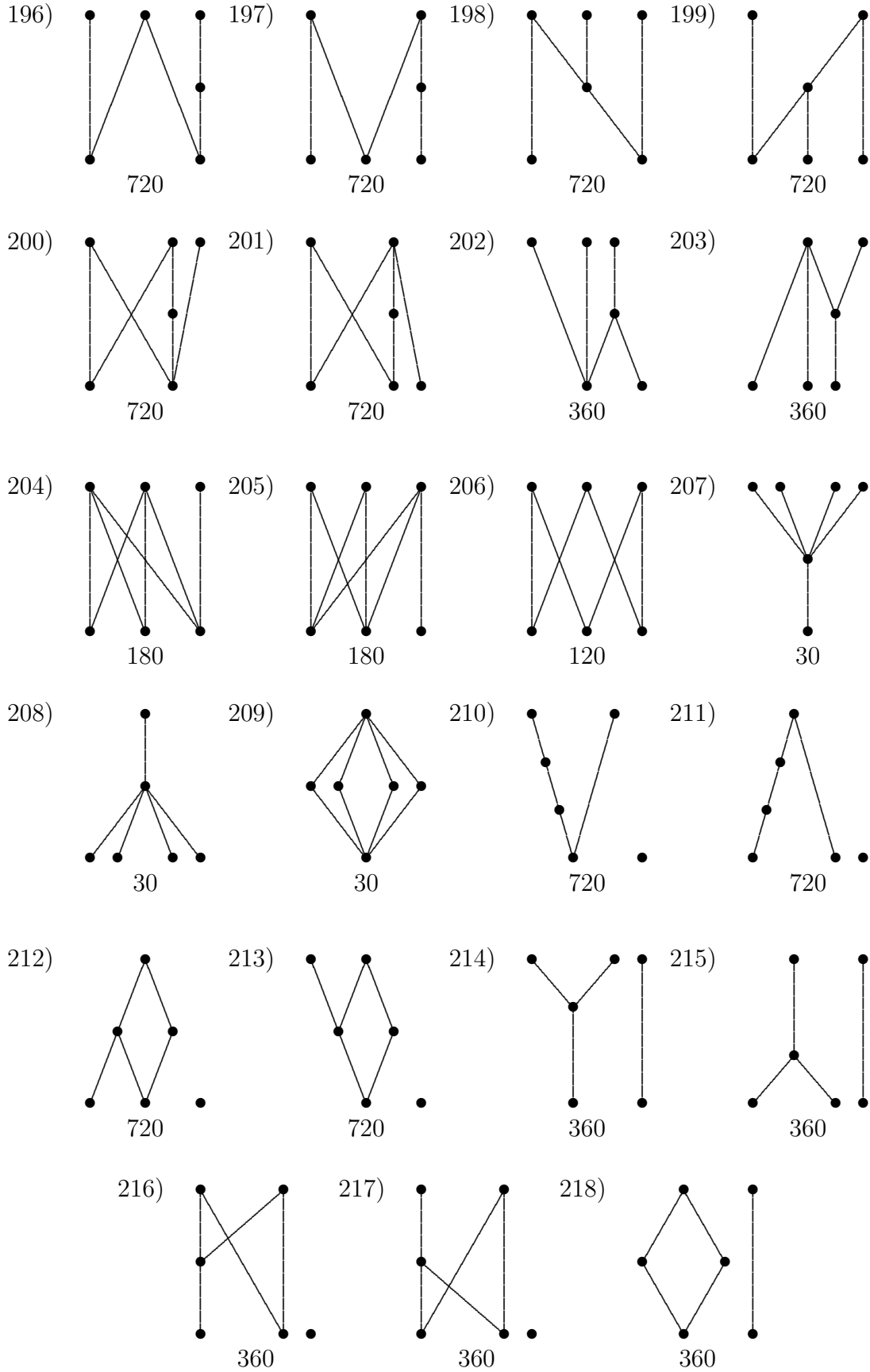
194)



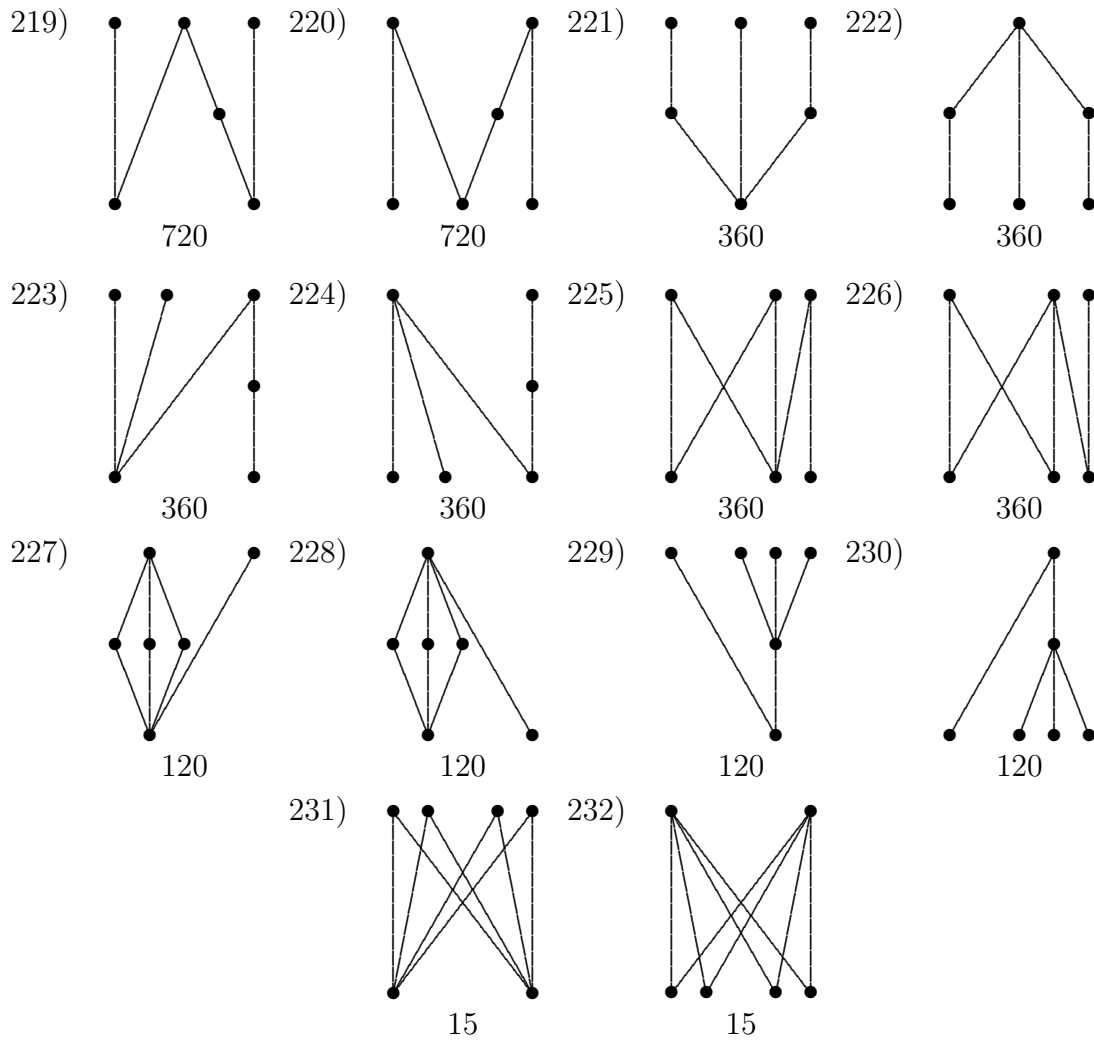
195)



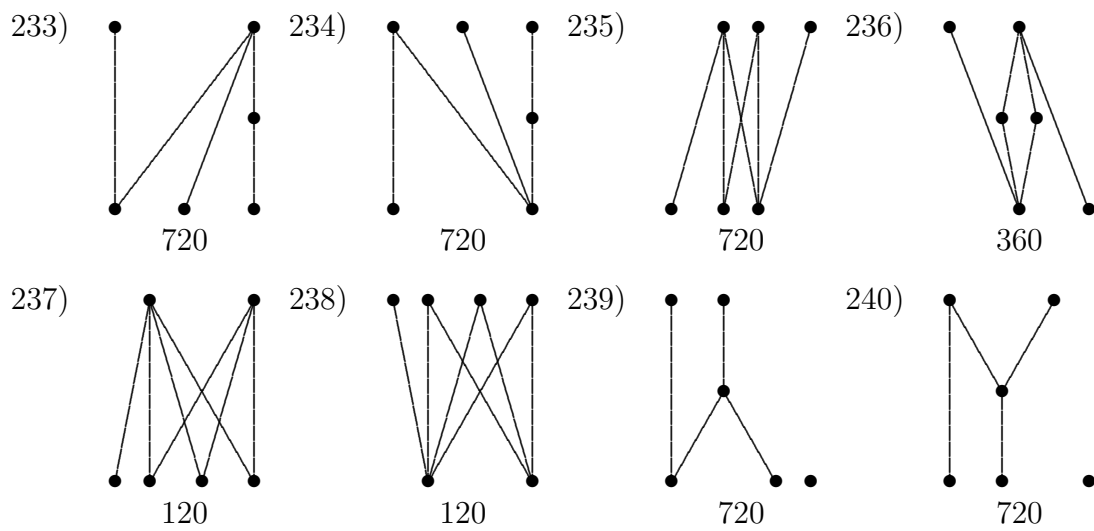
$$|RB(\mathbf{6})| = 18$$

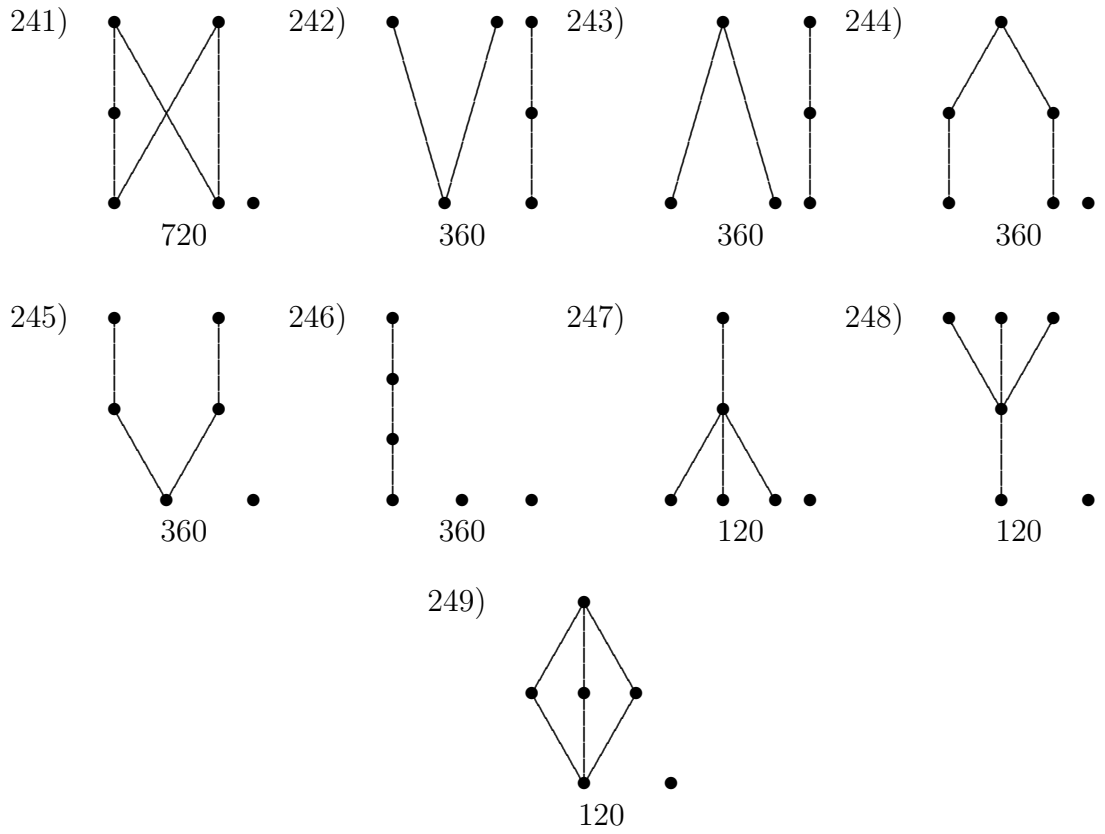


$$|RB(\mathbf{6})| = 19$$

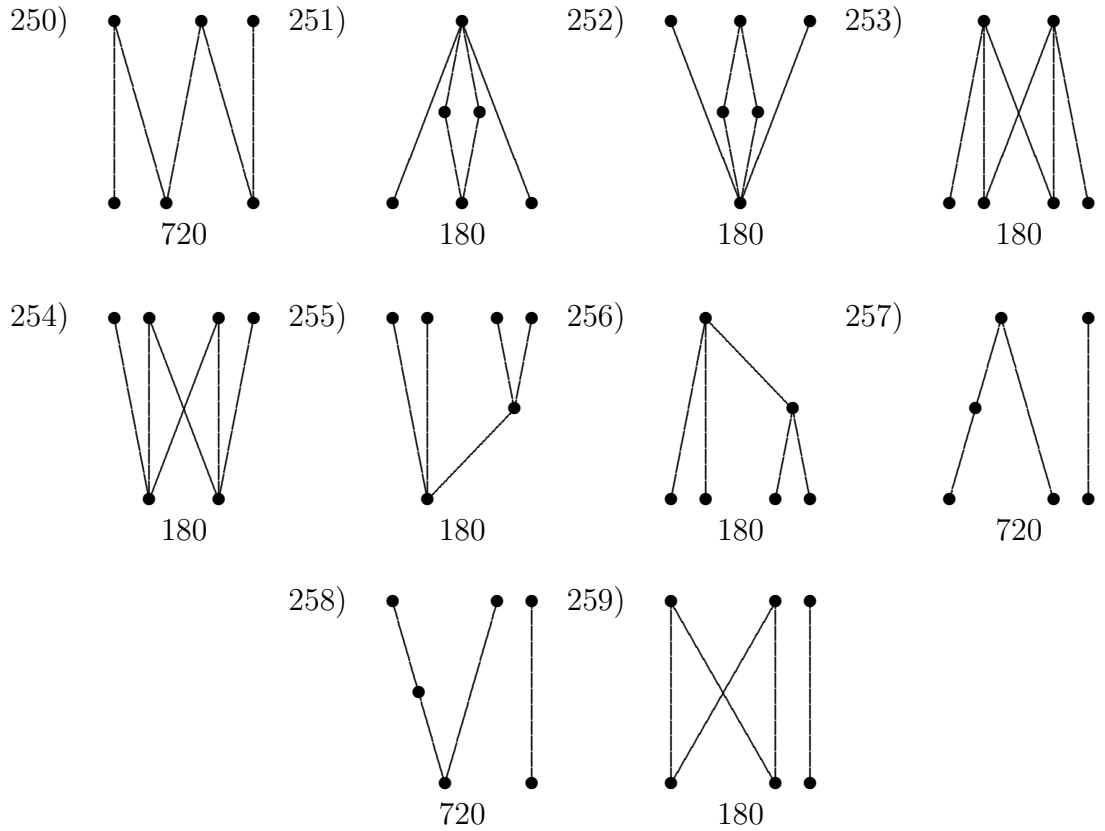


$$|RB(\mathbf{6})| = 20$$

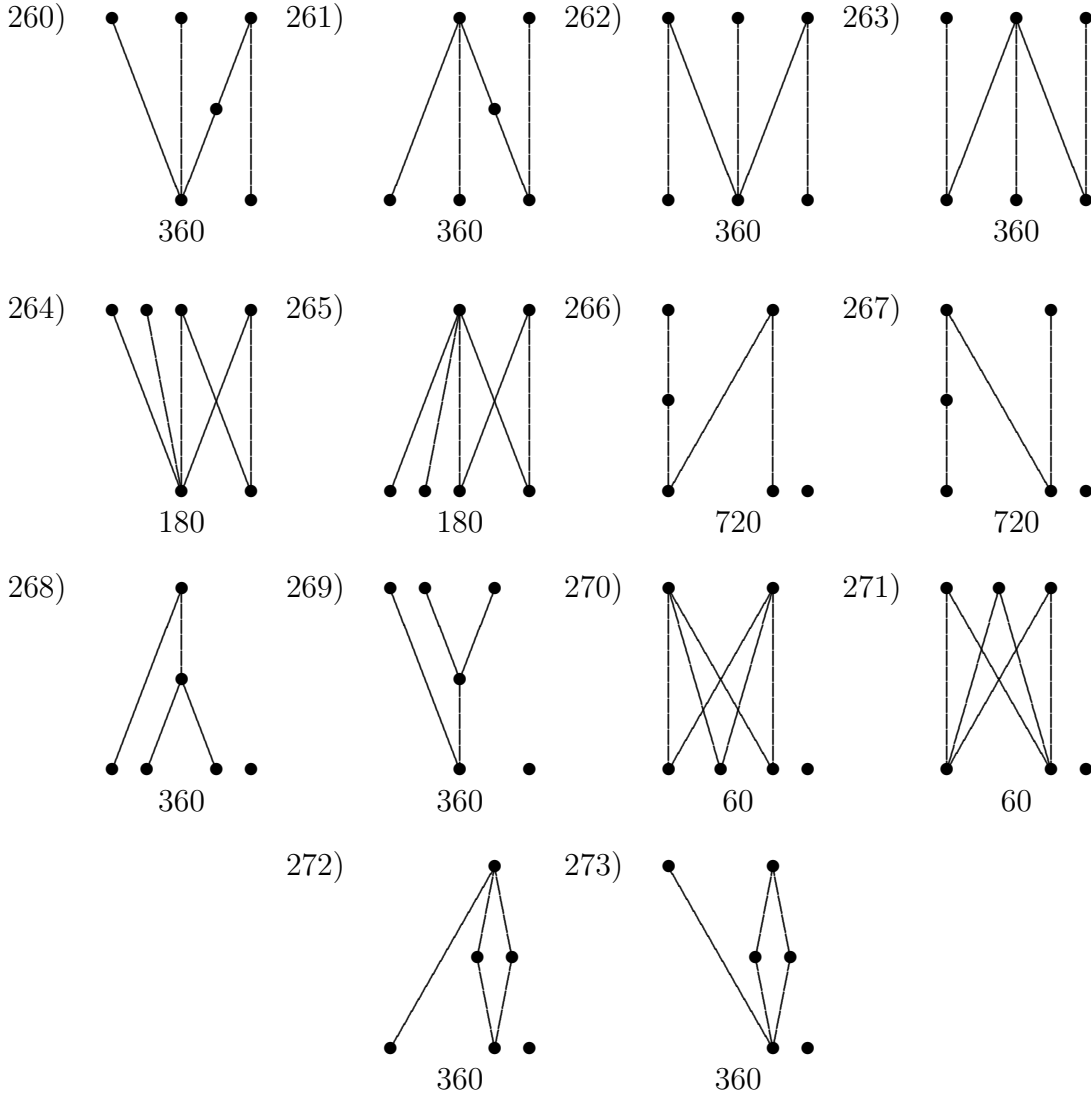




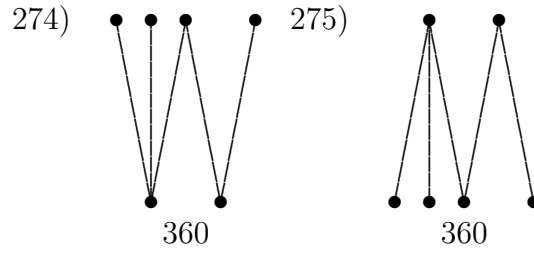
$|RB(\mathbf{6})| = 21$



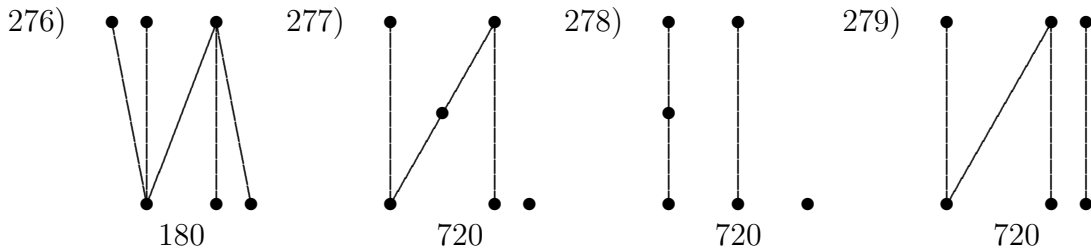
$$|RB(\mathbf{6})| = 22$$

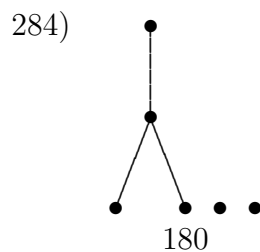
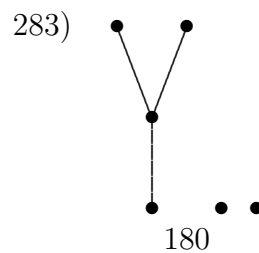
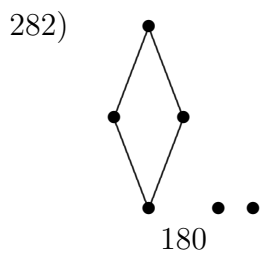
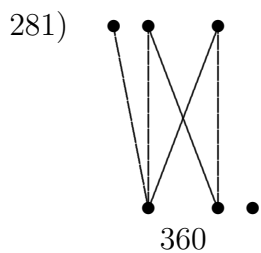
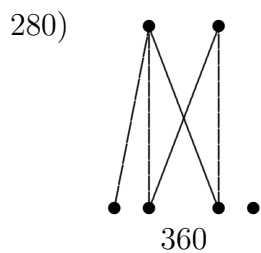


$$|RB(\mathbf{6})| = 23$$

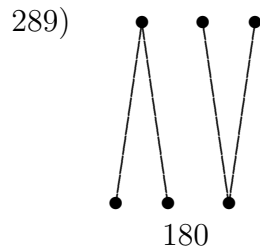
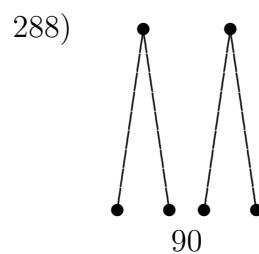
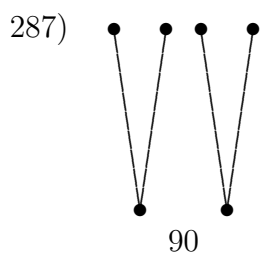
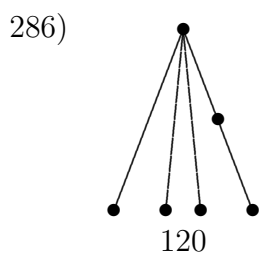
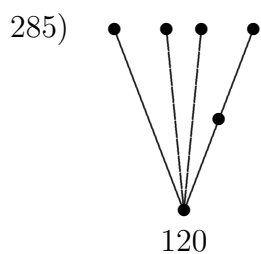


$$|RB(\mathbf{6})| = 24$$

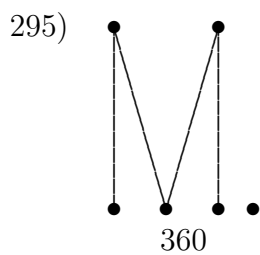
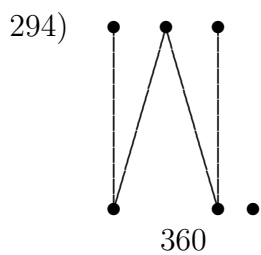
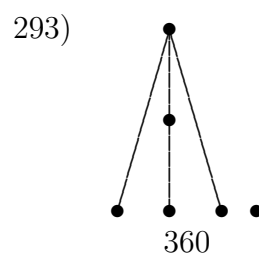
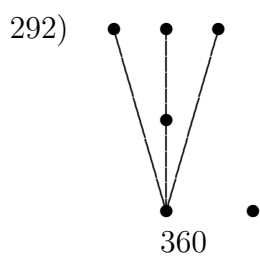
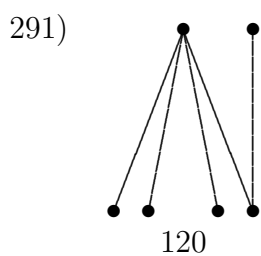
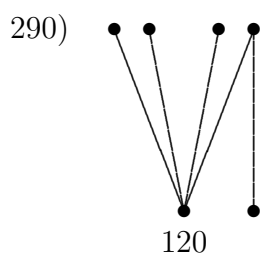




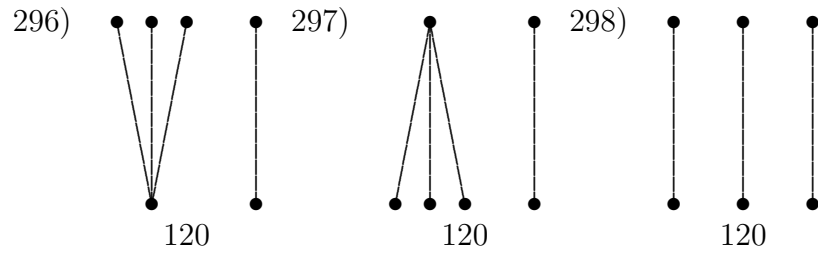
$|RB(\mathbf{6})| = 25$



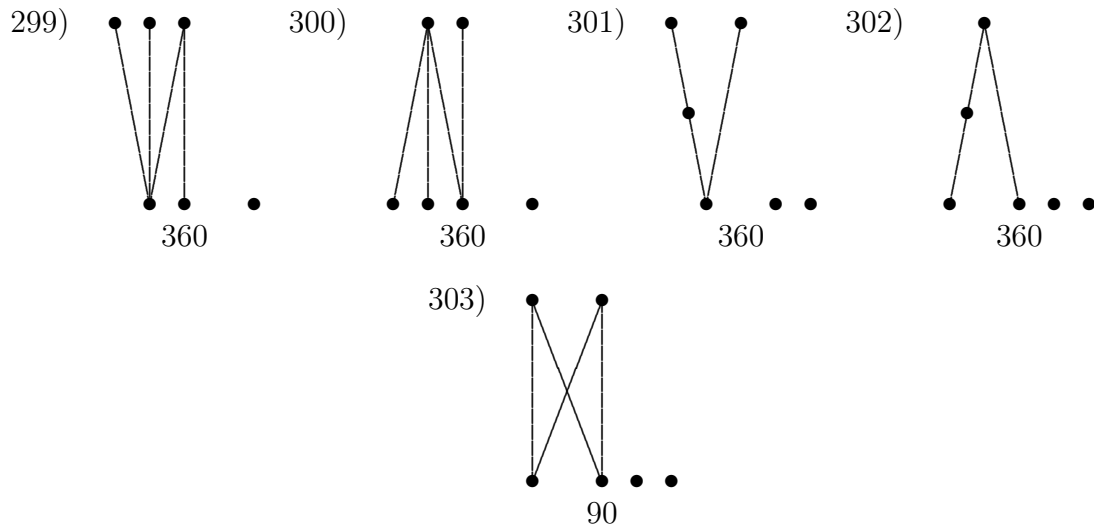
$|RB(\mathbf{6})| = 26$



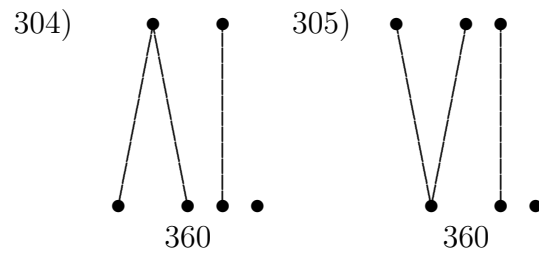
$$|RB(\mathbf{6})| = 27$$



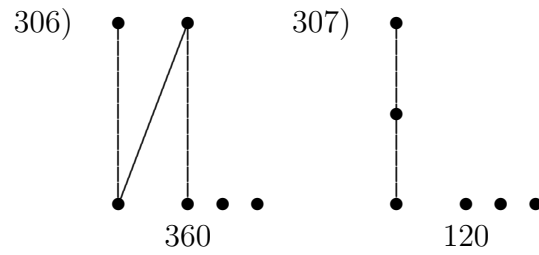
$$|RB(\mathbf{6})| = 28$$



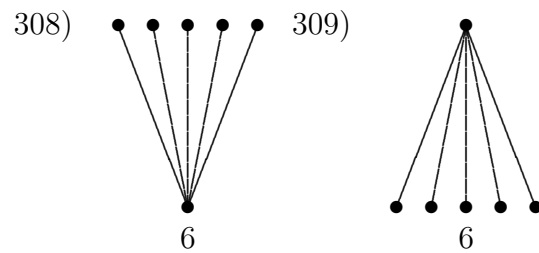
$$|RB(\mathbf{6})| = 30$$



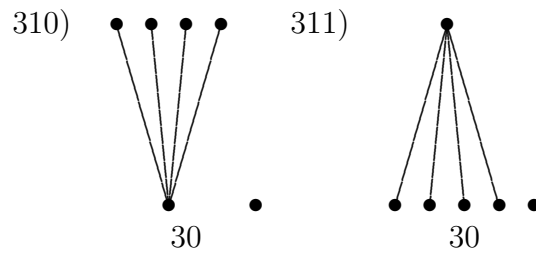
$$|RB(\mathbf{6})| = 32$$



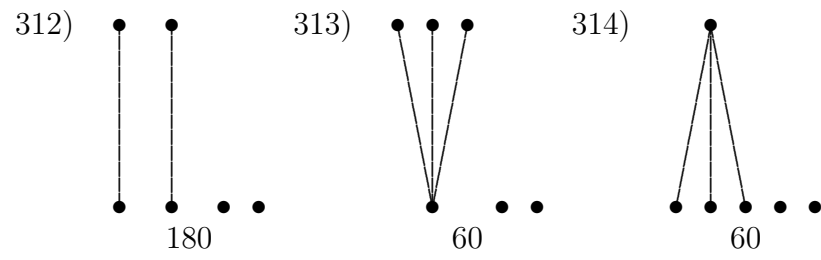
$$|RB(\mathbf{6})| = 33$$



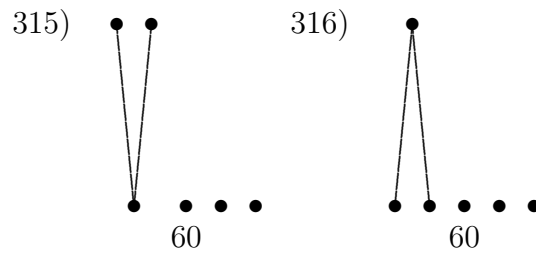
$$|RB(\mathbf{6})| = 34$$



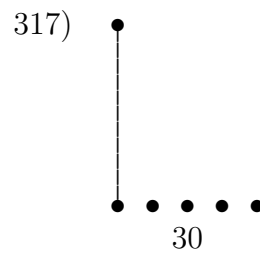
$$|RB(\mathbf{6})| = 36$$



$$|RB(\mathbf{6})| = 40$$



$$|RB(\mathbf{6})| = 48$$

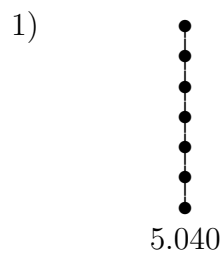


$$|RB(\mathbf{6})| = 64$$

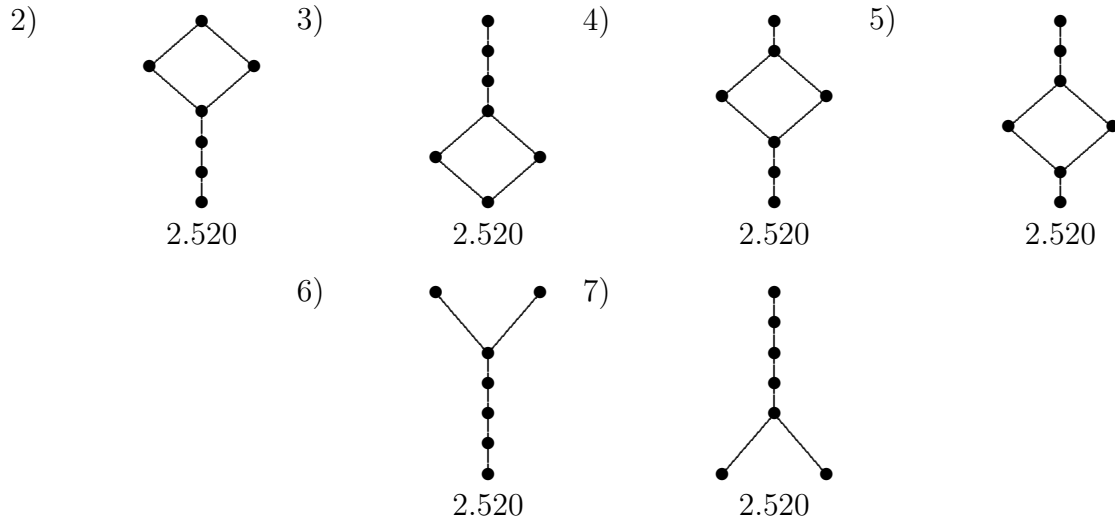


$PNI(\mathbf{7})$

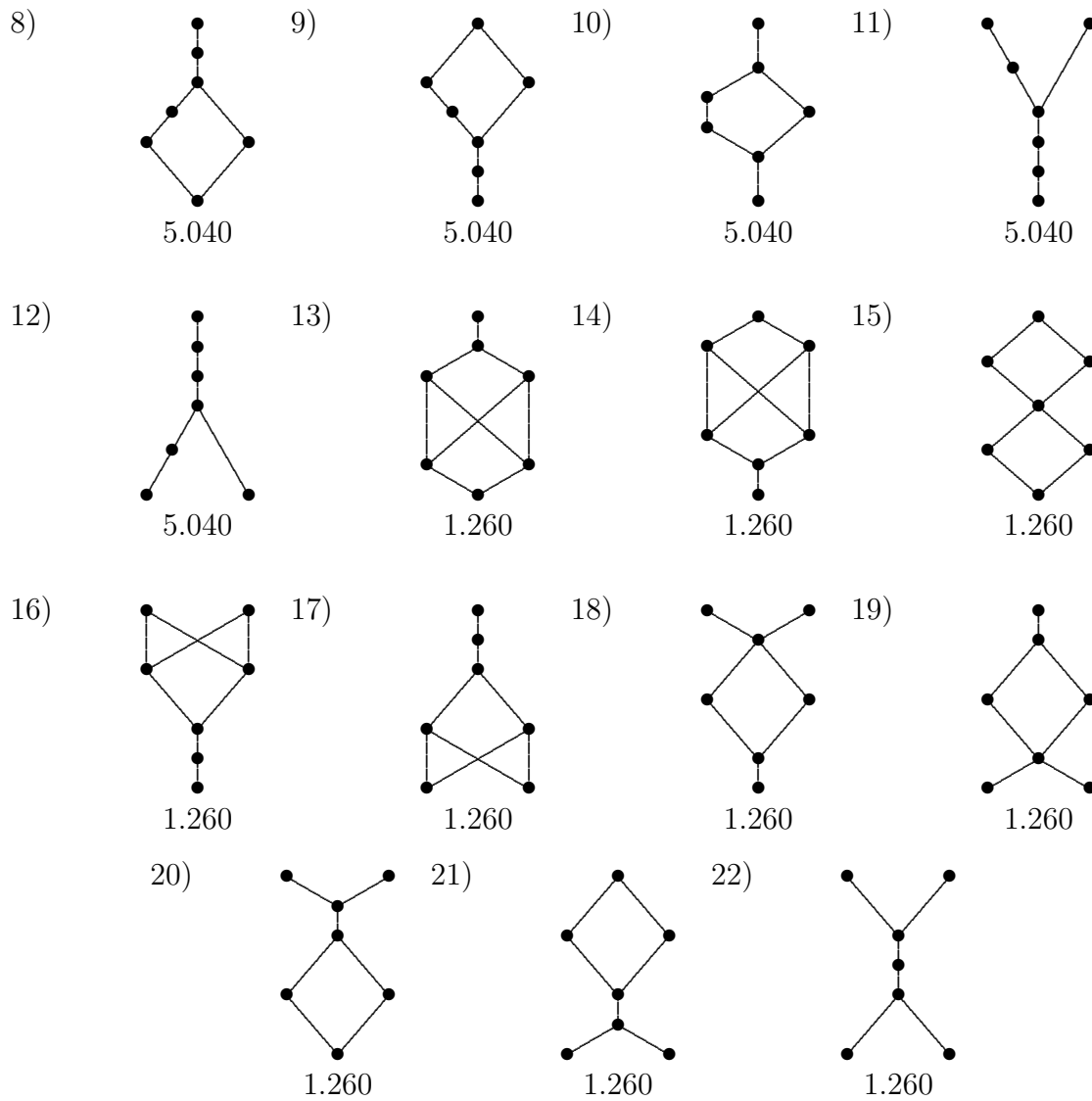
$$|RB(\mathbf{7})| = 8$$



$$|RB(\mathbf{7})| = 9$$

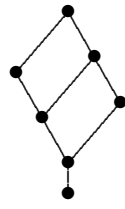


$$|RB(\mathbf{7})| = 10$$



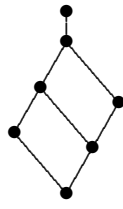
$$|RB(\mathbf{7})| = 11$$

23)



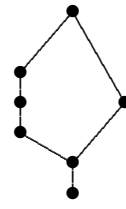
5.040

24)



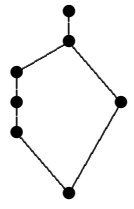
5.040

25)



5.040

26)



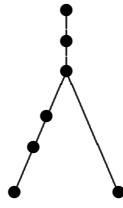
5.040

27)



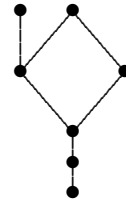
5.040

28)



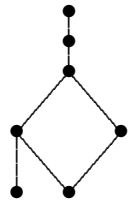
5.040

29)



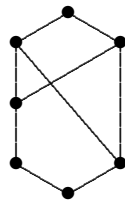
5.040

30)



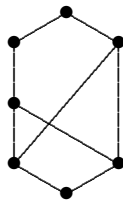
5.040

31)



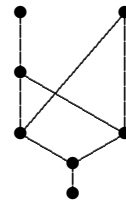
2.520

32)



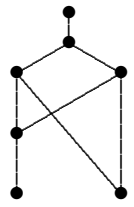
2.520

33)



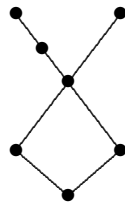
2.520

34)



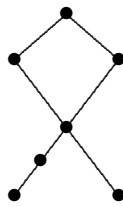
2.520

35)



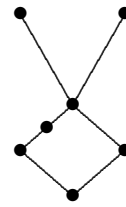
2.520

36)



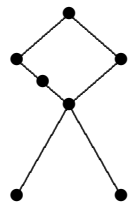
2.520

37)



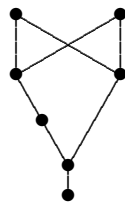
2.520

38)



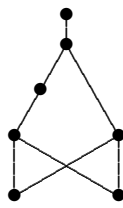
2.520

39)



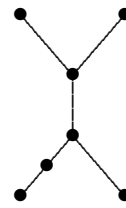
2.520

40)



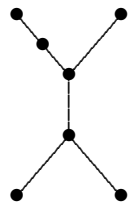
2.520

41)



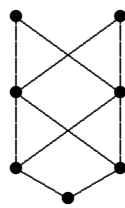
2.520

42)



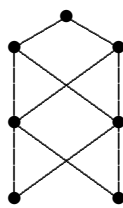
2.520

43)



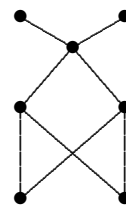
630

44)



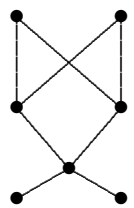
630

45)



630

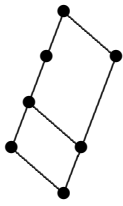
46)



630

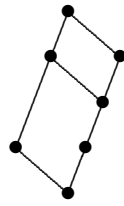
$$|RB(7)| = 12$$

47)



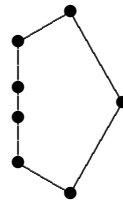
5.040

48)



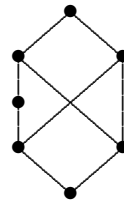
5.040

49)



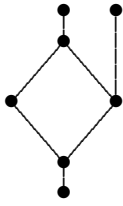
5.040

50)



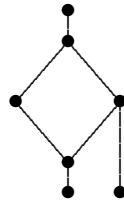
5.040

51)



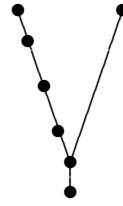
5.040

52)



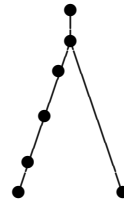
5.040

53)



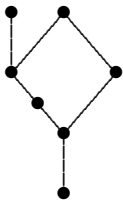
5.040

54)



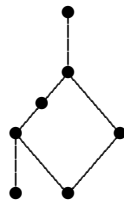
5.040

55)



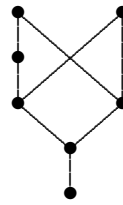
5.040

56)



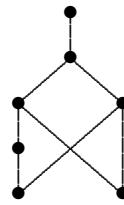
5.040

57)



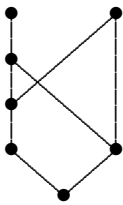
5.040

58)



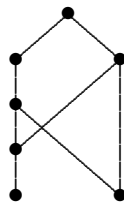
5.040

59)



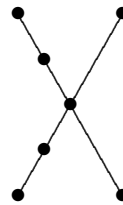
5.040

60)



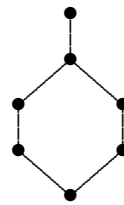
5.040

61)



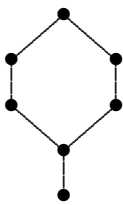
5.040

62)



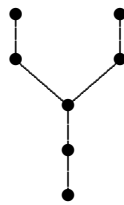
2.520

63)



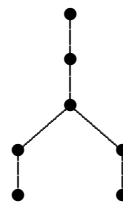
2.520

64)



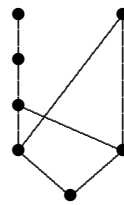
2.520

65)



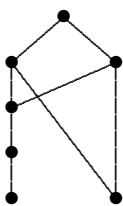
2.520

66)



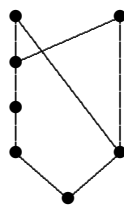
2.520

67)



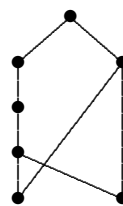
2.520

68)



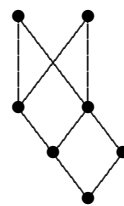
2.520

69)

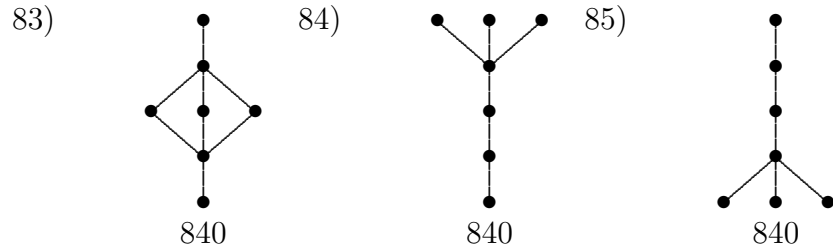
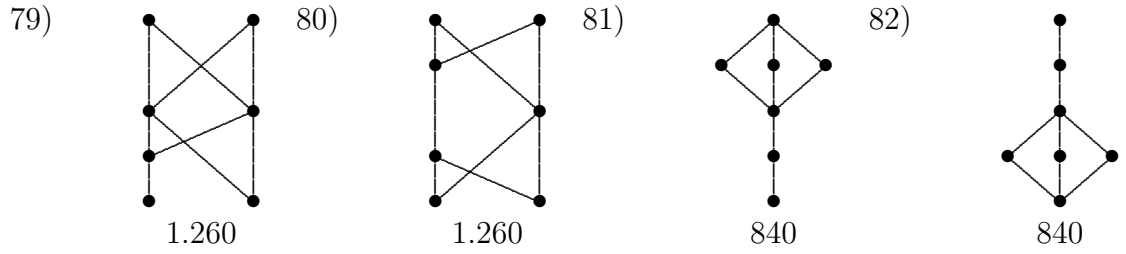
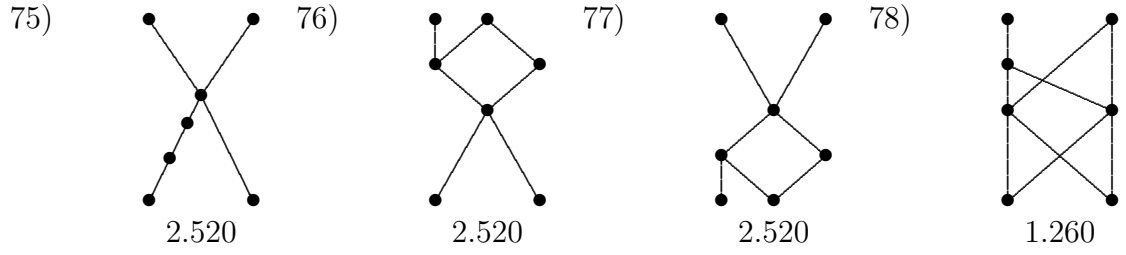
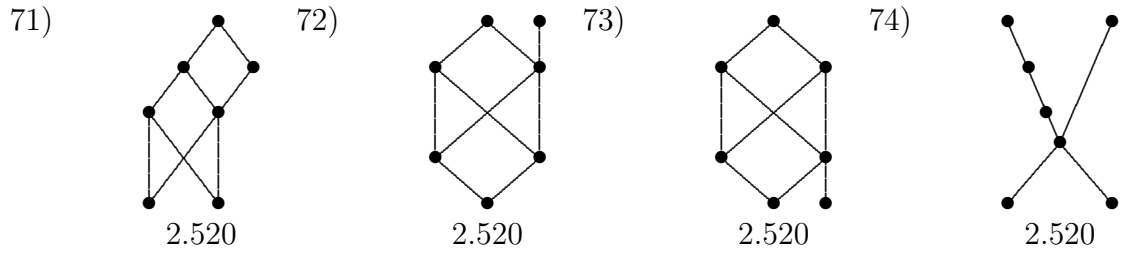


2.520

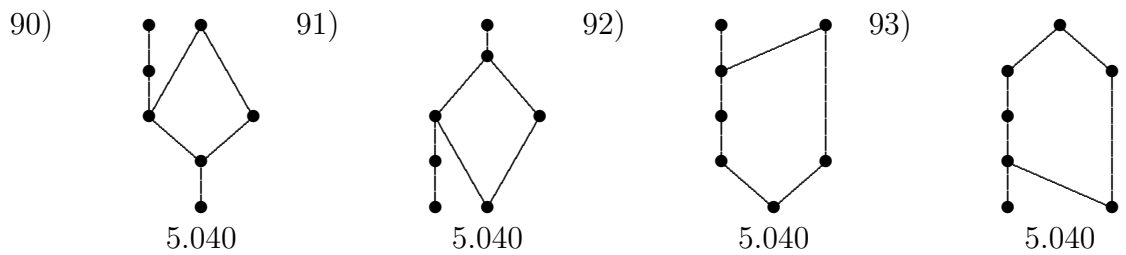
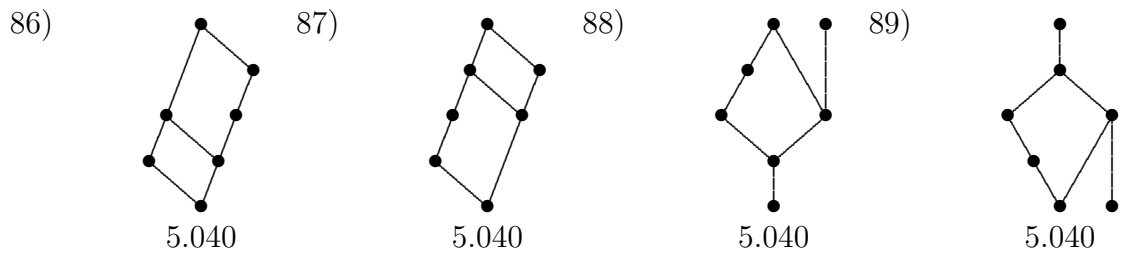
70)

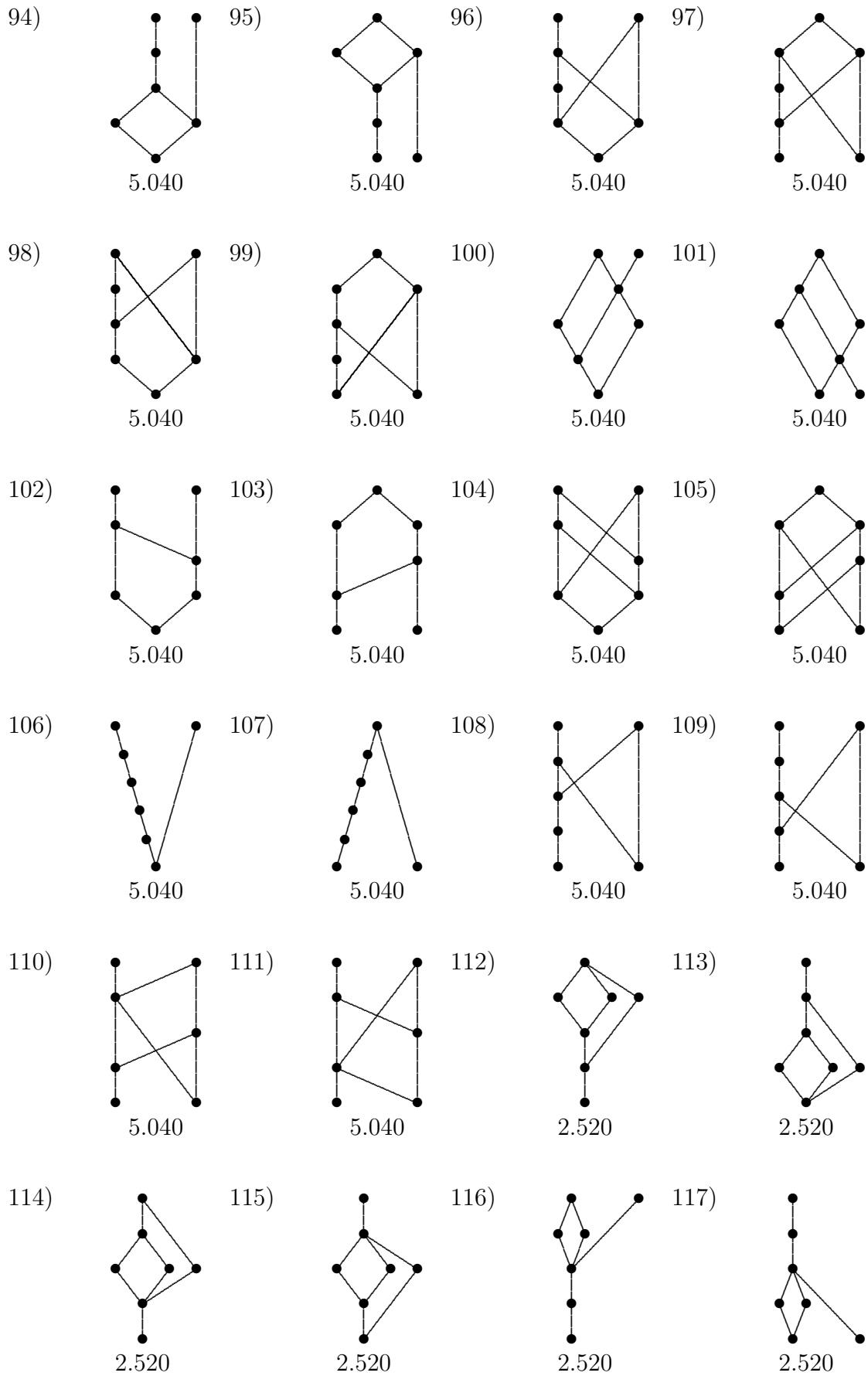


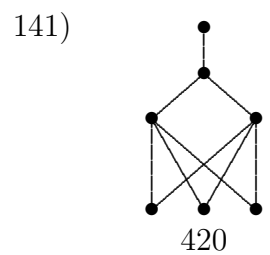
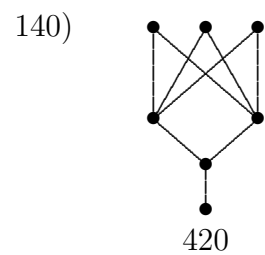
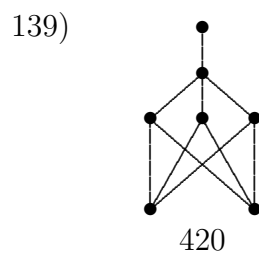
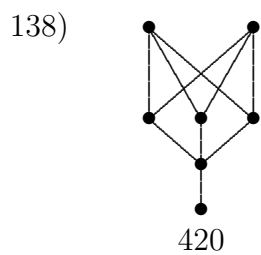
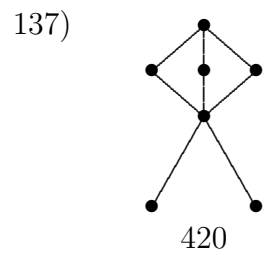
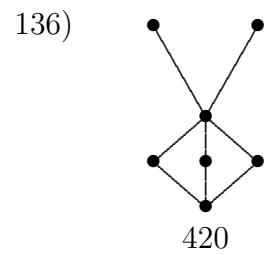
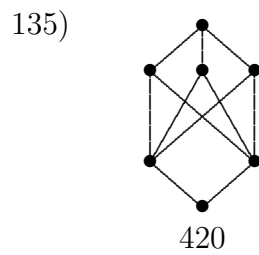
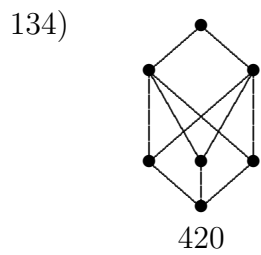
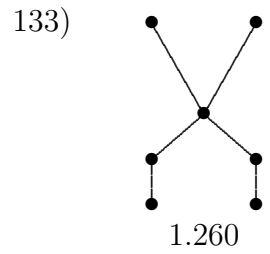
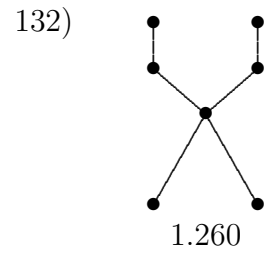
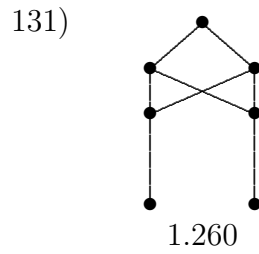
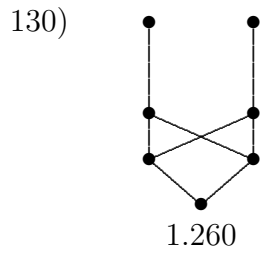
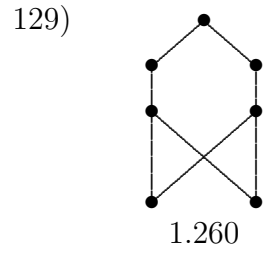
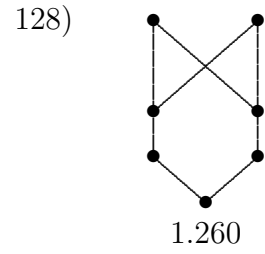
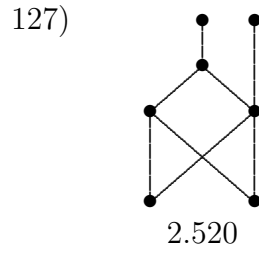
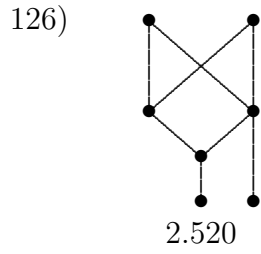
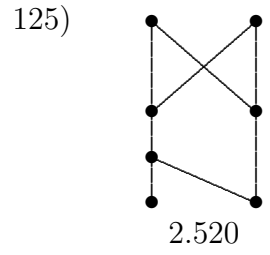
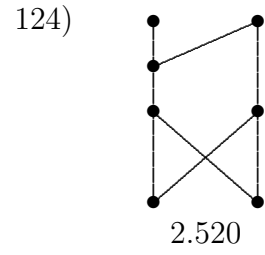
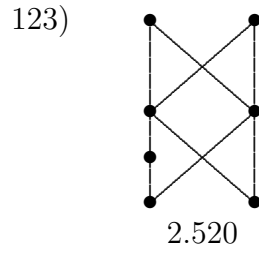
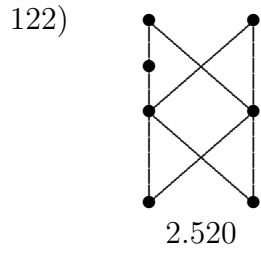
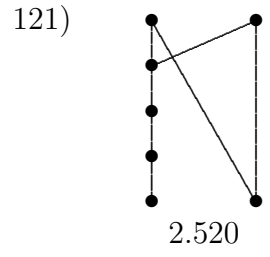
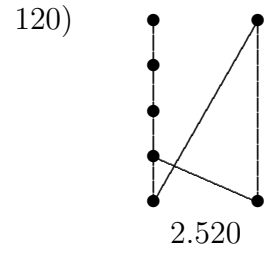
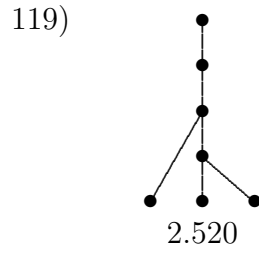
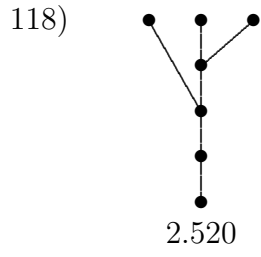
2.520



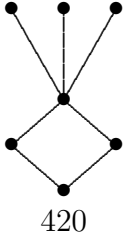
$|RB(\mathbf{7})| = 13$



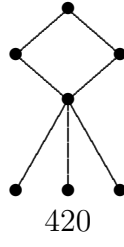




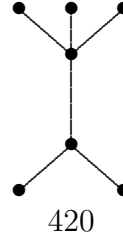
142)



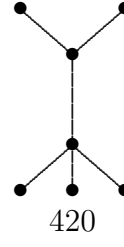
143)



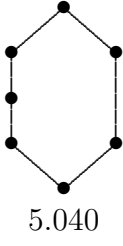
144)



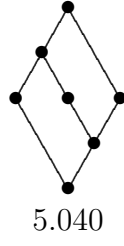
145)

 $|RB(7)| = 14$

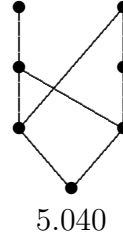
146)



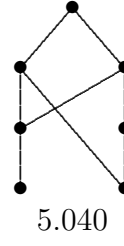
147)



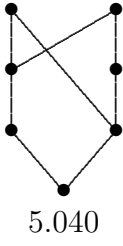
148)



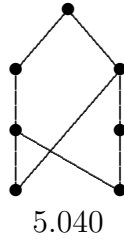
149)



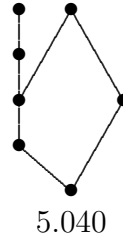
150)



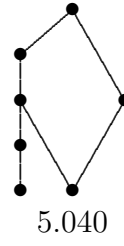
151)



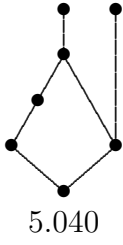
152)



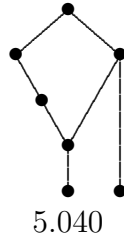
153)



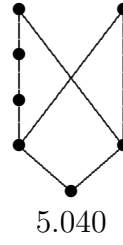
154)



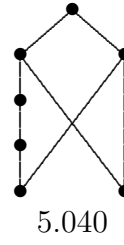
155)



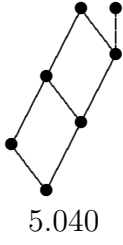
156)



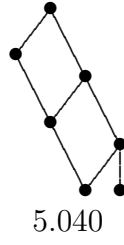
157)



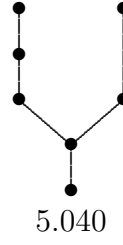
158)



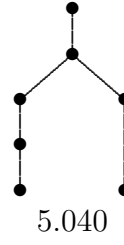
159)



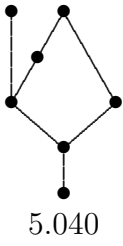
160)



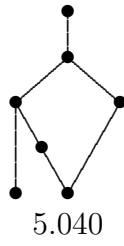
161)



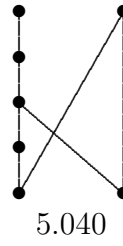
162)



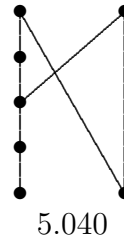
163)



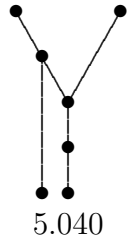
164)



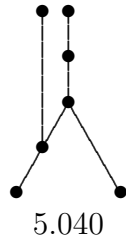
165)



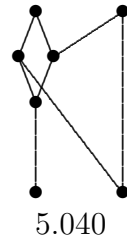
166)



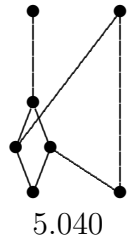
167)



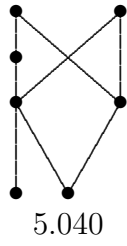
168)



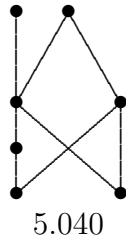
169)



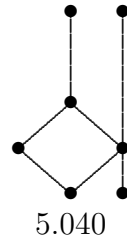
170)



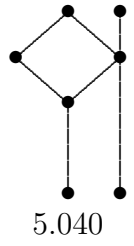
171)



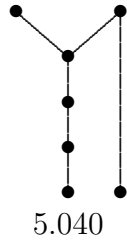
172)



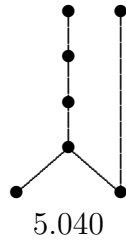
173)



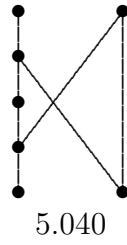
174)



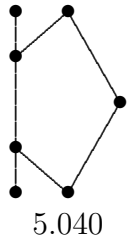
175)



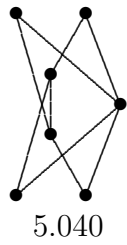
176)



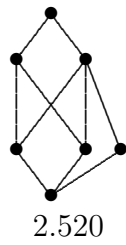
177)



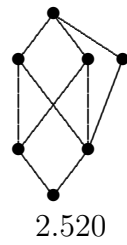
178)



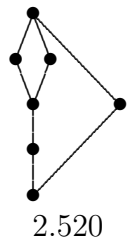
179)



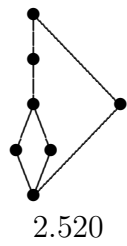
180)



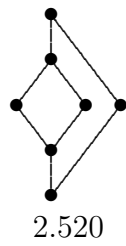
181)



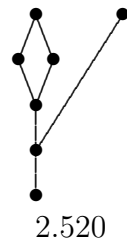
182)



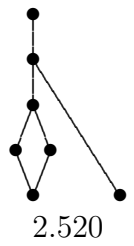
183)



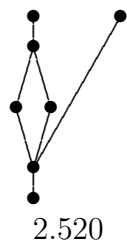
184)



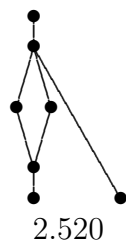
185)



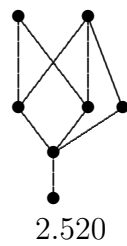
186)



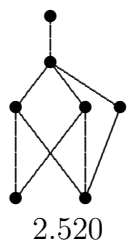
187)

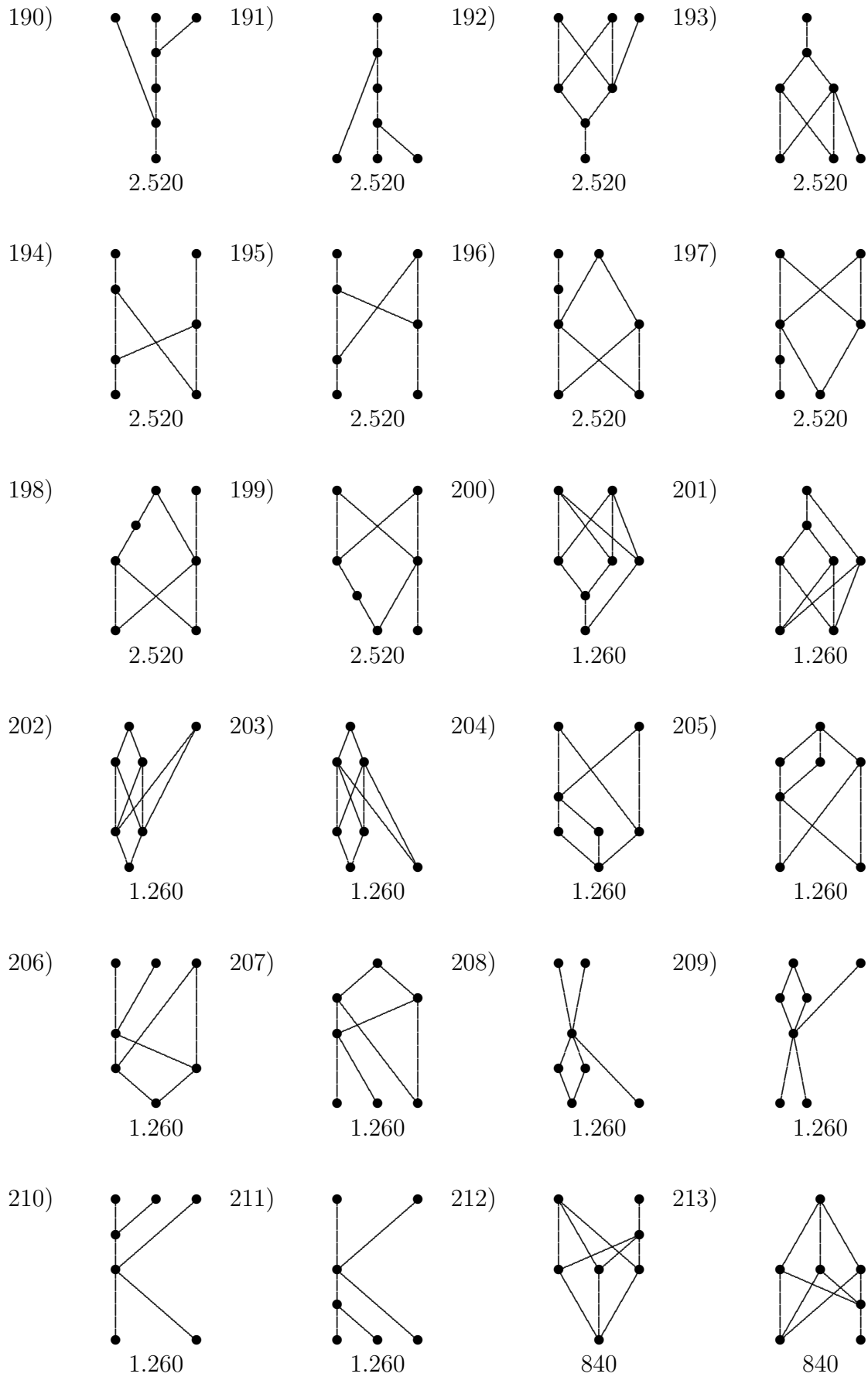


188)

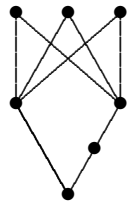


189)



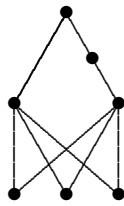


214)



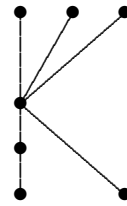
840

215)



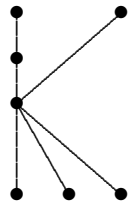
840

216)



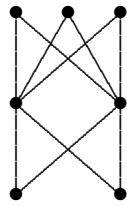
840

217)



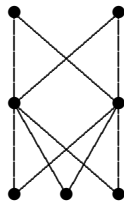
840

218)



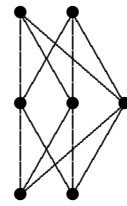
210

219)



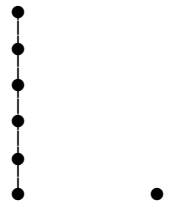
210

220)



210

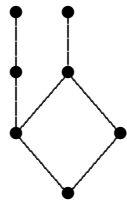
221)



5.040

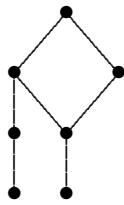
$|RB(\mathbf{7})| = 15$

222)



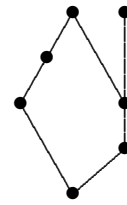
5.040

223)



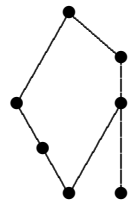
5.040

224)



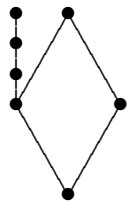
5.040

225)



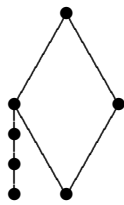
5.040

226)



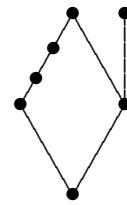
5.040

227)



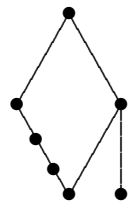
5.040

228)



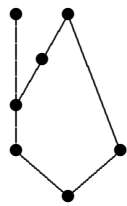
5.040

229)



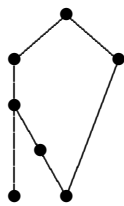
5.040

230)



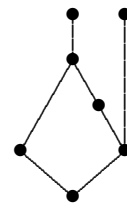
5.040

231)



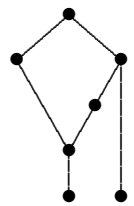
5.040

232)



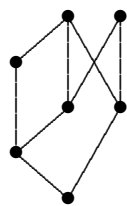
5.040

233)



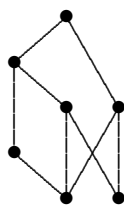
5.040

234)



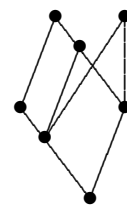
5.040

235)



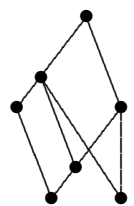
5.040

236)

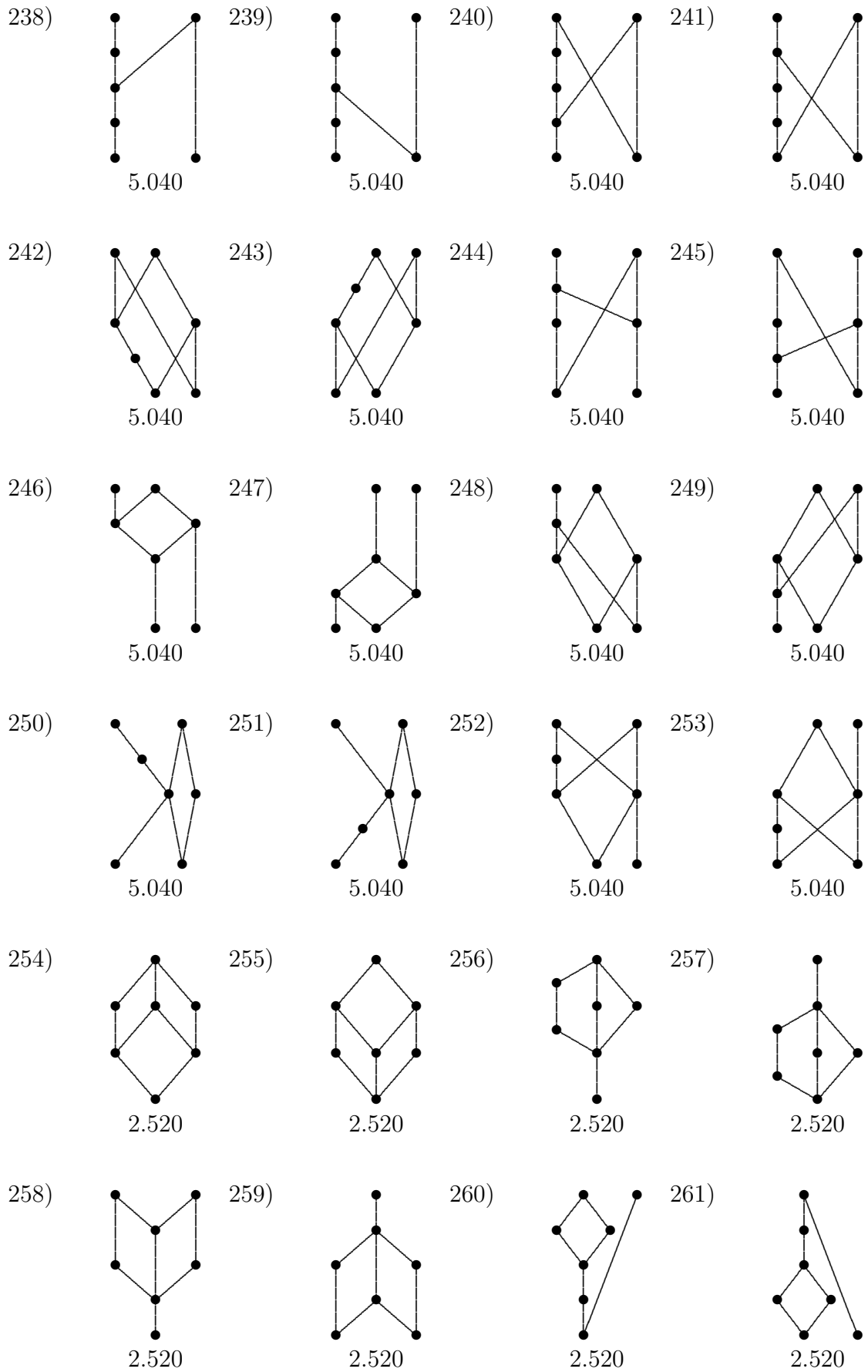


5.040

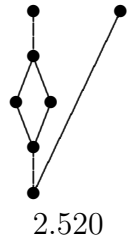
237)



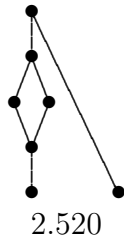
5.040



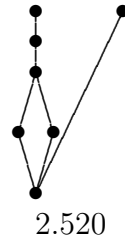
262)



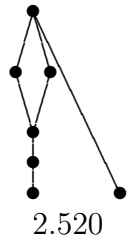
263)



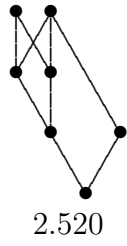
264)



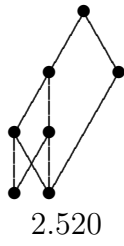
265)



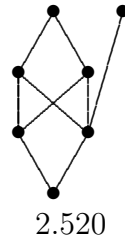
266)



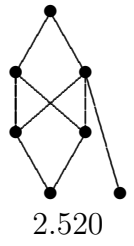
267)



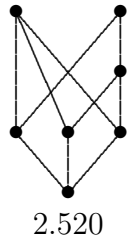
268)



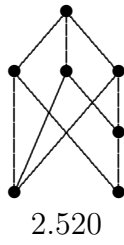
269)



270)



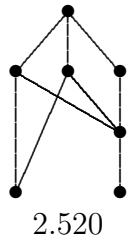
271)



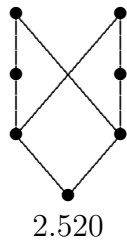
272)



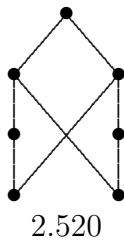
273)



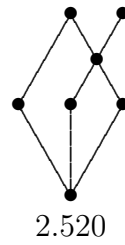
274)



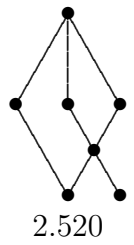
275)



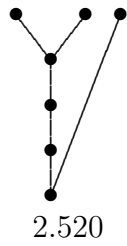
276)



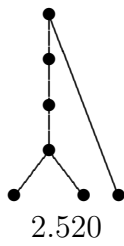
277)



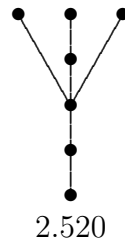
278)



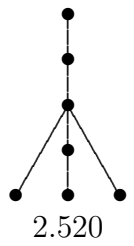
279)



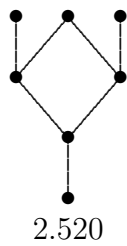
280)



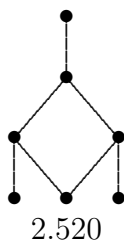
281)



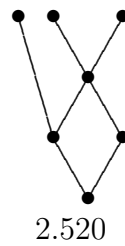
282)



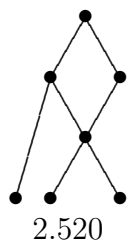
283)

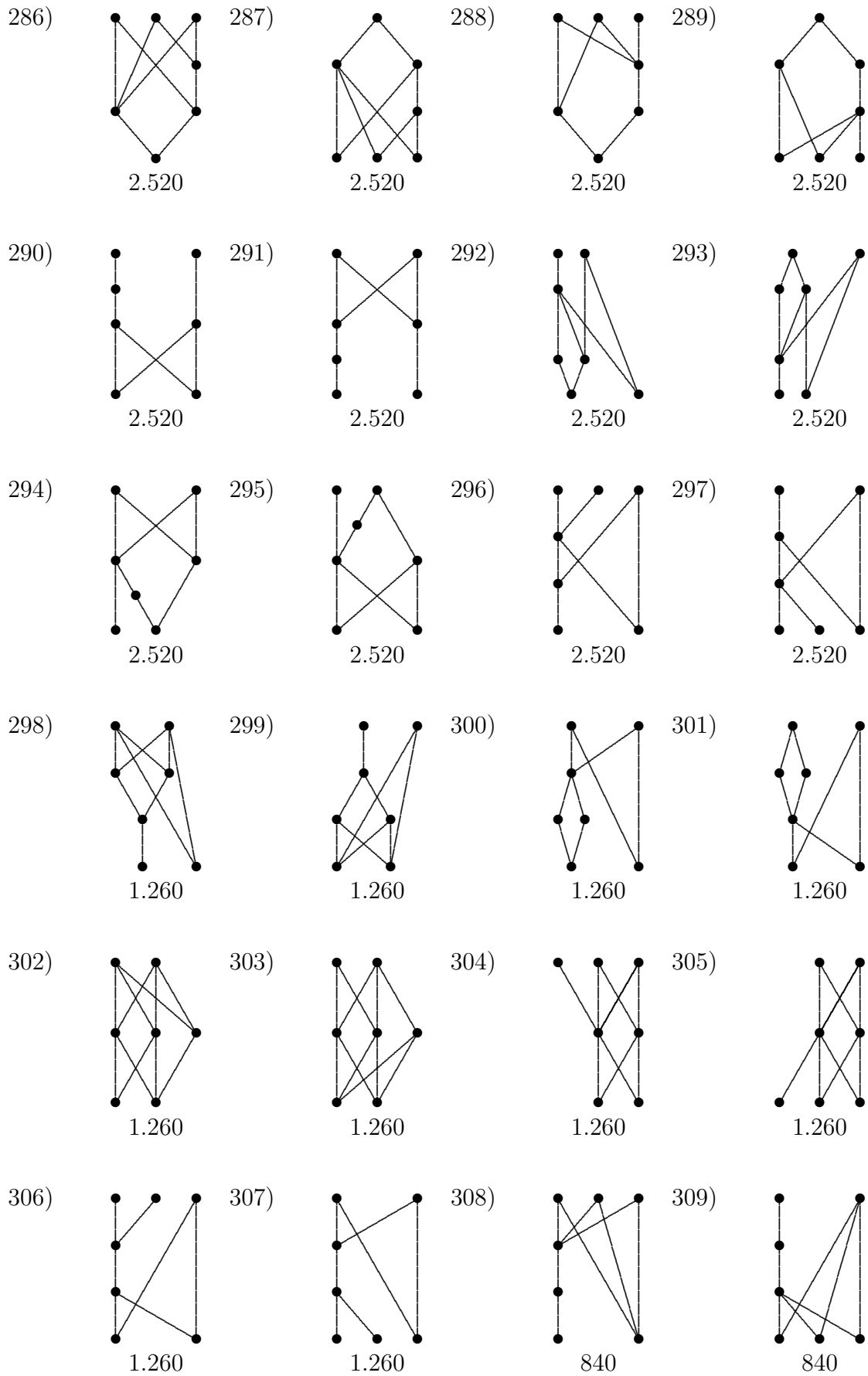


284)



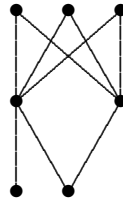
285)





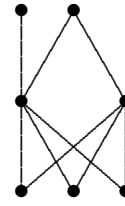
$$|RB(\mathbf{7})| = 16$$

310)



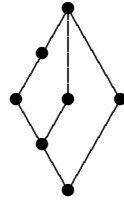
840

311)



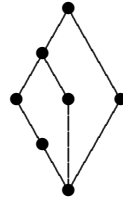
840

312)



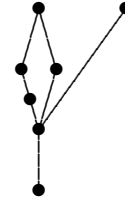
5.040

313)



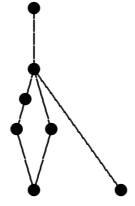
5.040

314)



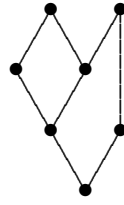
5.040

315)



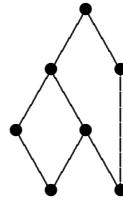
5.040

316)



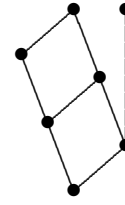
5.040

317)



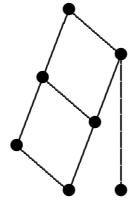
5.040

318)



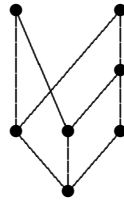
5.040

319)



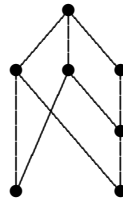
5.040

320)



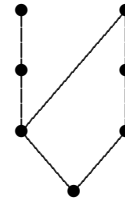
5.040

321)



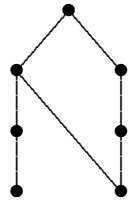
5.040

322)



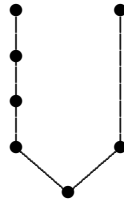
5.040

323)



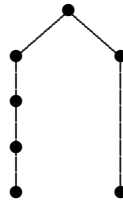
5.040

324)



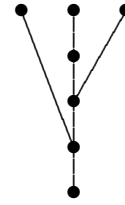
5.040

325)



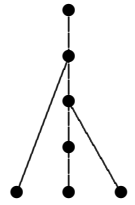
5.040

326)



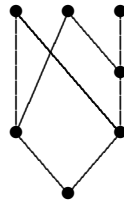
5.040

327)



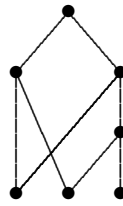
5.040

328)



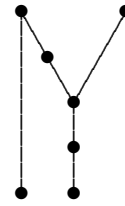
5.040

329)



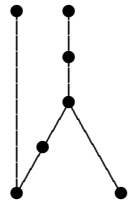
5.040

330)

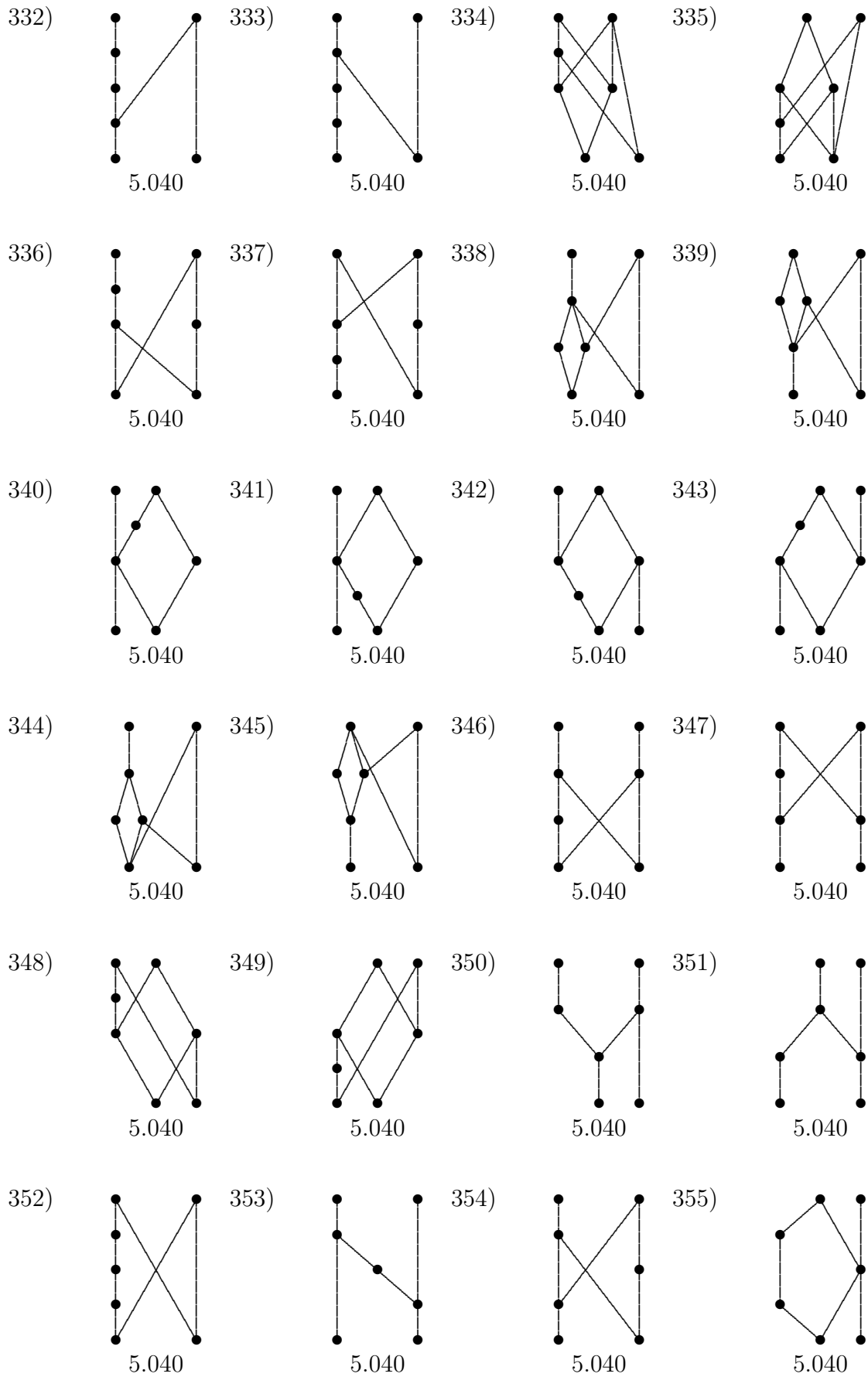


5.040

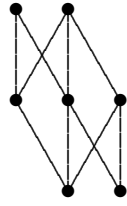
331)



5.040

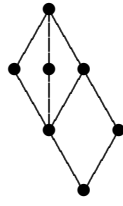


356)



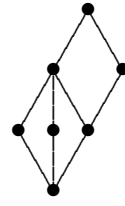
5.040

357)



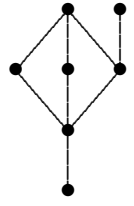
2.520

358)



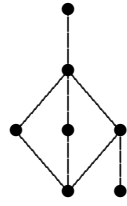
2.520

359)



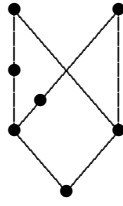
2.520

360)



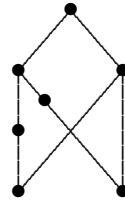
2.520

361)



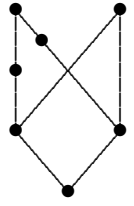
2.520

362)



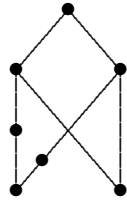
2.520

363)



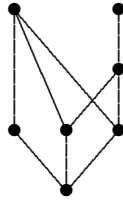
2.520

364)



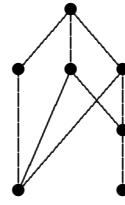
2.520

365)



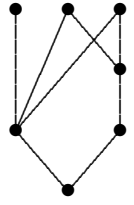
2.520

366)



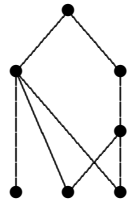
2.520

367)



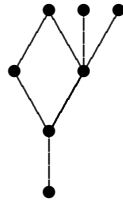
2.520

368)



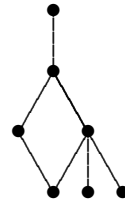
2.520

369)



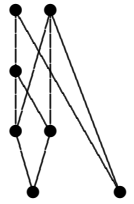
2.520

370)



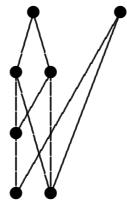
2.520

371)



2.520

372)



2.520

373)



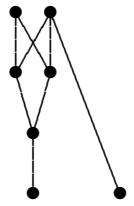
2.520

374)



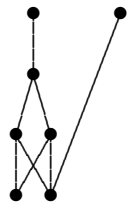
2.520

375)



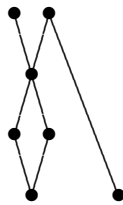
2.520

376)



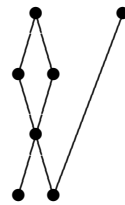
2.520

377)



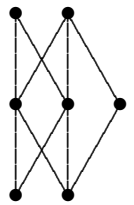
2.520

378)

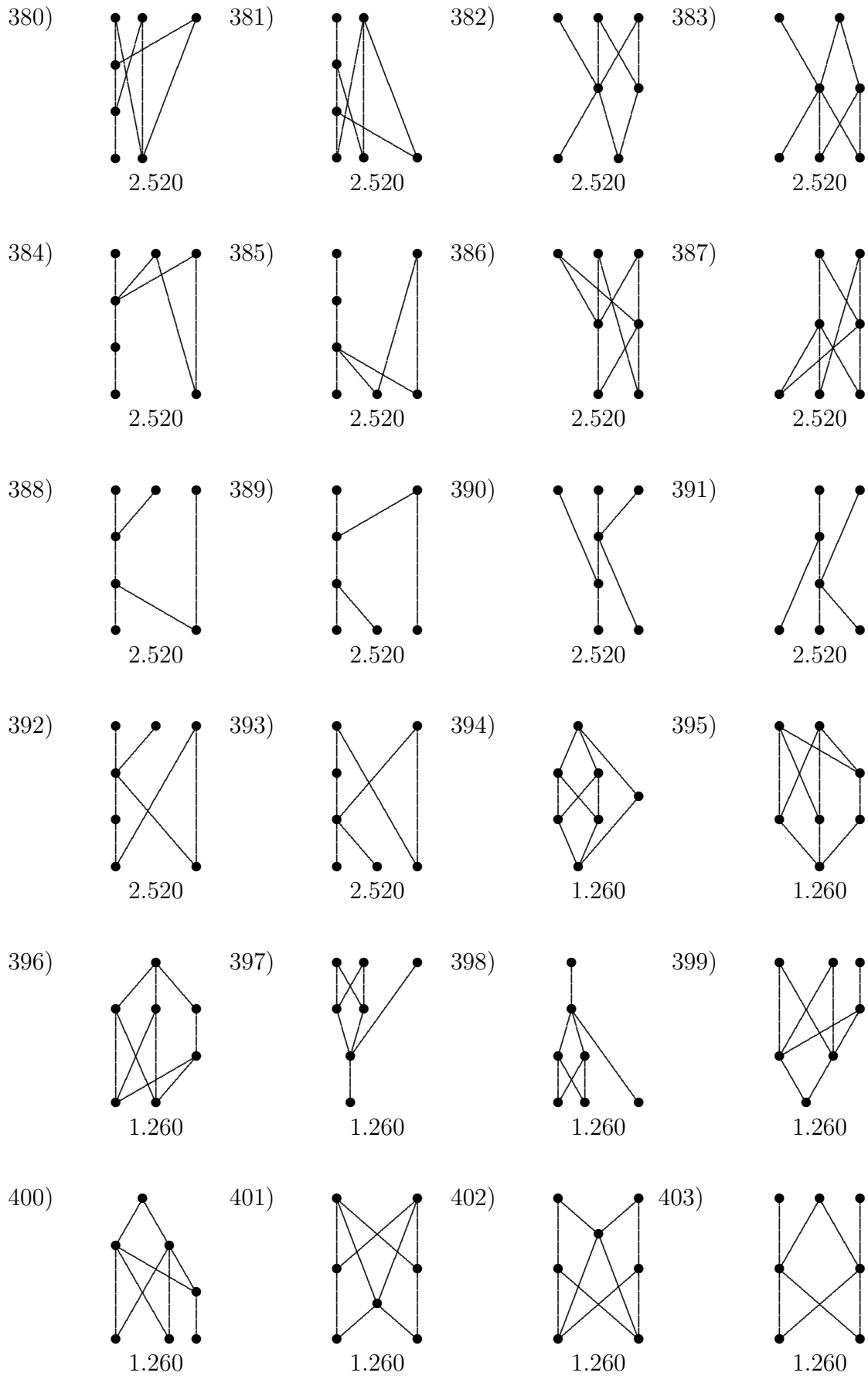


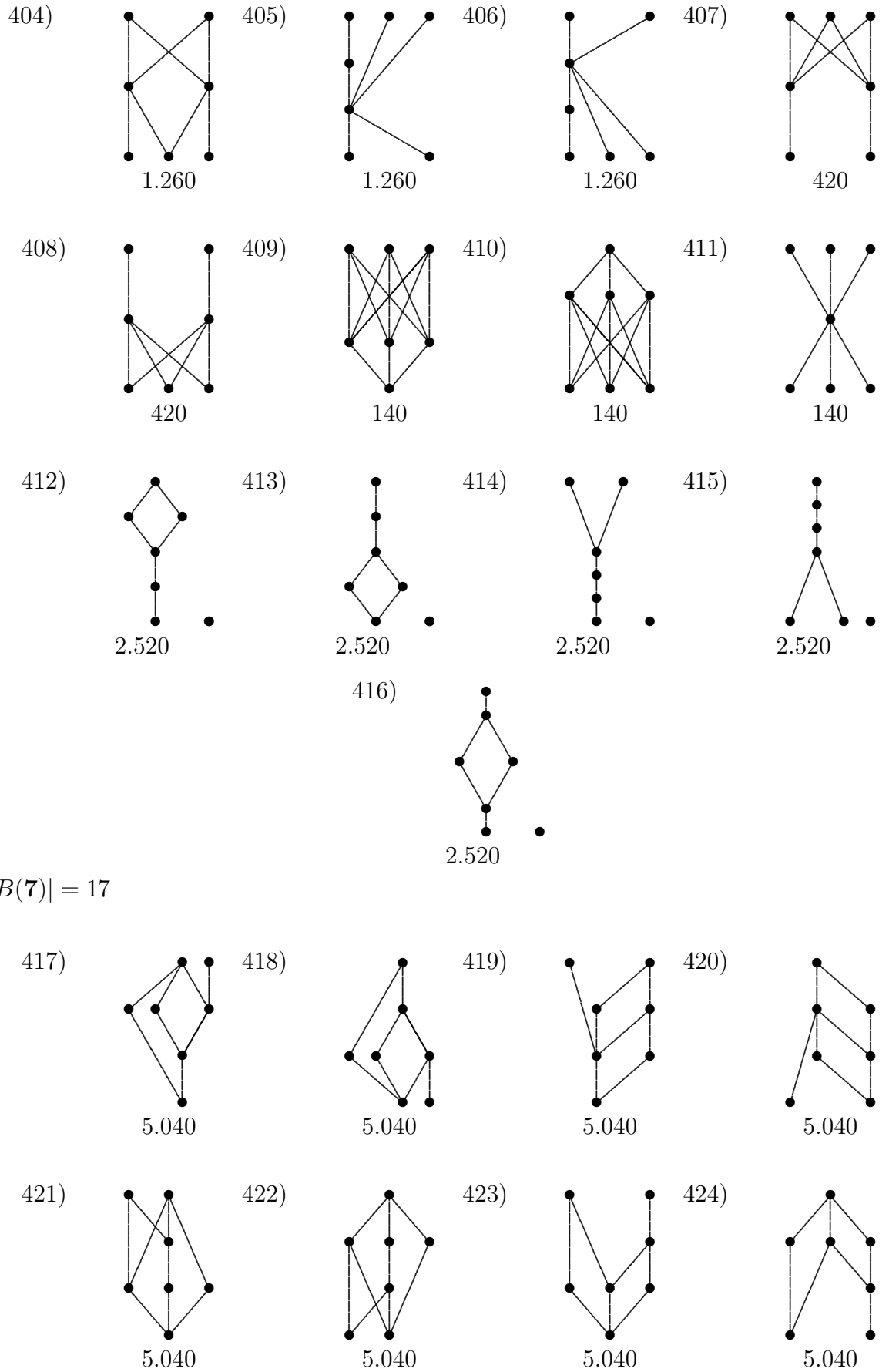
2.520

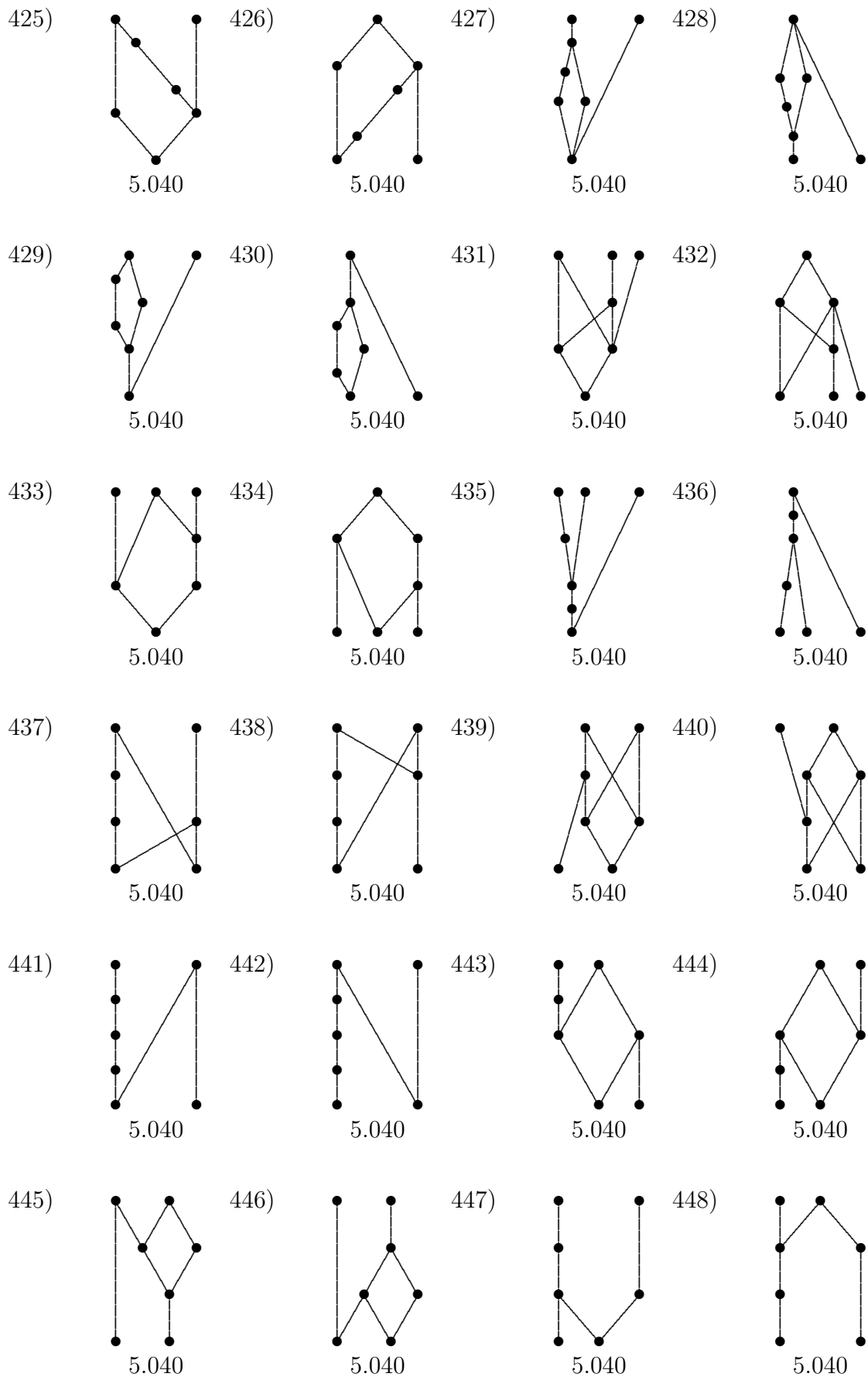
379)



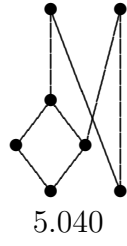
2.520



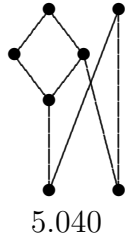




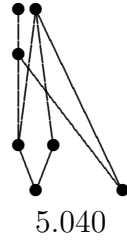
449)



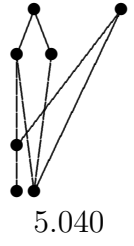
450)



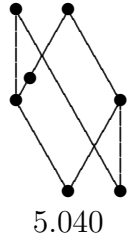
451)



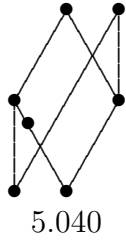
452)



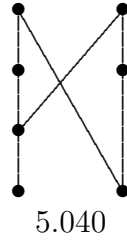
453)



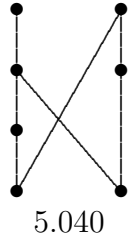
454)



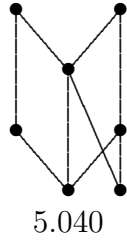
455)



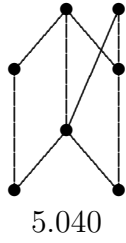
456)



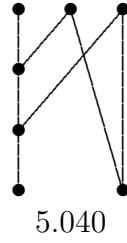
457)



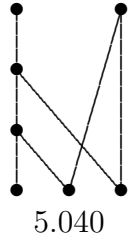
458)



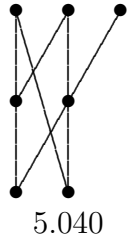
459)



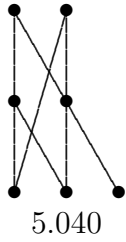
460)



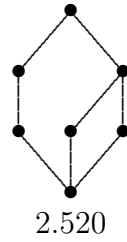
461)



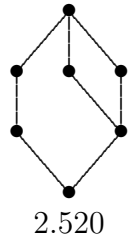
462)



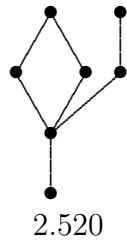
463)



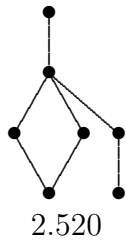
464)



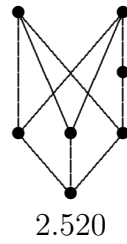
465)



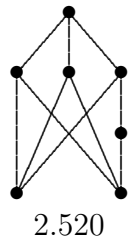
466)



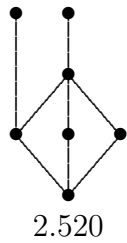
467)



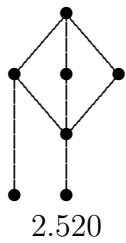
468)



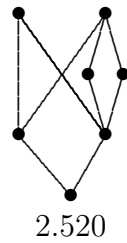
469)



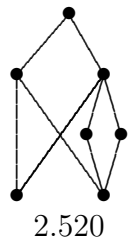
470)

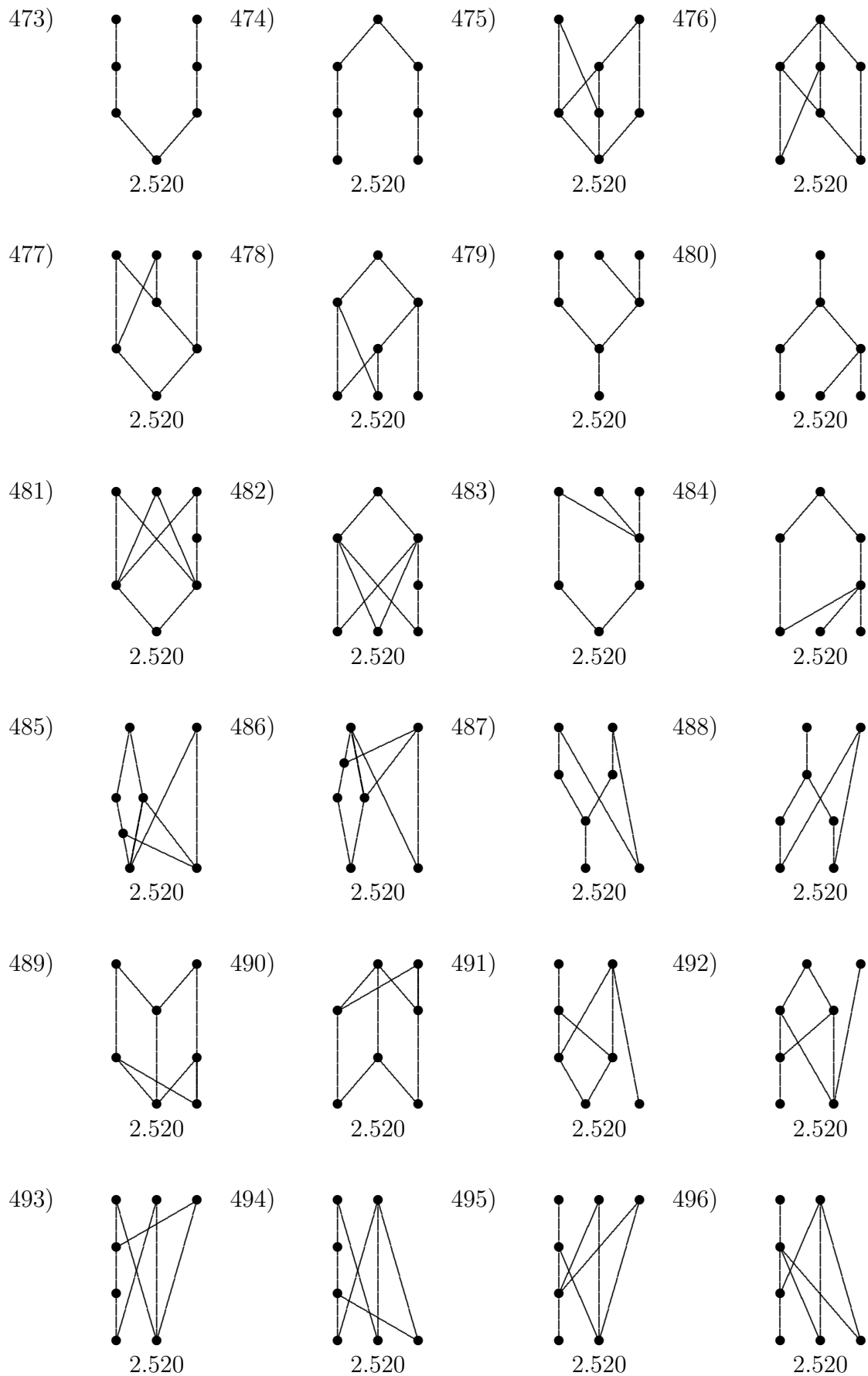


471)

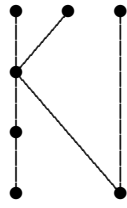


472)



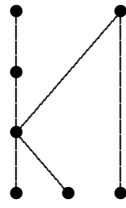


497)



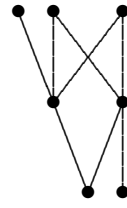
2.520

498)



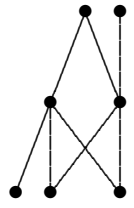
2.520

499)



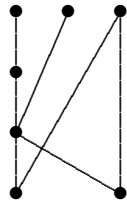
2.520

500)



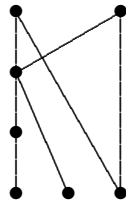
2.520

501)



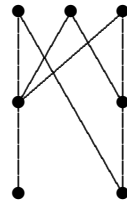
2.520

502)



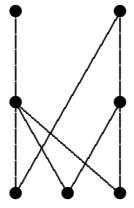
2.520

503)



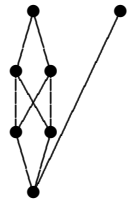
2.520

504)



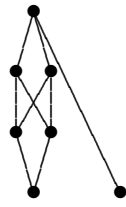
2.520

505)



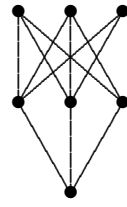
1.260

506)



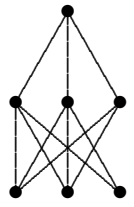
1.260

507)



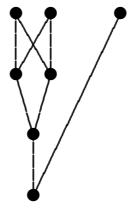
1.260

508)



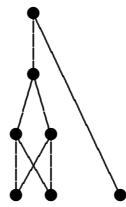
1.260

509)



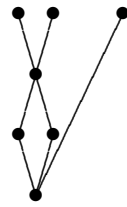
1.260

510)



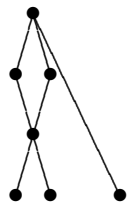
1.260

511)



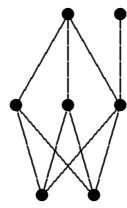
1.260

512)



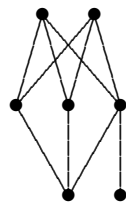
1.260

513)



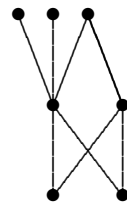
1.260

514)



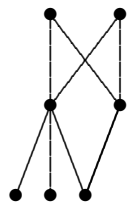
1.260

515)



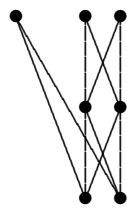
1.260

516)



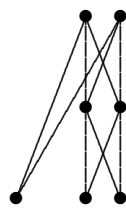
1.260

517)



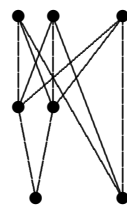
630

518)



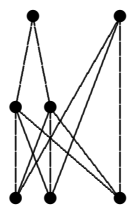
630

519)



420

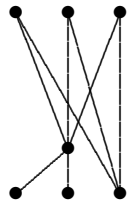
520)



420

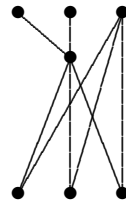
$$|RB(7)| = 18$$

521)



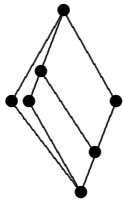
420

522)



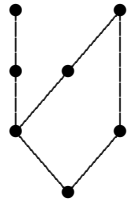
420

523)



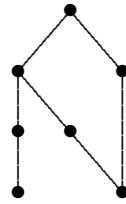
5.040

524)



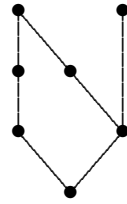
5.040

525)



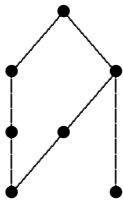
5.040

526)



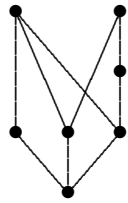
5.040

527)



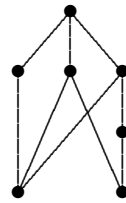
5.040

528)



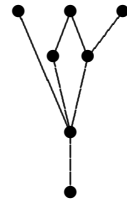
5.040

529)



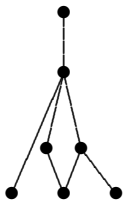
5.040

530)



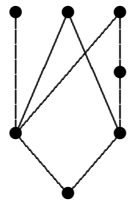
5.040

531)



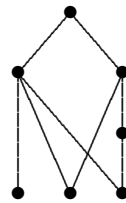
5.040

532)



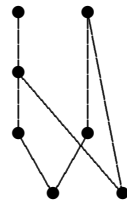
5.040

533)



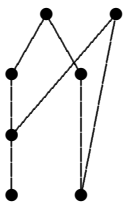
5.040

534)



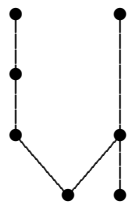
5.040

535)



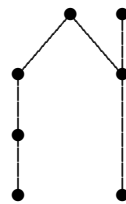
5.040

536)



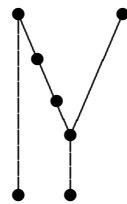
5.040

537)



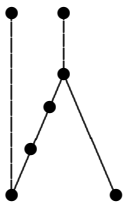
5.040

538)



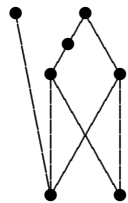
5.040

539)



5.040

540)



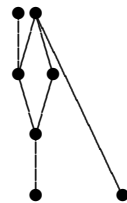
5.040

541)



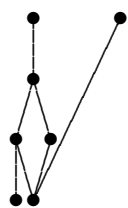
5.040

542)



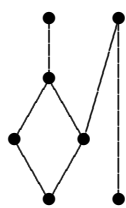
5.040

543)



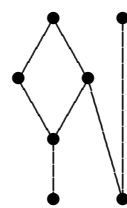
5.040

544)



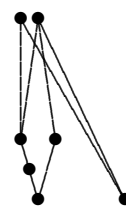
5.040

545)



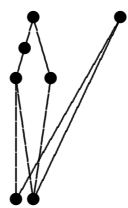
5.040

546)



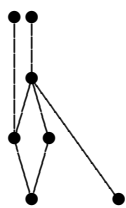
5.040

547)



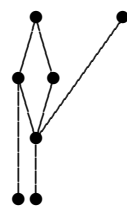
5.040

548)



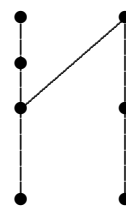
5.040

549)



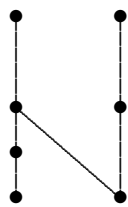
5.040

550)



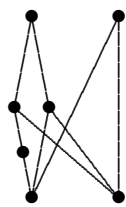
5.040

551)



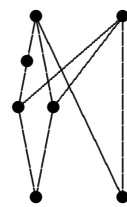
5.040

552)



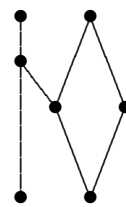
5.040

553)



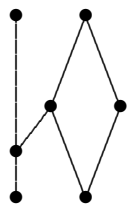
5.040

554)



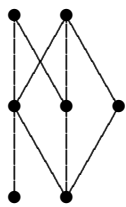
5.040

555)



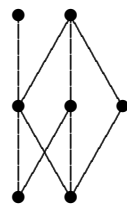
5.040

556)



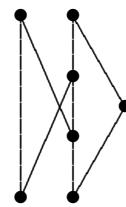
5.040

557)



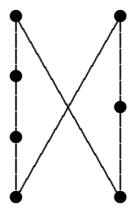
5.040

558)



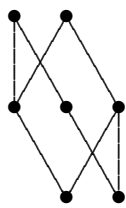
5.040

559)



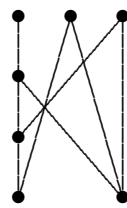
5.040

560)



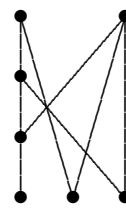
5.040

561)



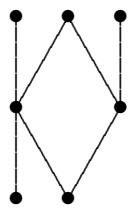
5.040

562)



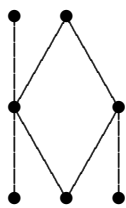
5.040

563)



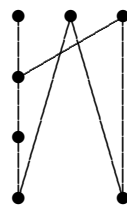
5.040

564)



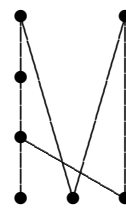
5.040

565)



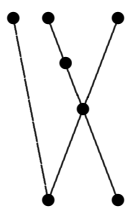
5.040

566)



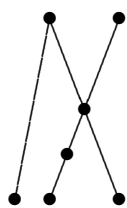
5.040

567)



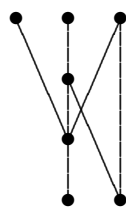
5.040

568)



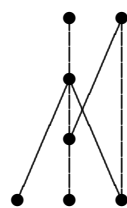
5.040

569)



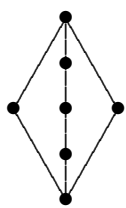
5.040

570)



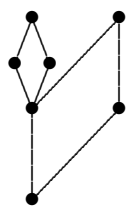
5.040

571)



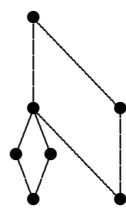
2.520

572)



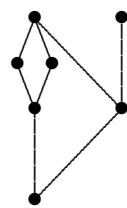
2.520

573)



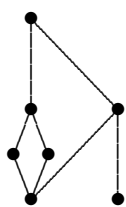
2.520

574)



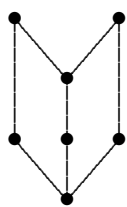
2.520

575)



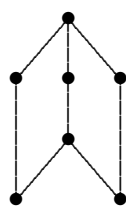
2.520

576)



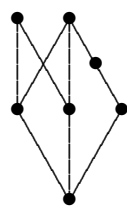
2.520

577)



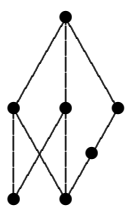
2.520

578)



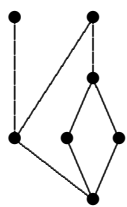
2.520

579)



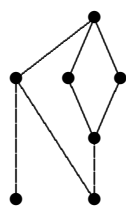
2.520

580)



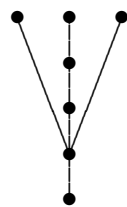
2.520

581)



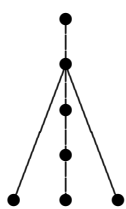
2.520

582)



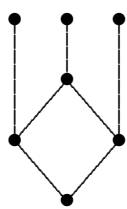
2.520

583)



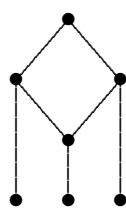
2.520

584)



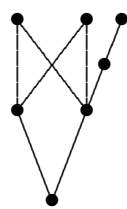
2.520

585)



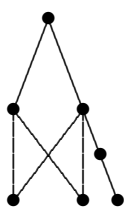
2.520

586)



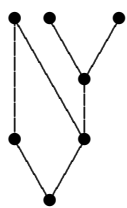
2.520

587)



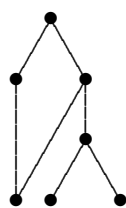
2.520

588)



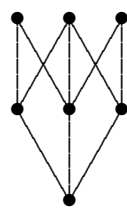
2.520

589)



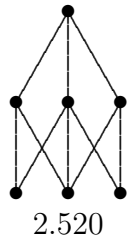
2.520

590)

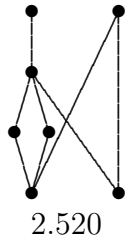


2.520

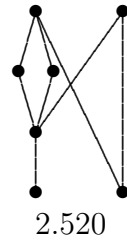
591)



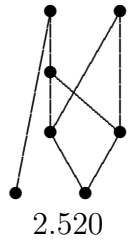
592)



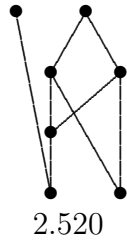
593)



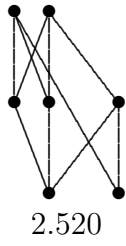
594)



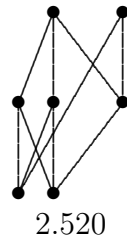
595)



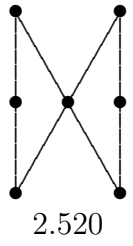
596)



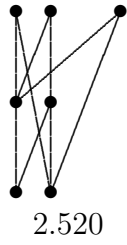
597)



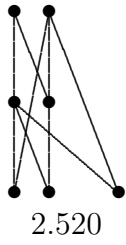
598)



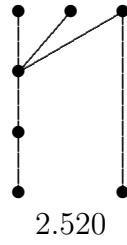
599)



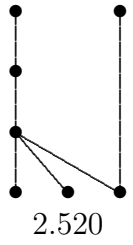
600)



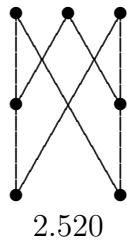
601)



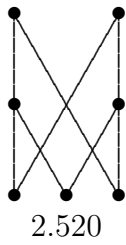
602)



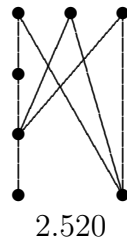
603)



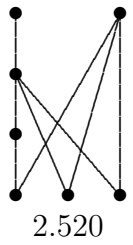
604)



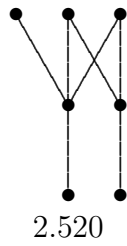
605)



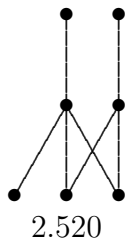
606)



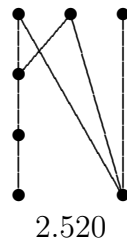
607)



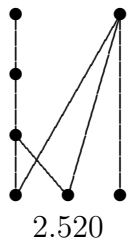
608)



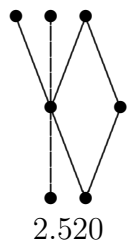
609)



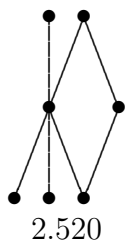
610)



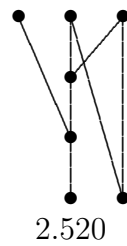
611)



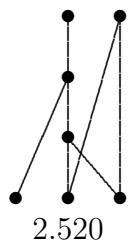
612)

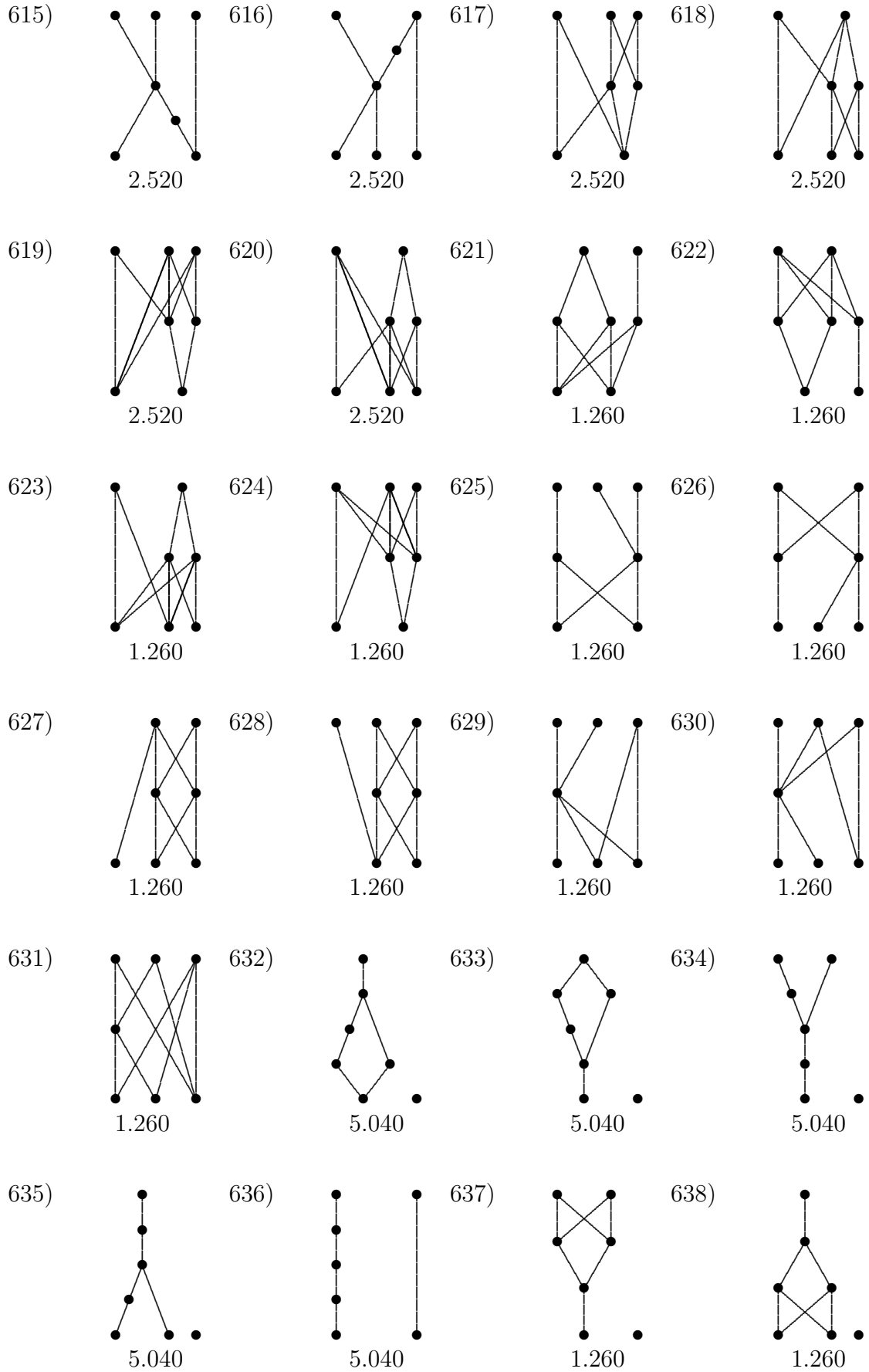


613)

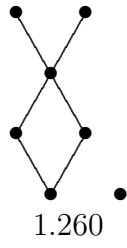


614)

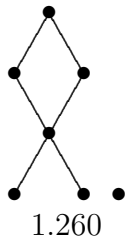




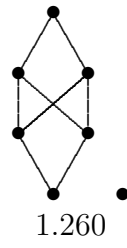
639)



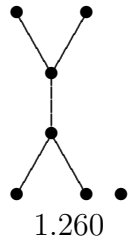
640)



641)

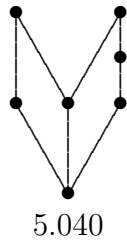


642)

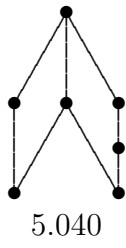


$|RB(\mathbf{7})| = 19$

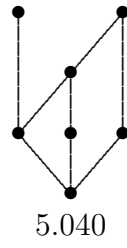
643)



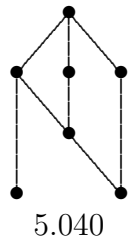
644)



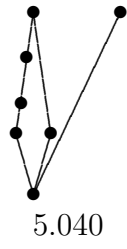
645)



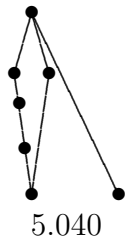
646)



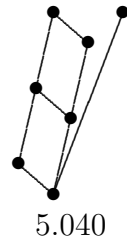
647)



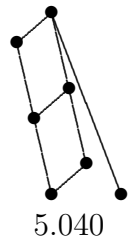
648)



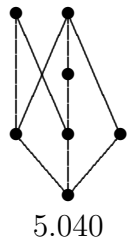
649)



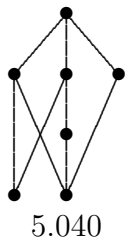
650)



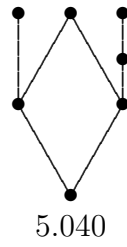
651)



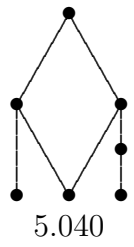
652)



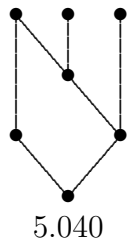
653)



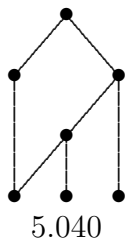
654)



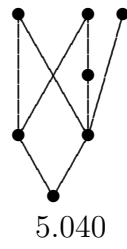
655)



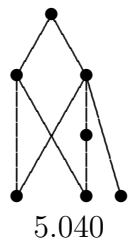
656)



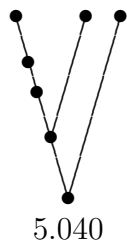
657)



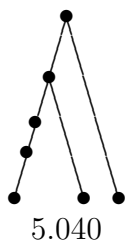
658)



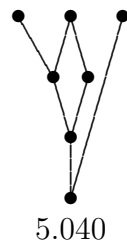
659)



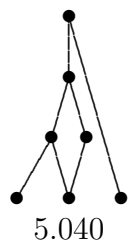
660)

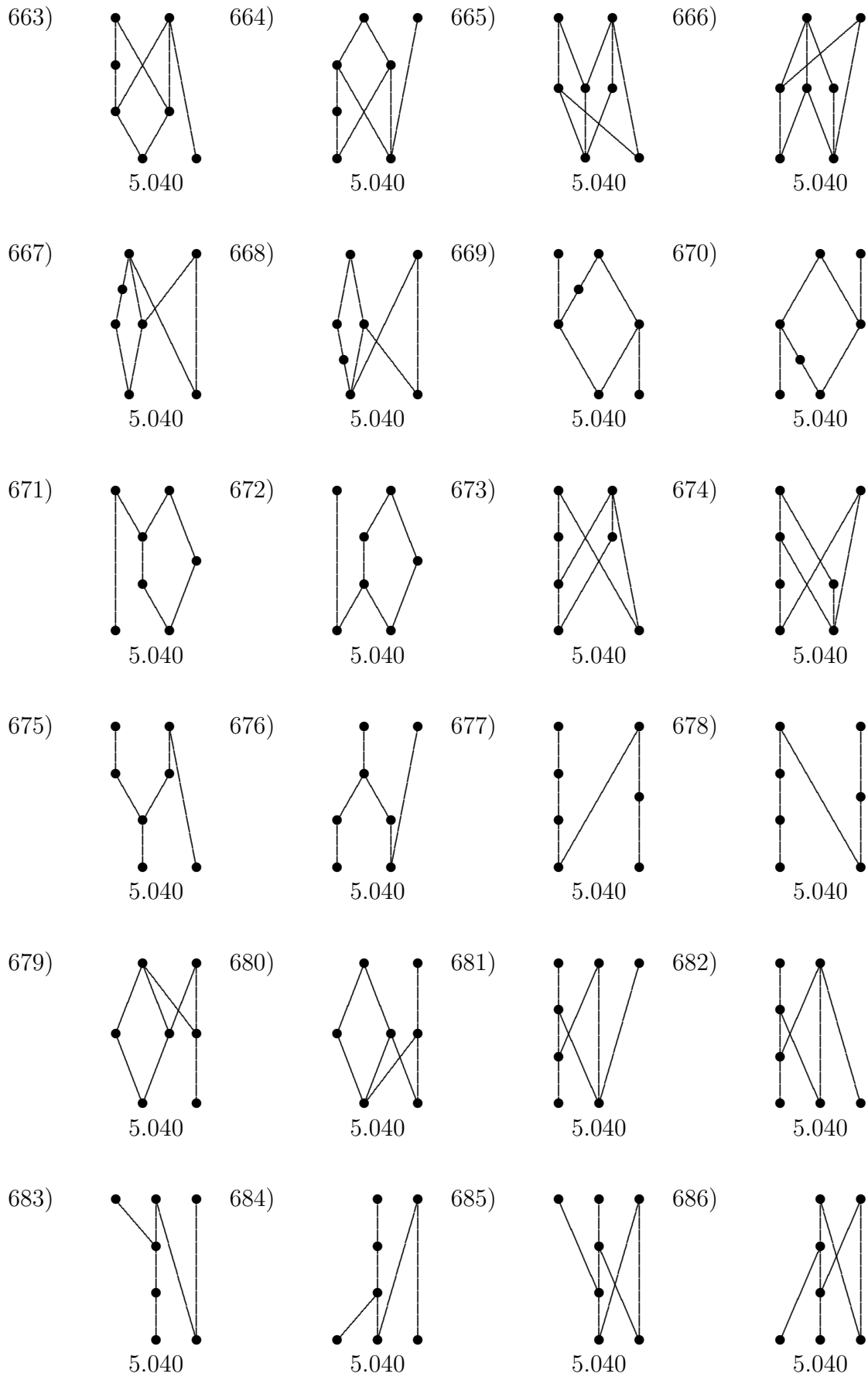


661)

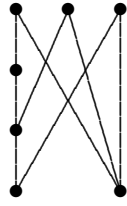


662)



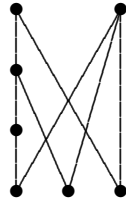


687)



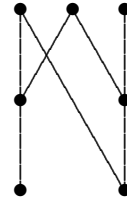
5.040

688)



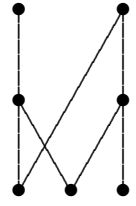
5.040

689)



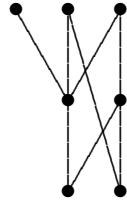
5.040

690)



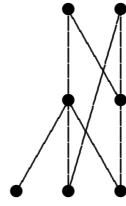
5.040

691)



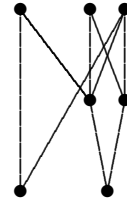
5.040

692)



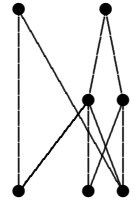
5.040

693)



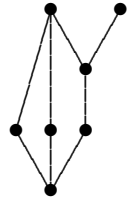
5.040

694)



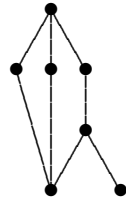
5.040

695)



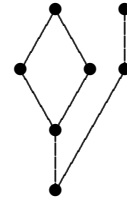
2.520

696)



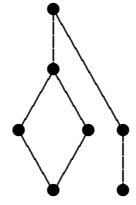
2.520

697)



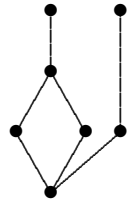
2.520

698)



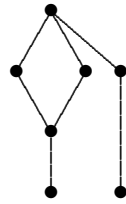
2.520

699)



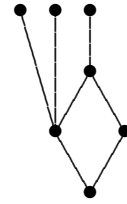
2.520

700)



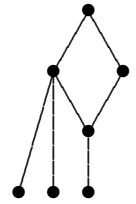
2.520

701)



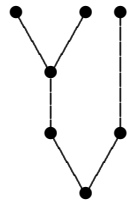
2.520

702)



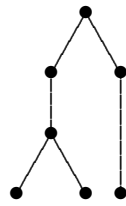
2.520

703)



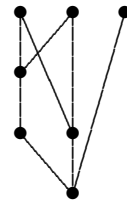
2.520

704)



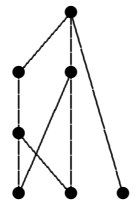
2.520

705)



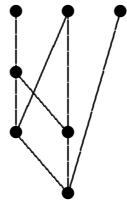
2.520

706)



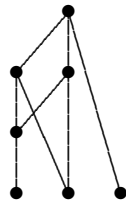
2.520

707)



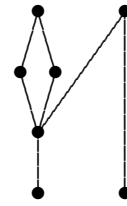
2.520

708)



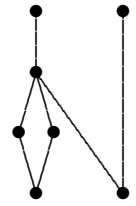
2.520

709)

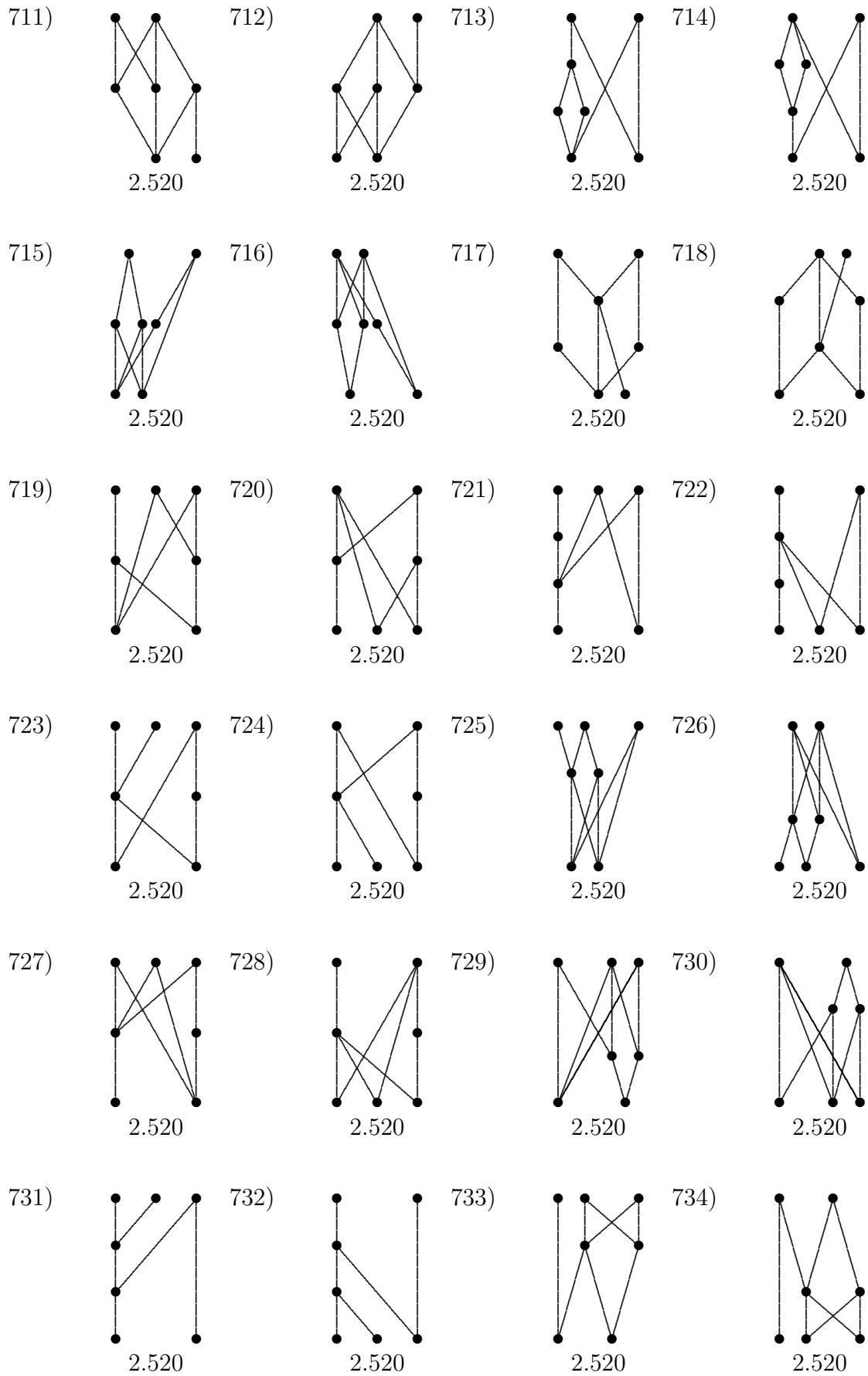


2.520

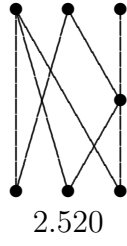
710)



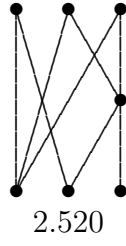
2.520



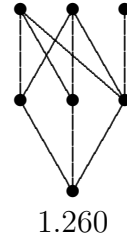
735)



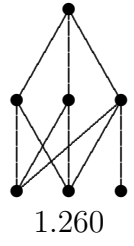
736)



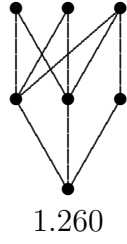
737)



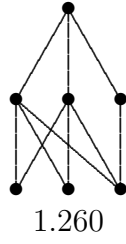
738)



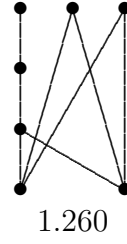
739)



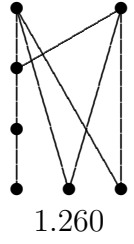
740)



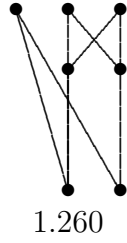
741)



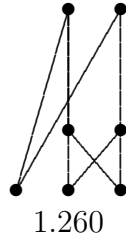
742)



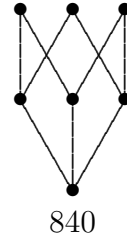
743)



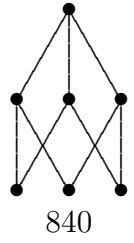
744)



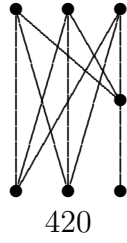
745)



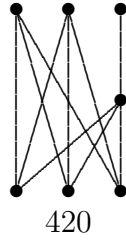
746)



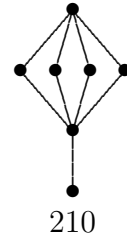
747)



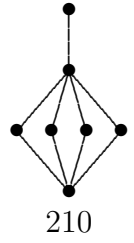
748)



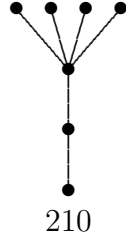
749)



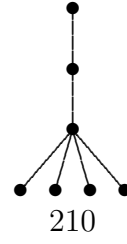
750)



751)

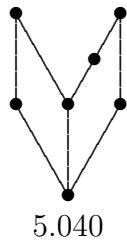


752)

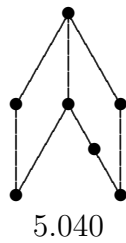


$|RB(7)| = 20$

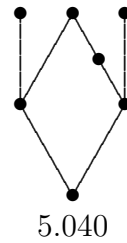
753)



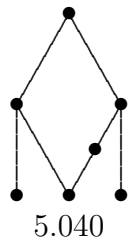
754)

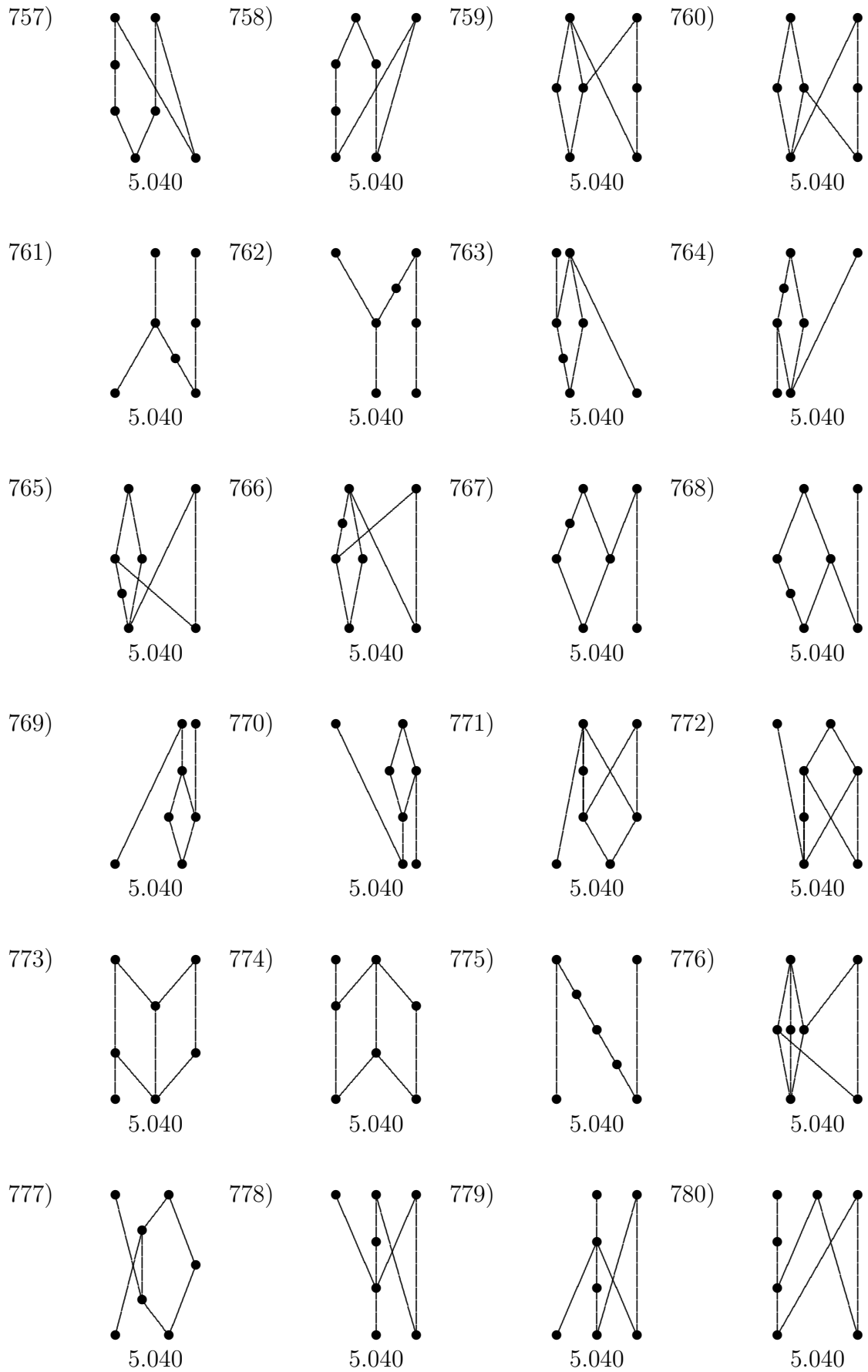


755)

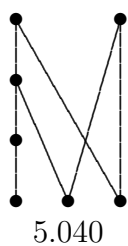


756)

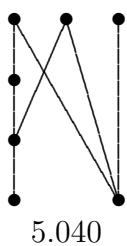




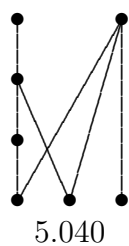
781)



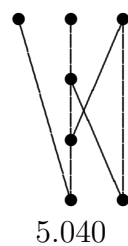
782)



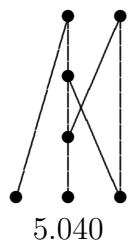
783)



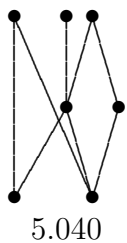
784)



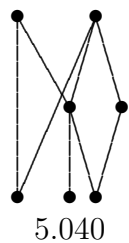
785)



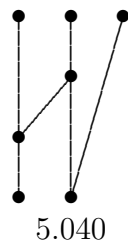
786)



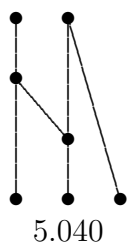
787)



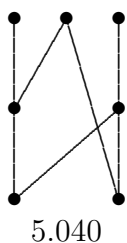
788)



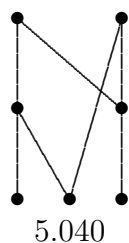
789)



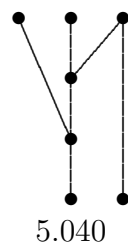
790)



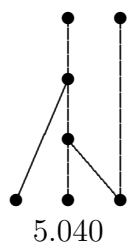
791)



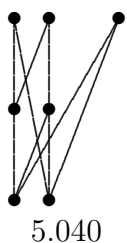
792)



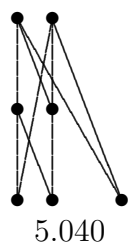
793)



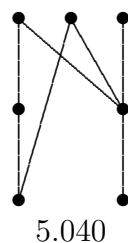
794)



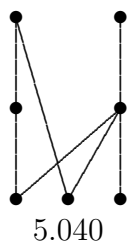
795)



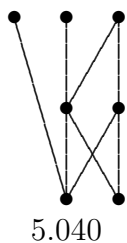
796)



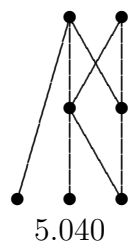
797)



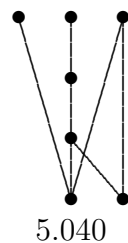
798)



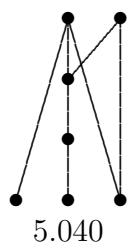
799)



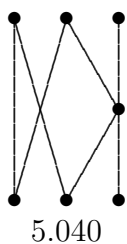
800)



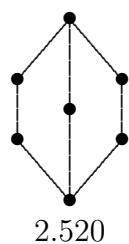
801)



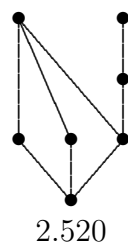
802)



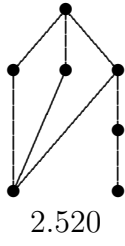
803)



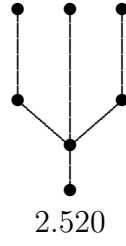
804)



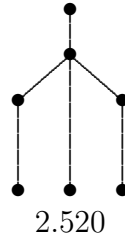
805)



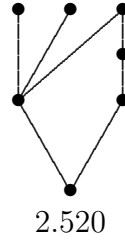
806)



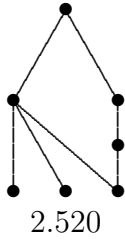
807)



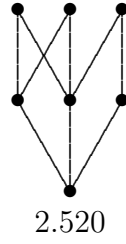
808)



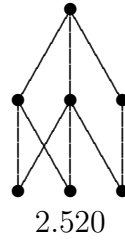
809)



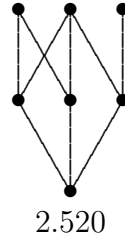
810)



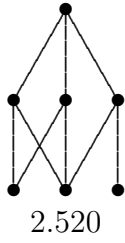
811)



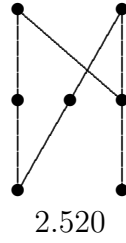
812)



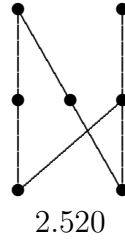
813)



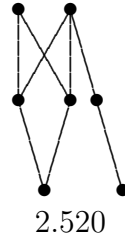
814)



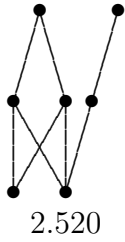
815)



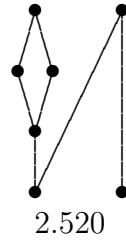
816)



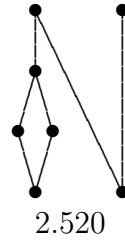
817)



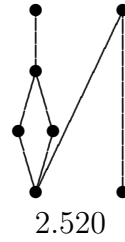
818)



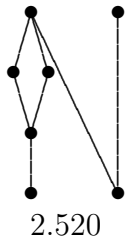
819)



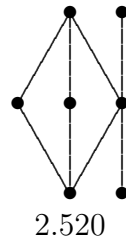
820)



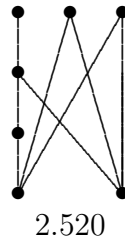
821)



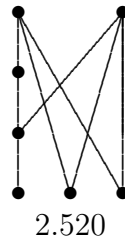
822)



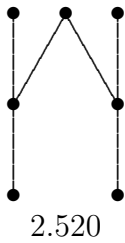
823)



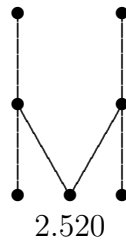
824)



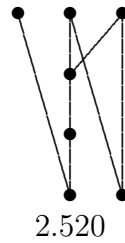
825)



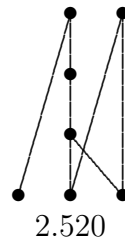
826)



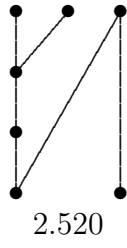
827)



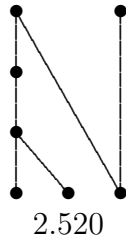
828)



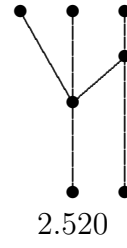
829)



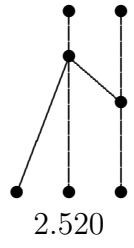
830)



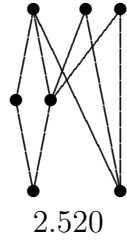
831)



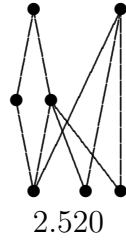
832)



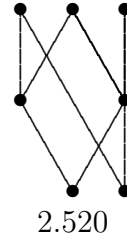
833)



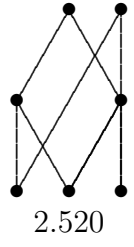
834)



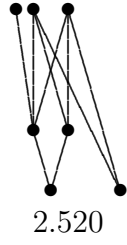
835)



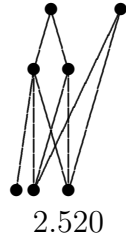
836)



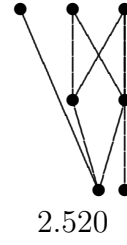
837)



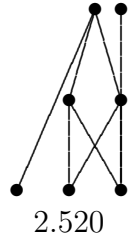
838)



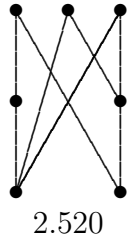
839)



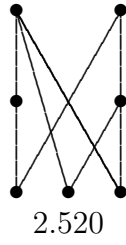
840)



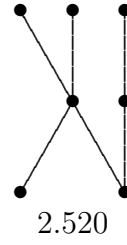
841)



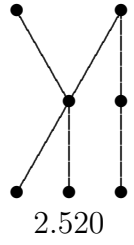
842)



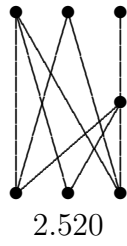
843)



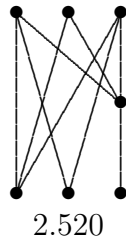
844)



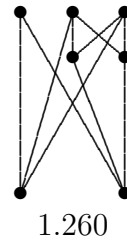
845)



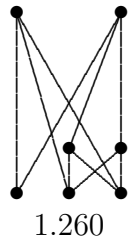
846)



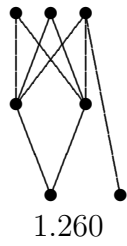
847)



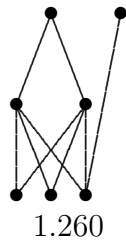
848)



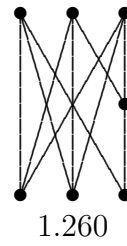
849)



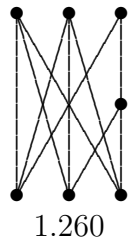
850)

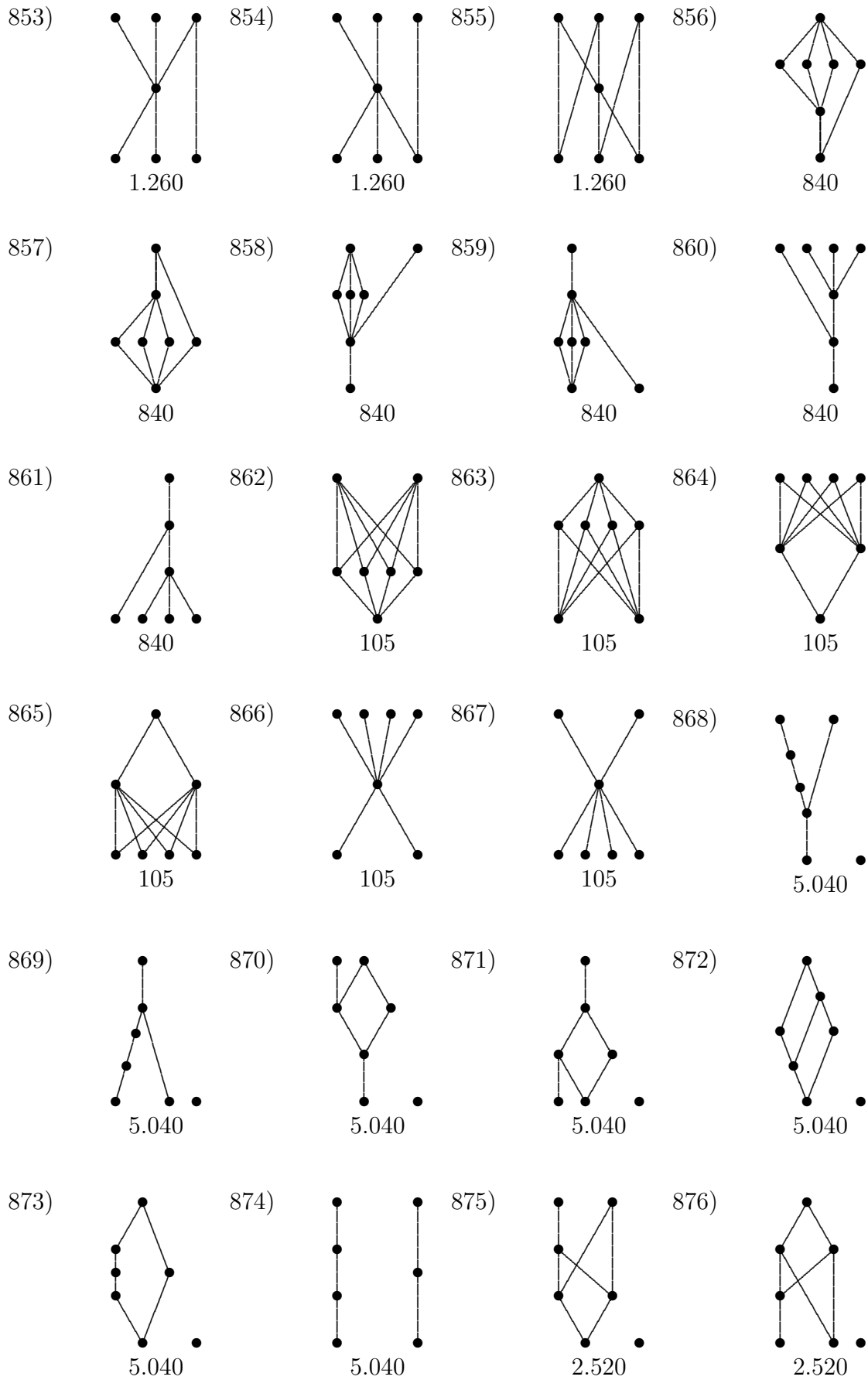


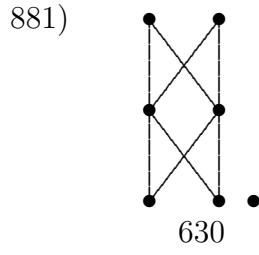
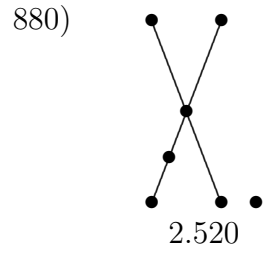
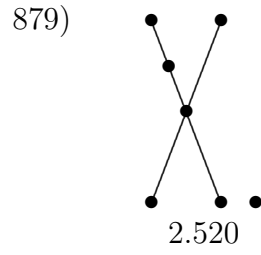
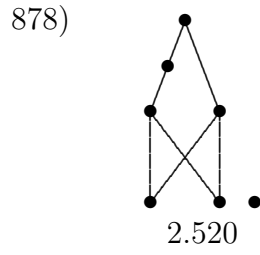
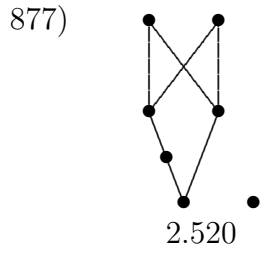
851)



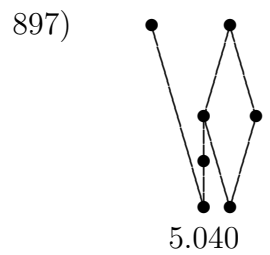
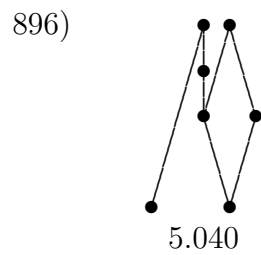
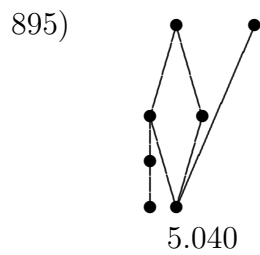
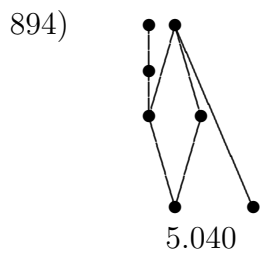
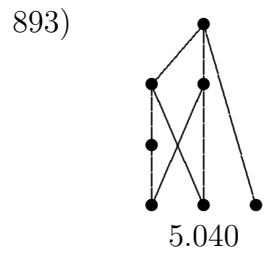
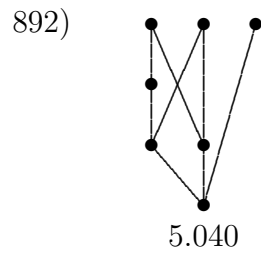
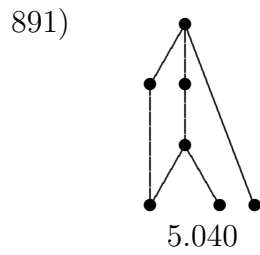
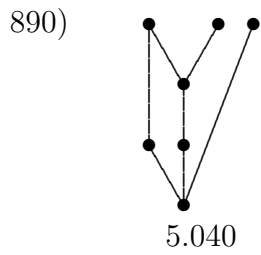
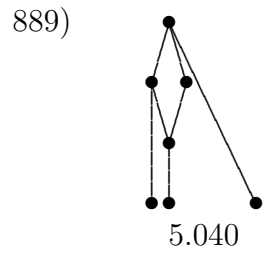
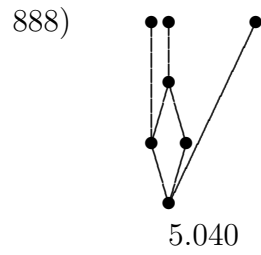
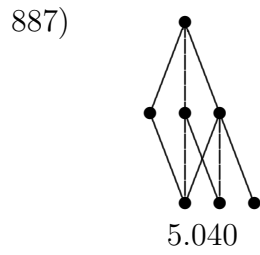
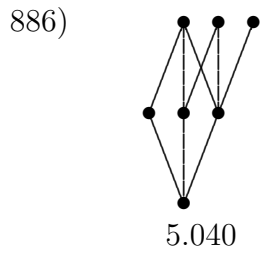
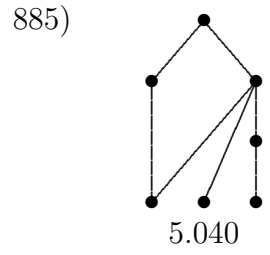
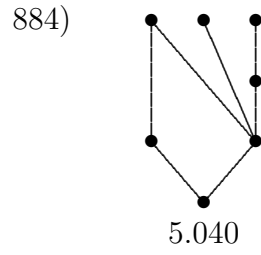
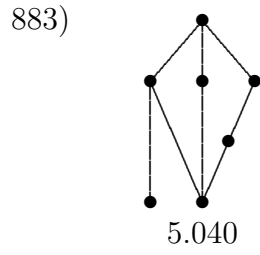
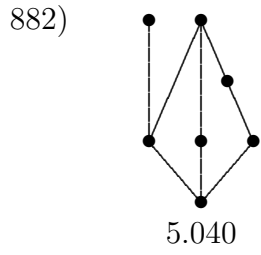
852)

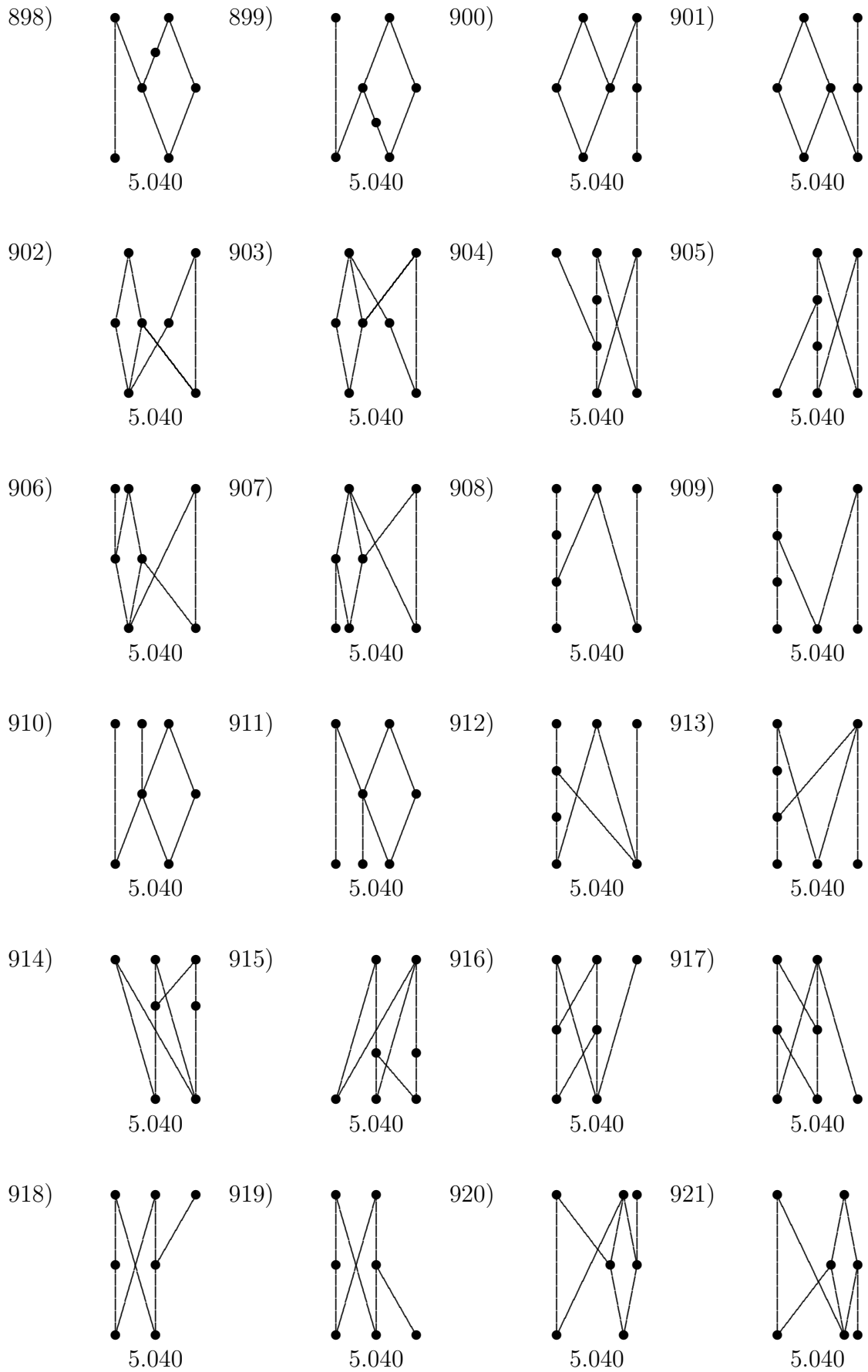




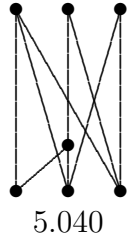


$|RB(\mathbf{7})| = 21$

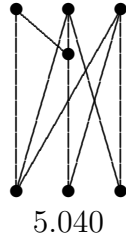




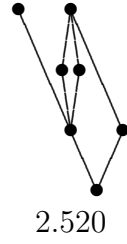
922)



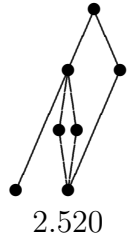
923)



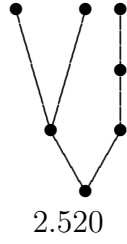
924)



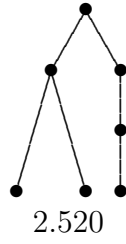
925)



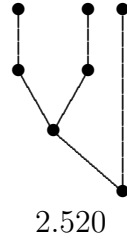
926)



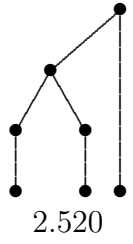
927)



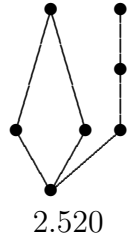
928)



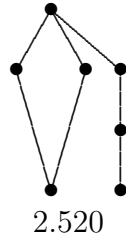
929)



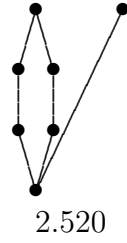
930)



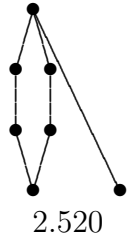
931)



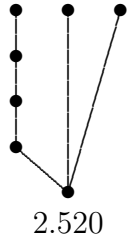
932)



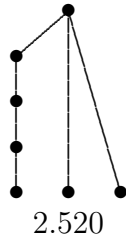
933)



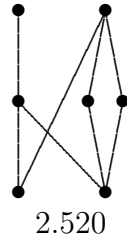
934)



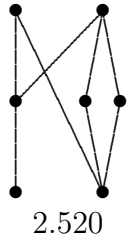
935)



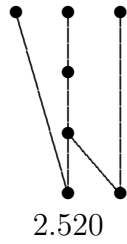
936)



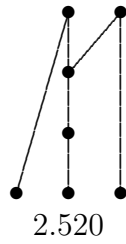
937)



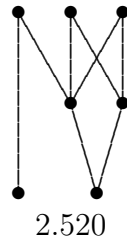
938)



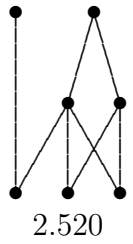
939)



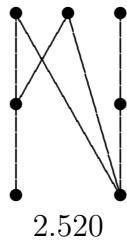
940)



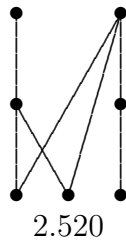
941)



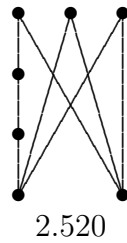
942)



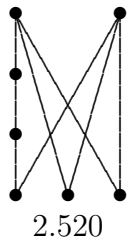
943)

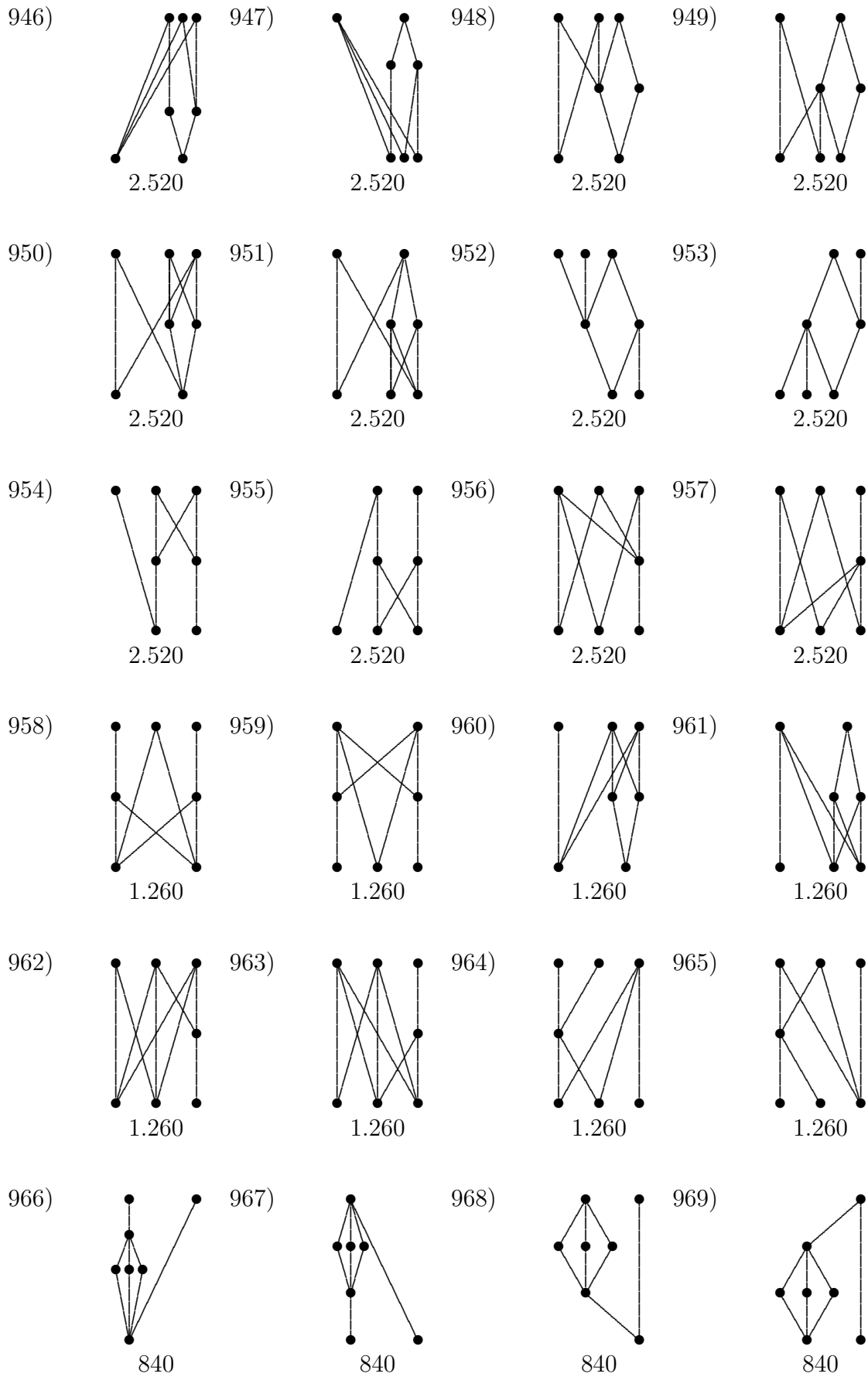


944)

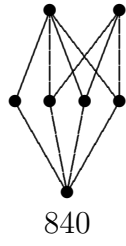


945)

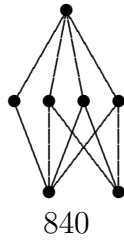




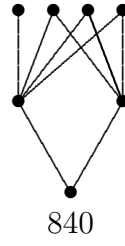
970)



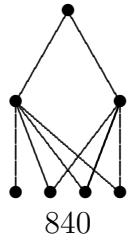
971)



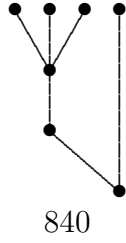
972)



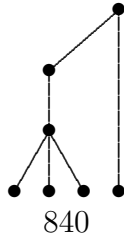
973)



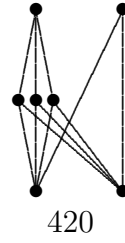
974)



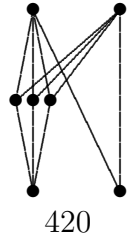
975)



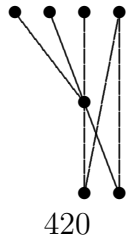
976)



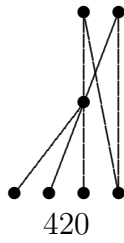
977)



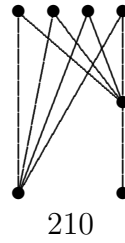
978)



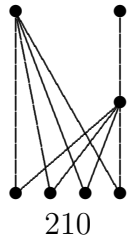
979)



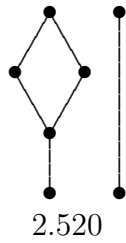
980)



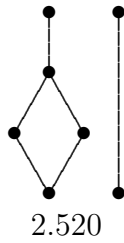
981)



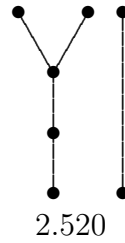
982)



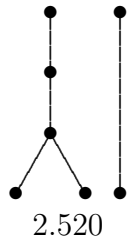
983)



984)

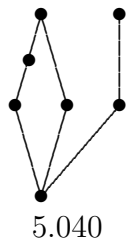


985)

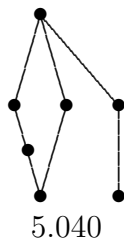


$|RB(\mathbf{7})| = 22$

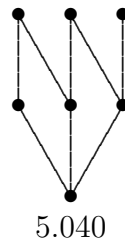
986)



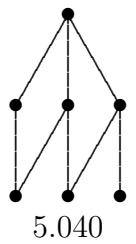
987)



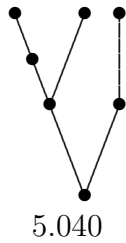
988)



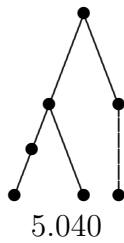
989)



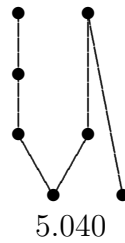
990)



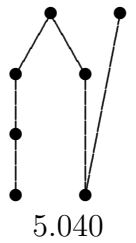
991)

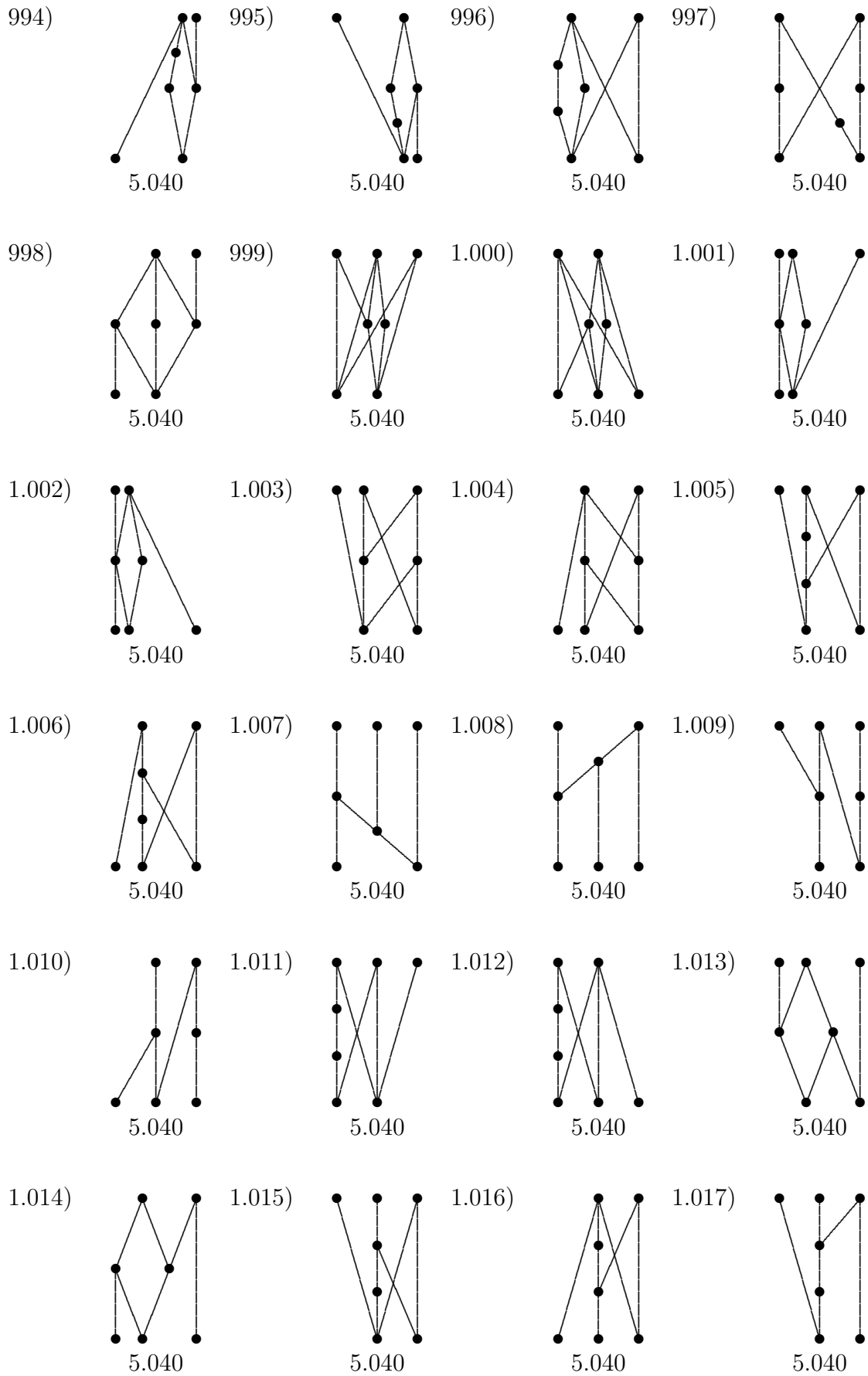


992)

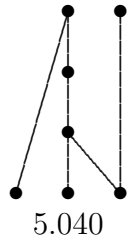


993)

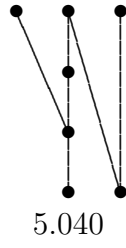




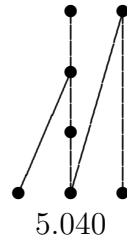
1.018)



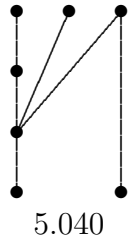
1.019)



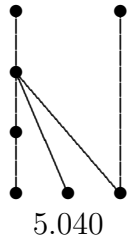
1.020)



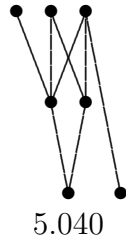
1.021)



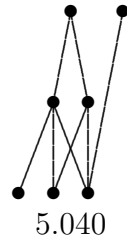
1.022)



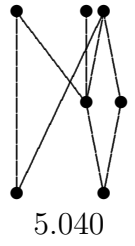
1.023)



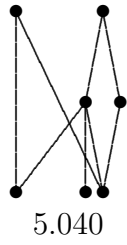
1.024)



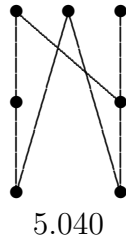
1.025)



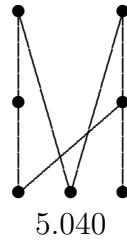
1.026)



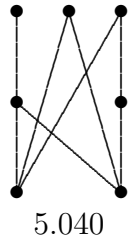
1.027)



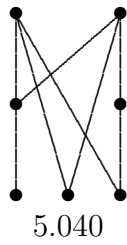
1.028)



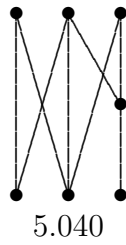
1.029)



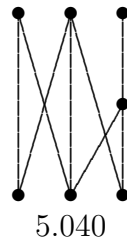
1.030)



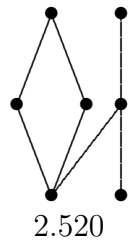
1.031)



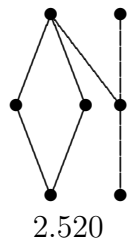
1.032)



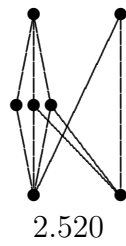
1.033)



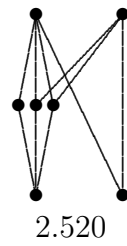
1.034)



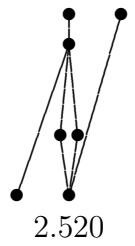
1.035)



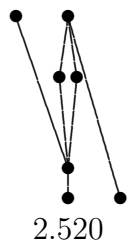
1.036)



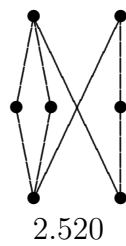
1.037)



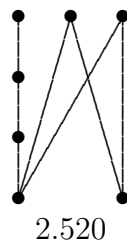
1.038)



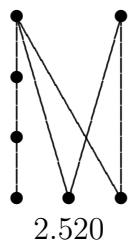
1.039)

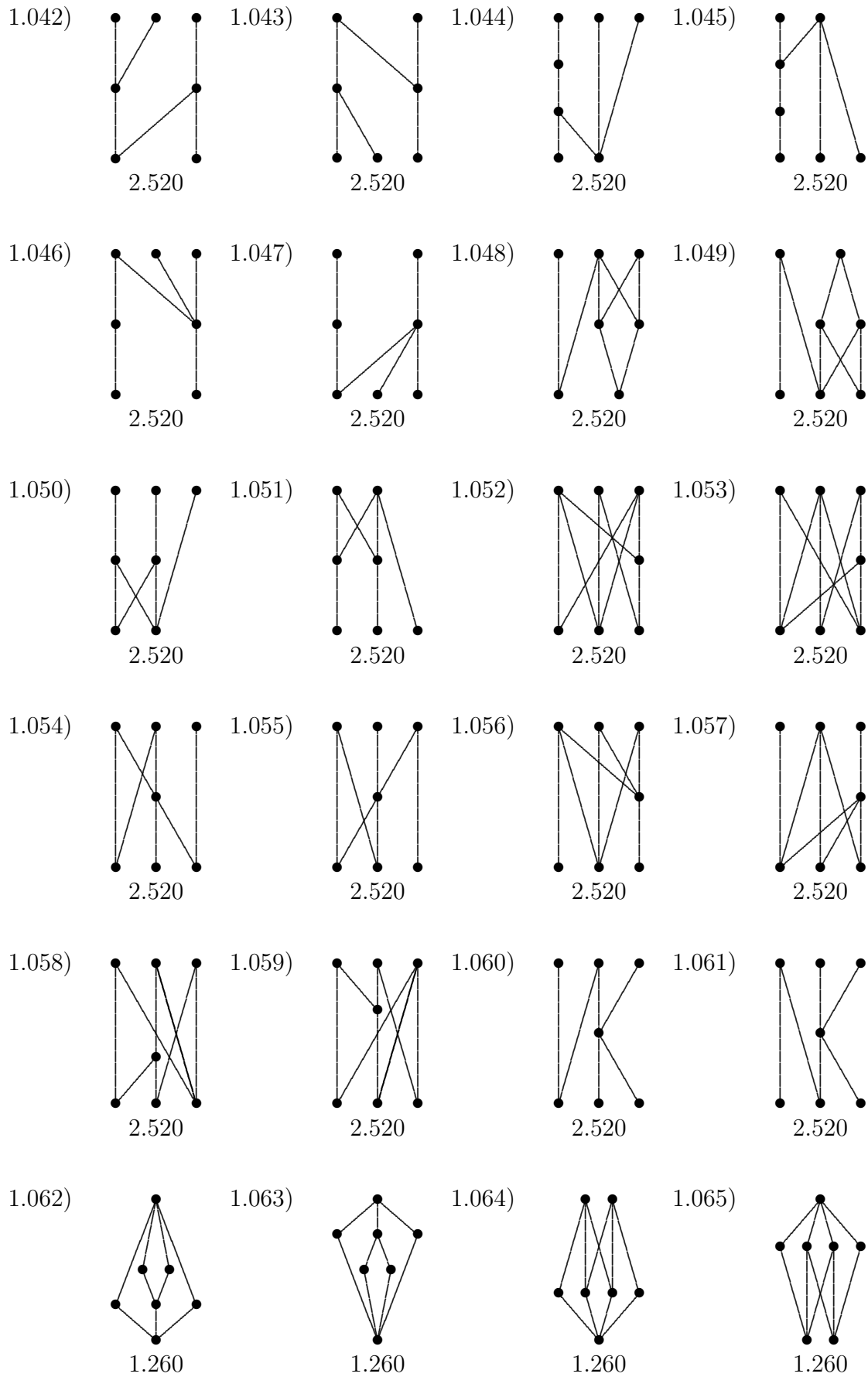


1.040)

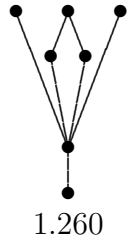


1.041)

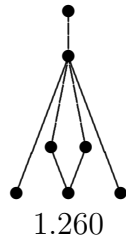




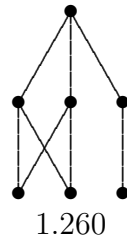
1.066)



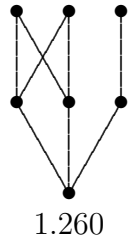
1.067)



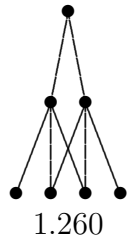
1.068)



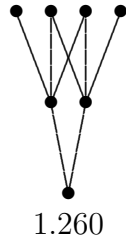
1.069)



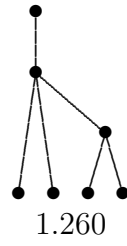
1.070)



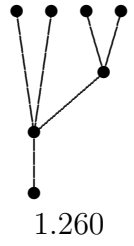
1.071)



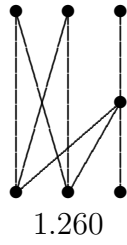
1.072)



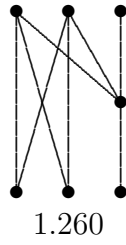
1.073)



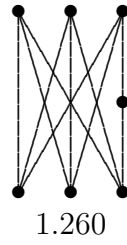
1.074)



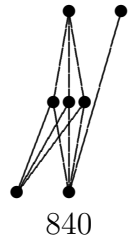
1.075)



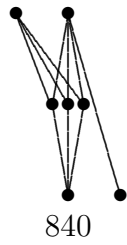
1.076)



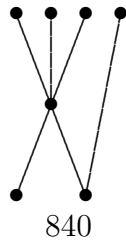
1.077)



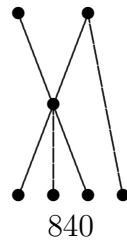
1.078)



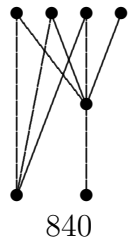
1.079)



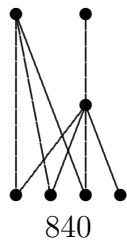
1.080)



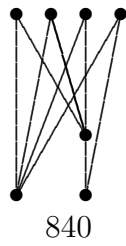
1.081)



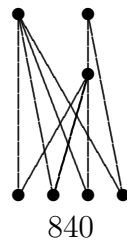
1.082)



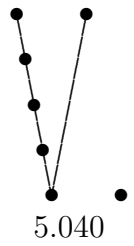
1.083)



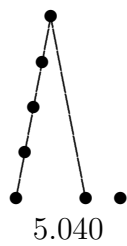
1.084)



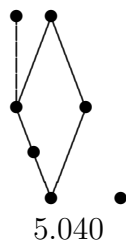
1.085)



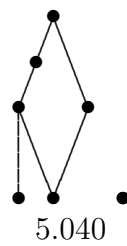
1.086)



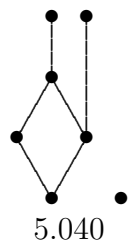
1.087)

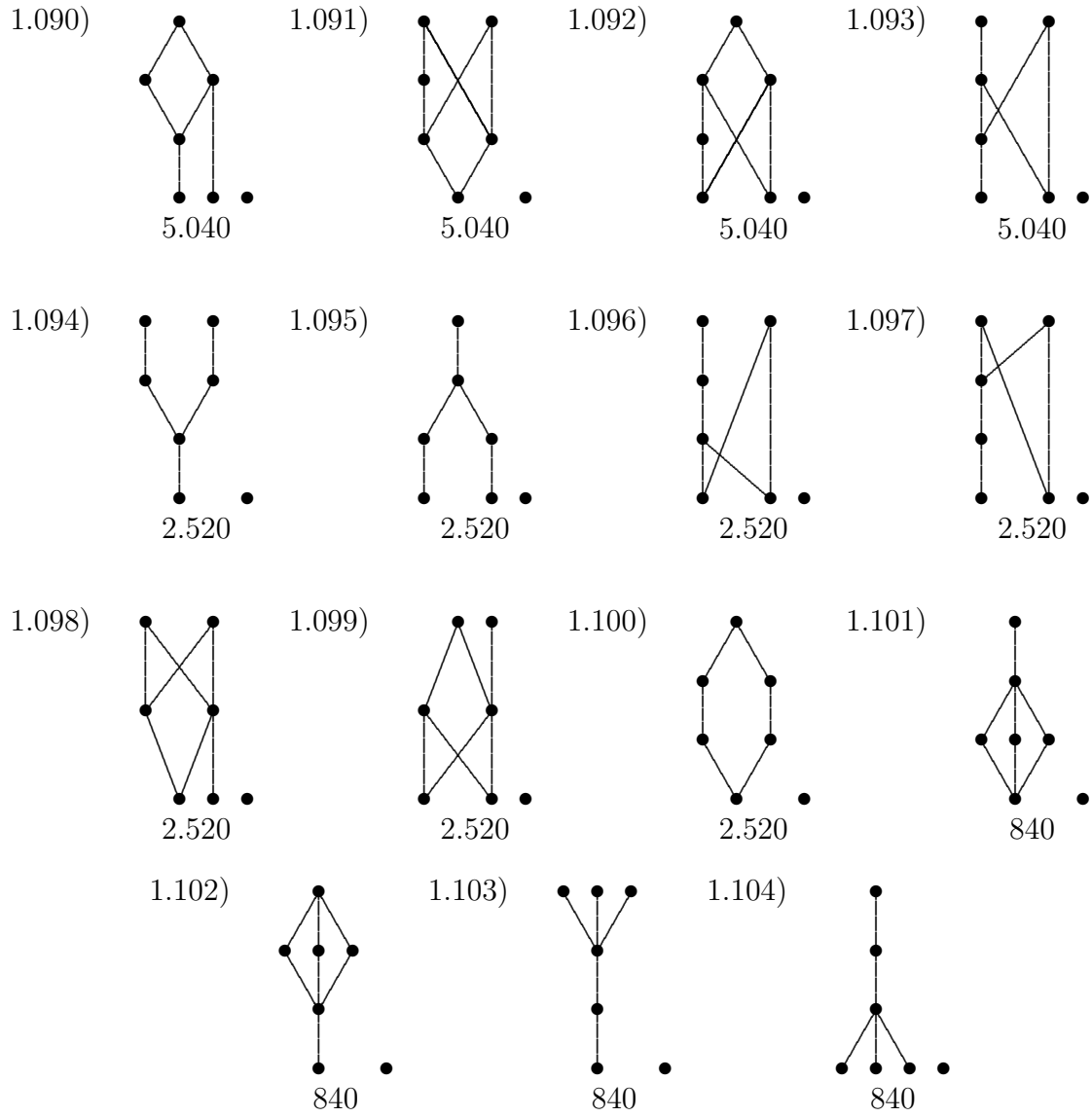


1.088)

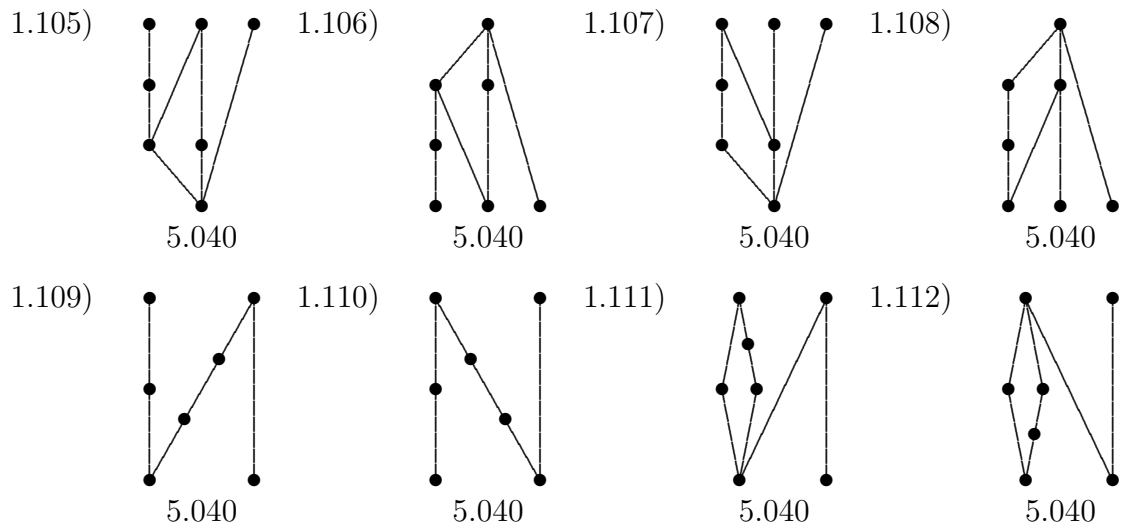


1.089)

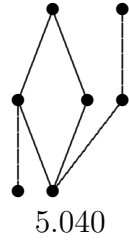




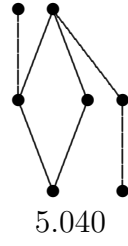
$$|RB(7)| = 23$$



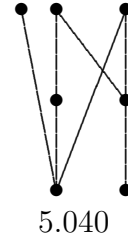
1.113)



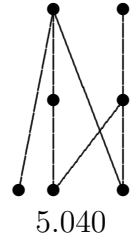
1.114)



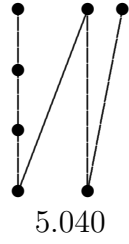
1.115)



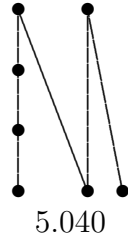
1.116)



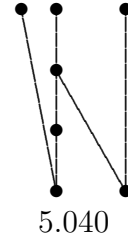
1.117)



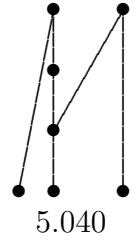
1.118)



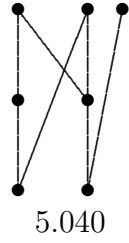
1.119)



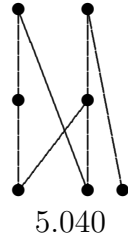
1.120)



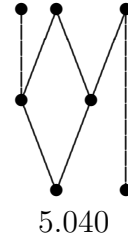
1.121)



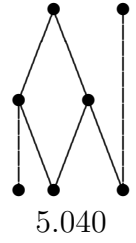
1.122)



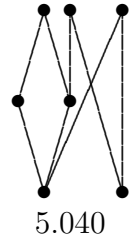
1.123)



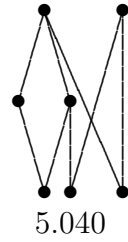
1.124)



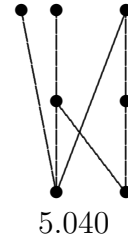
1.125)



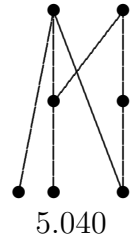
1.126)



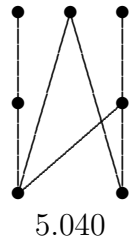
1.127)



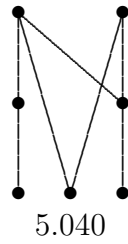
1.128)



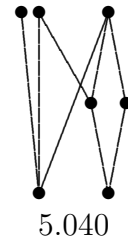
1.129)



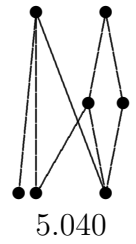
1.130)



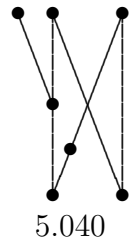
1.131)



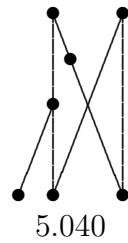
1.132)



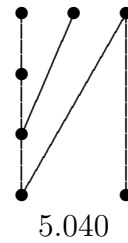
1.133)



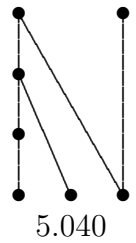
1.134)

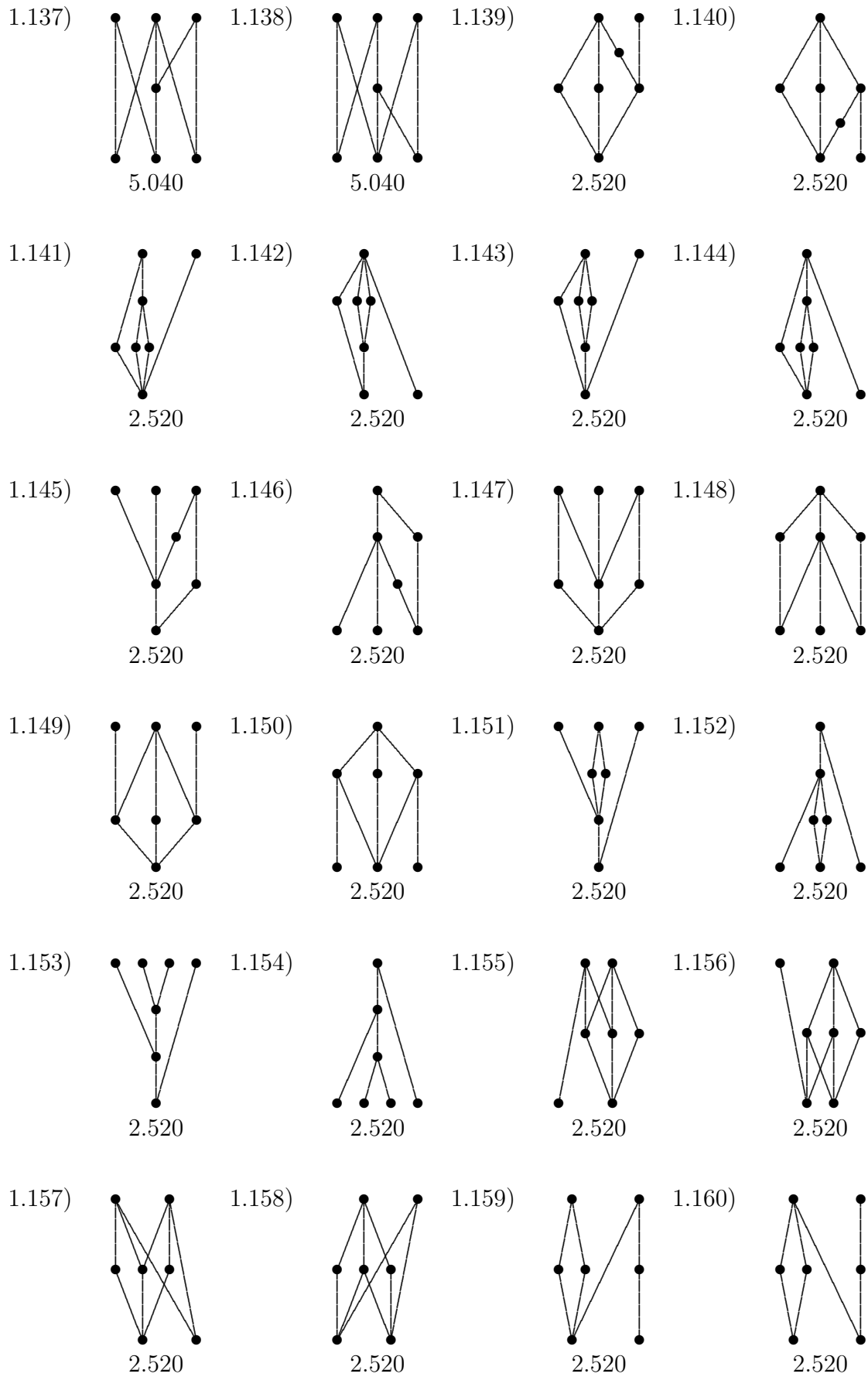


1.135)

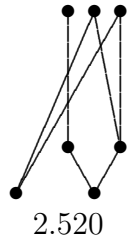


1.136)

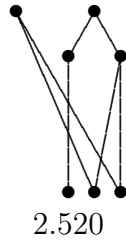




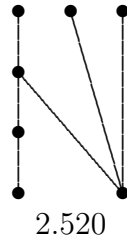
1.161)



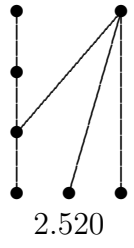
1.162)



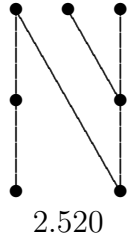
1.163)



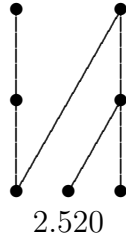
1.164)



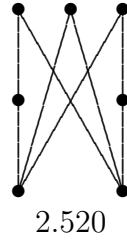
1.165)



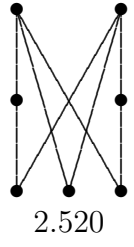
1.166)



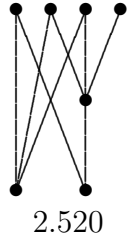
1.167)



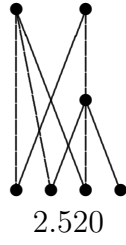
1.168)



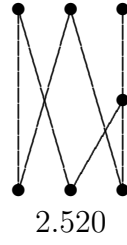
1.169)



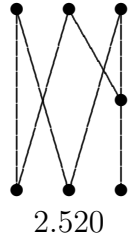
1.170)



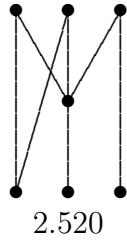
1.171)



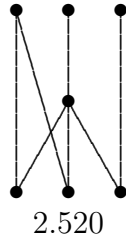
1.172)



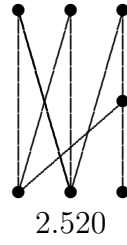
1.173)



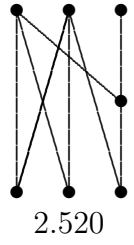
1.174)



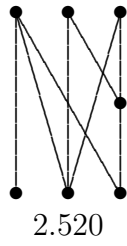
1.175)



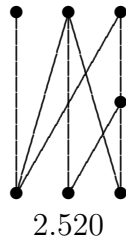
1.176)



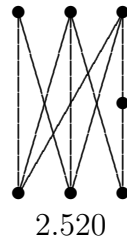
1.177)



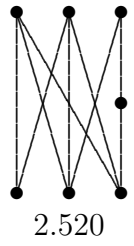
1.178)



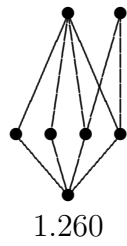
1.179)



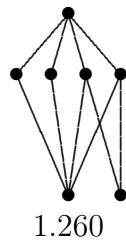
1.180)



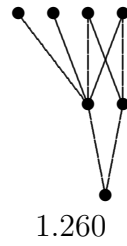
1.181)



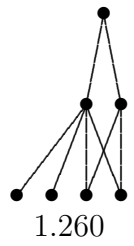
1.182)

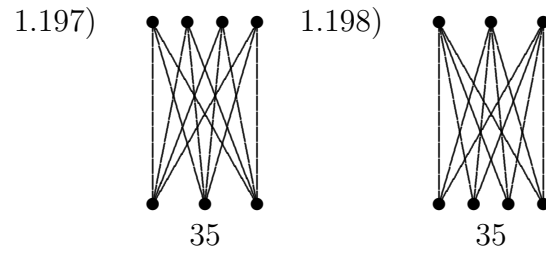
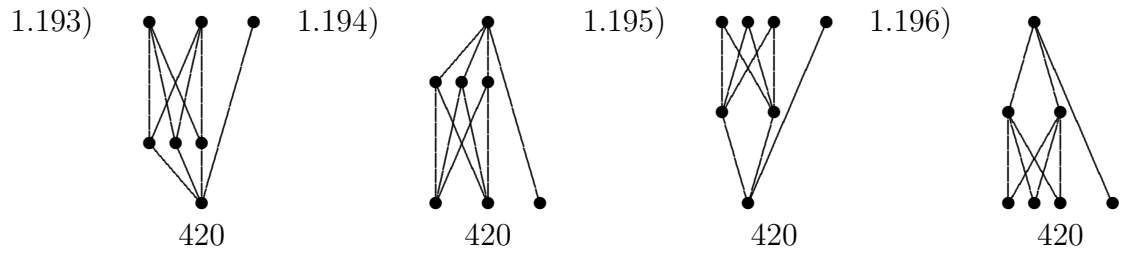
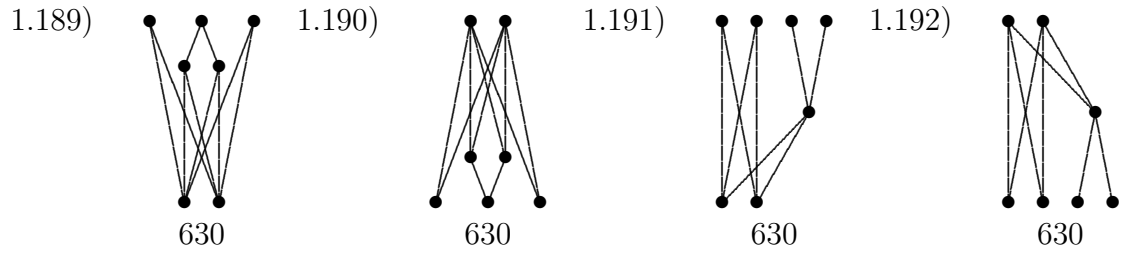
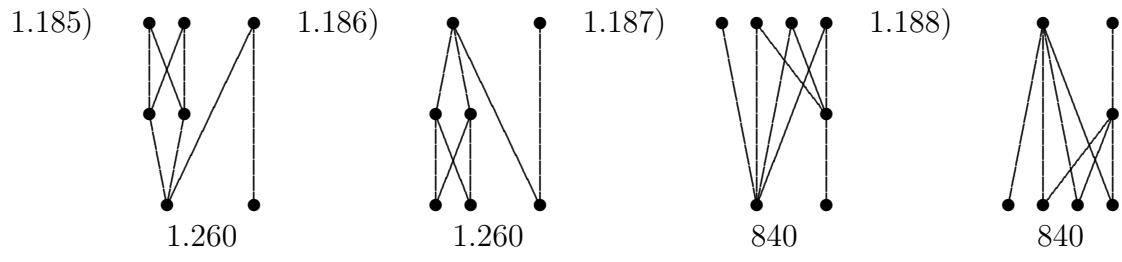


1.183)

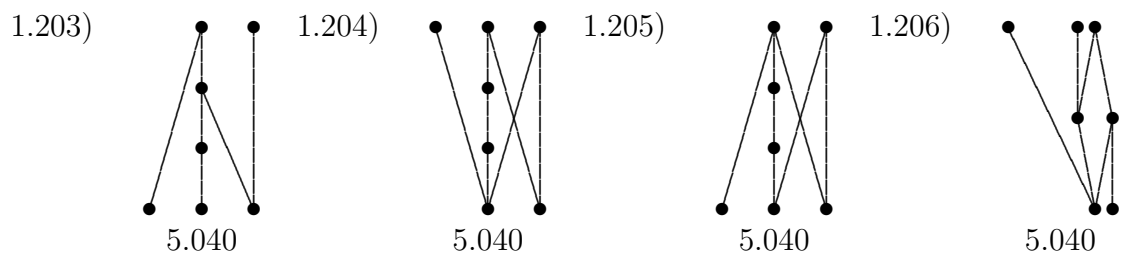
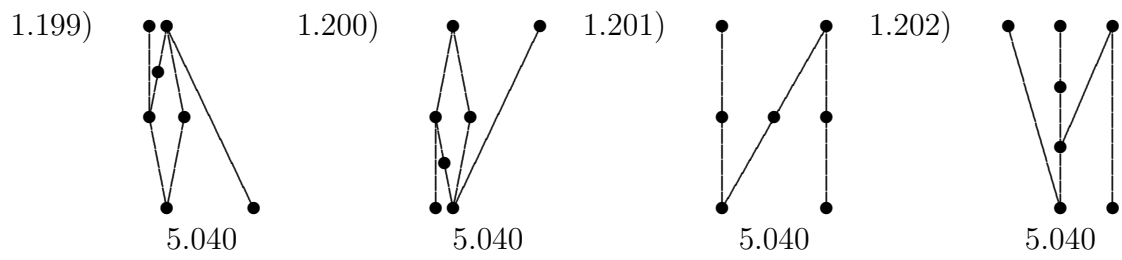


1.184)

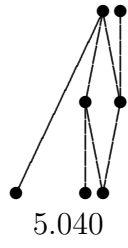




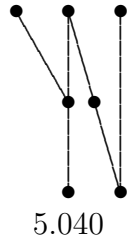
$$|RB(7)| = 24$$



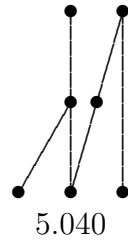
1.207)



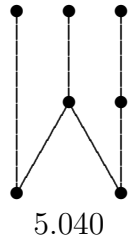
1.208)



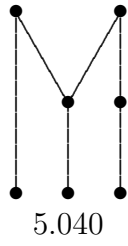
1.209)



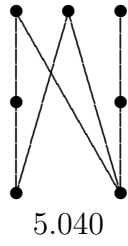
1.210)



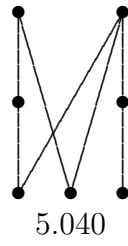
1.211)



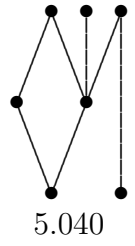
1.212)



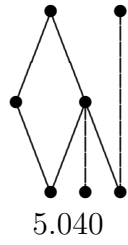
1.213)



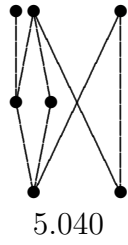
1.214)



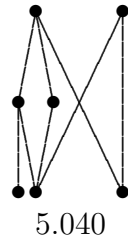
1.215)



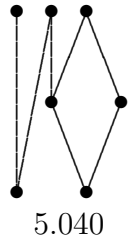
1.216)



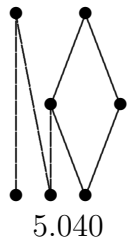
1.217)



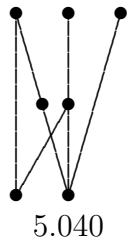
1.218)



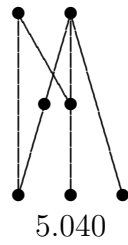
1.219)



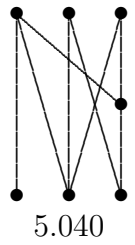
1.220)



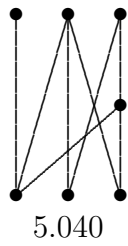
1.221)



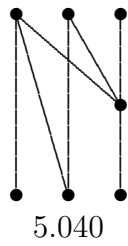
1.222)



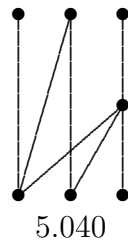
1.223)



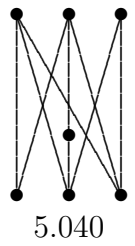
1.224)



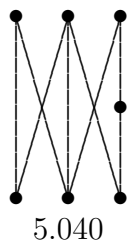
1.225)



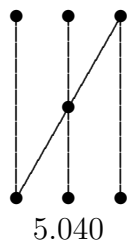
1.226)



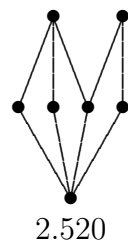
1.227)



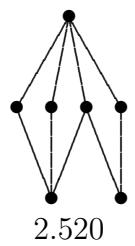
1.228)

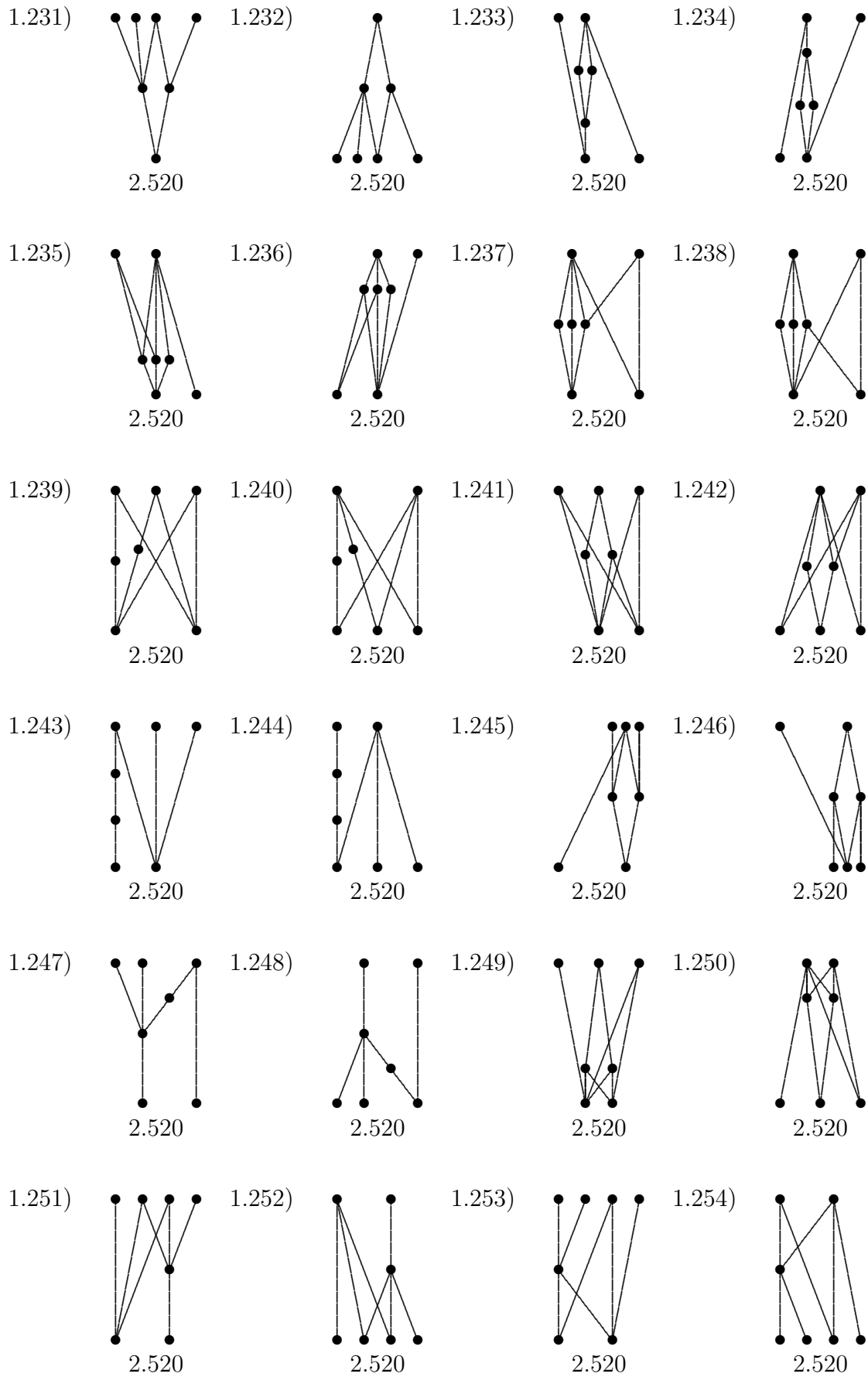


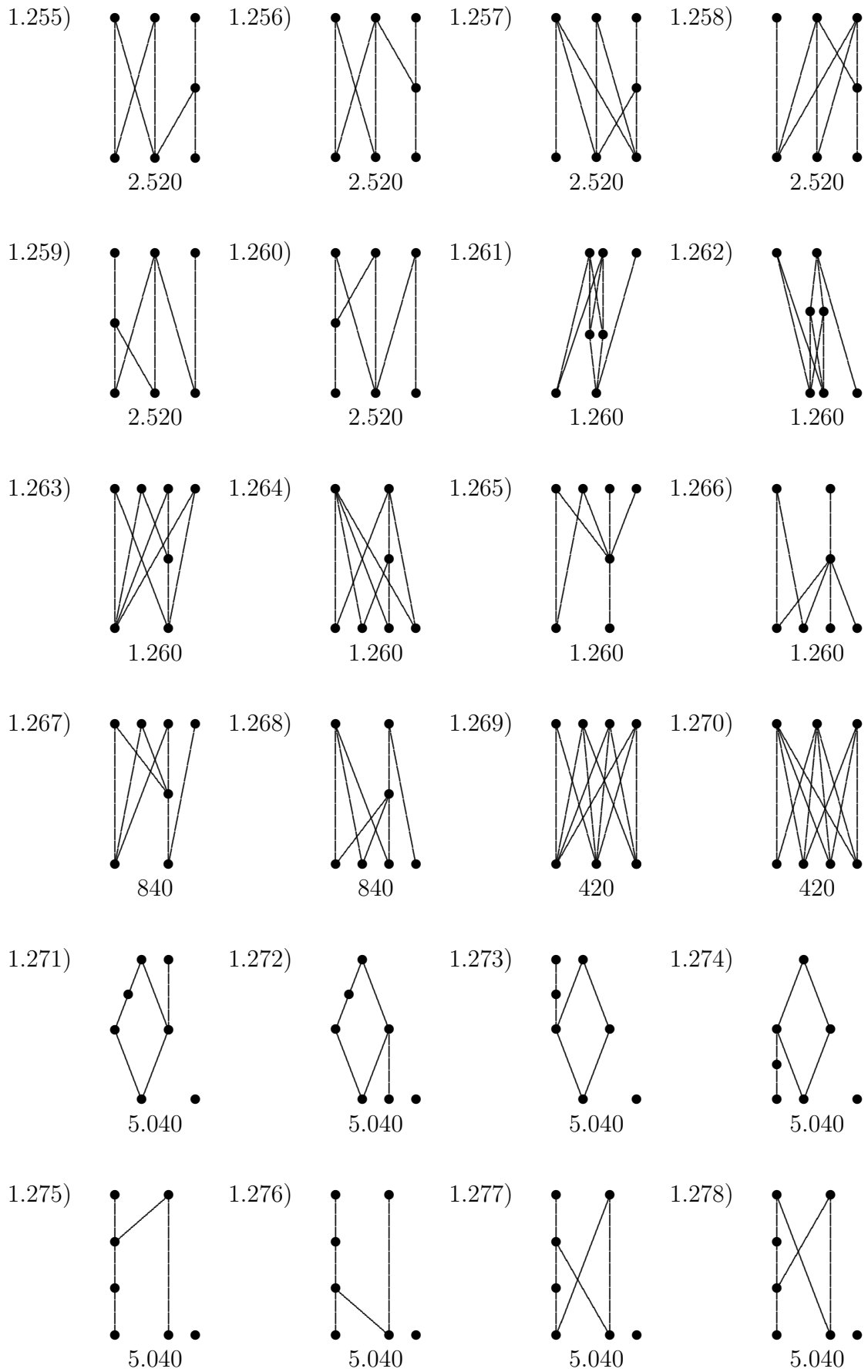
1.229)

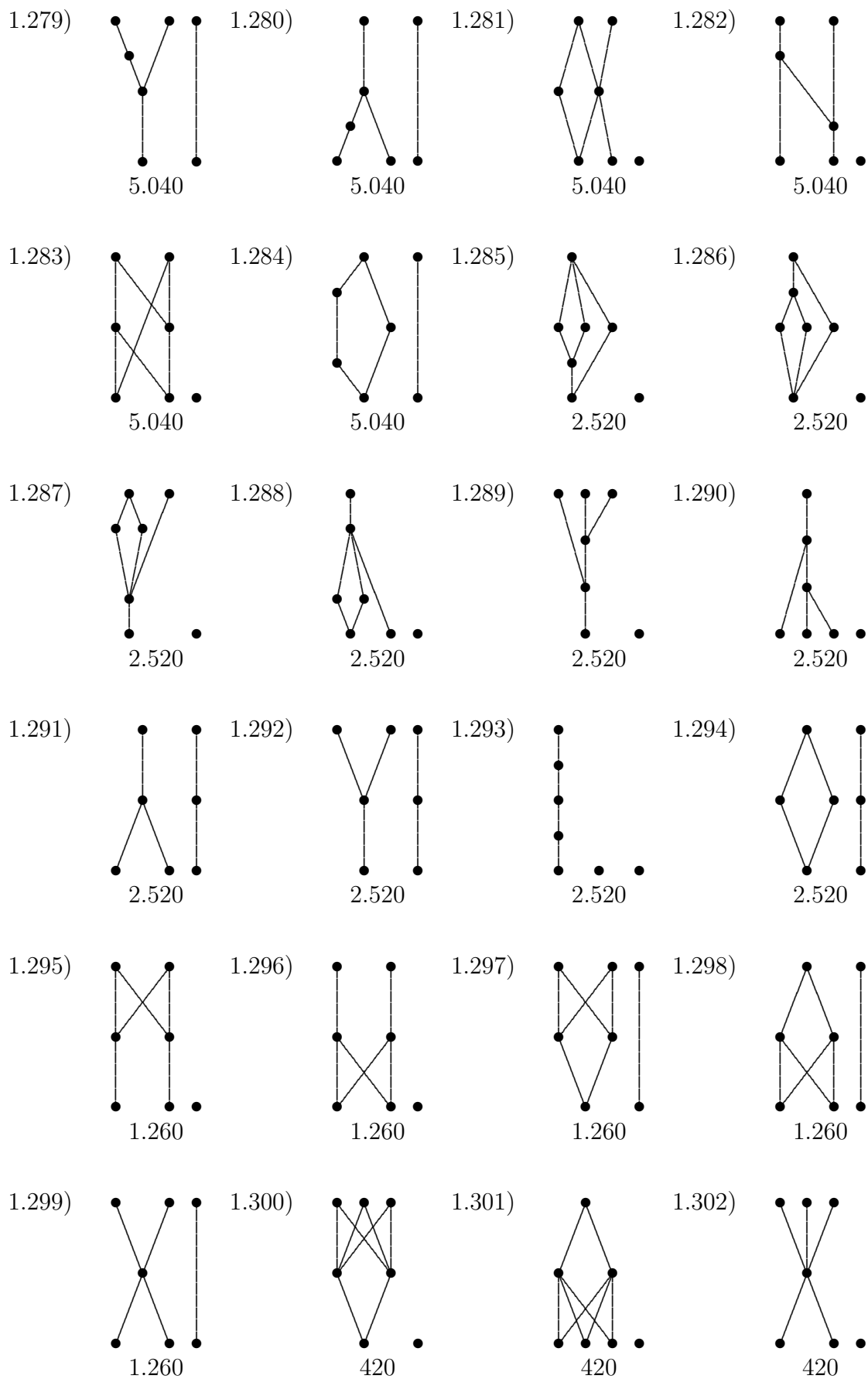


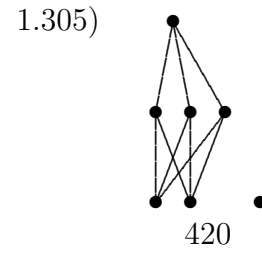
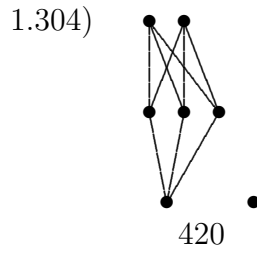
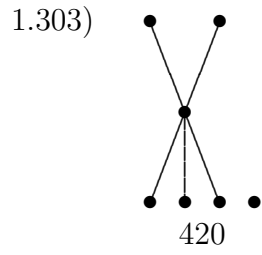
1.230)



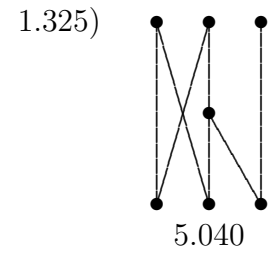
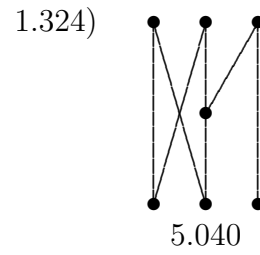
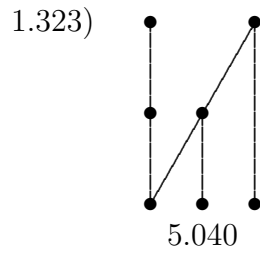
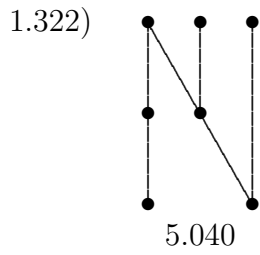
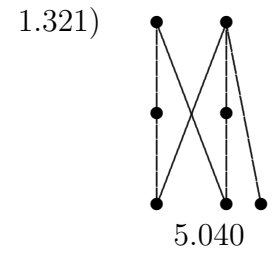
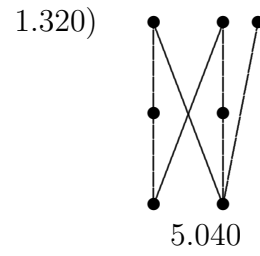
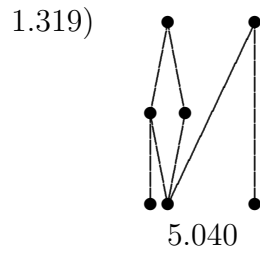
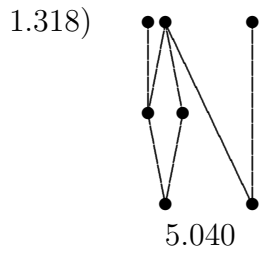
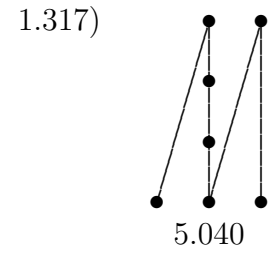
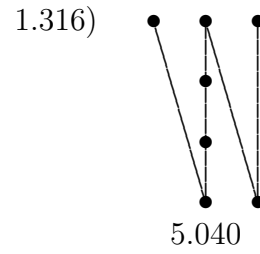
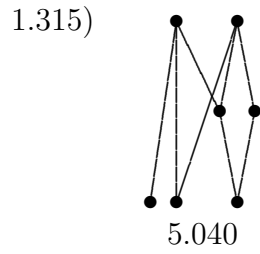
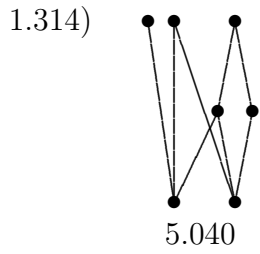
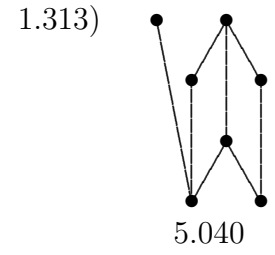
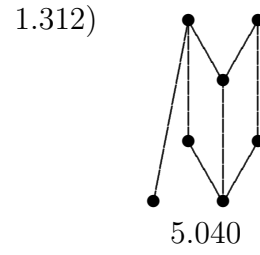
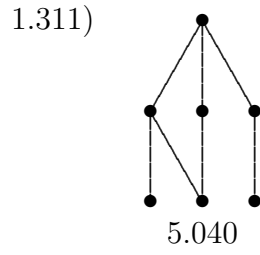
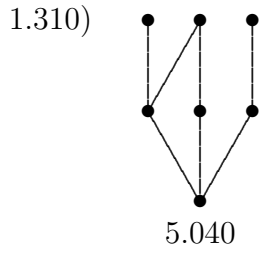
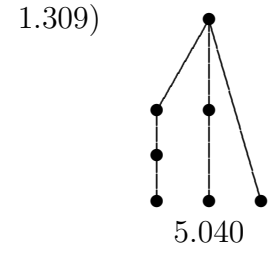
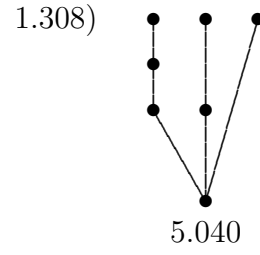
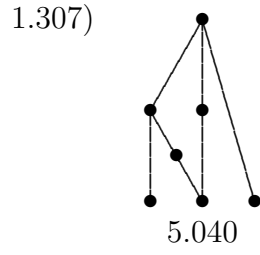
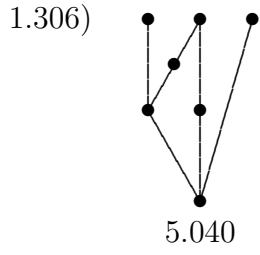


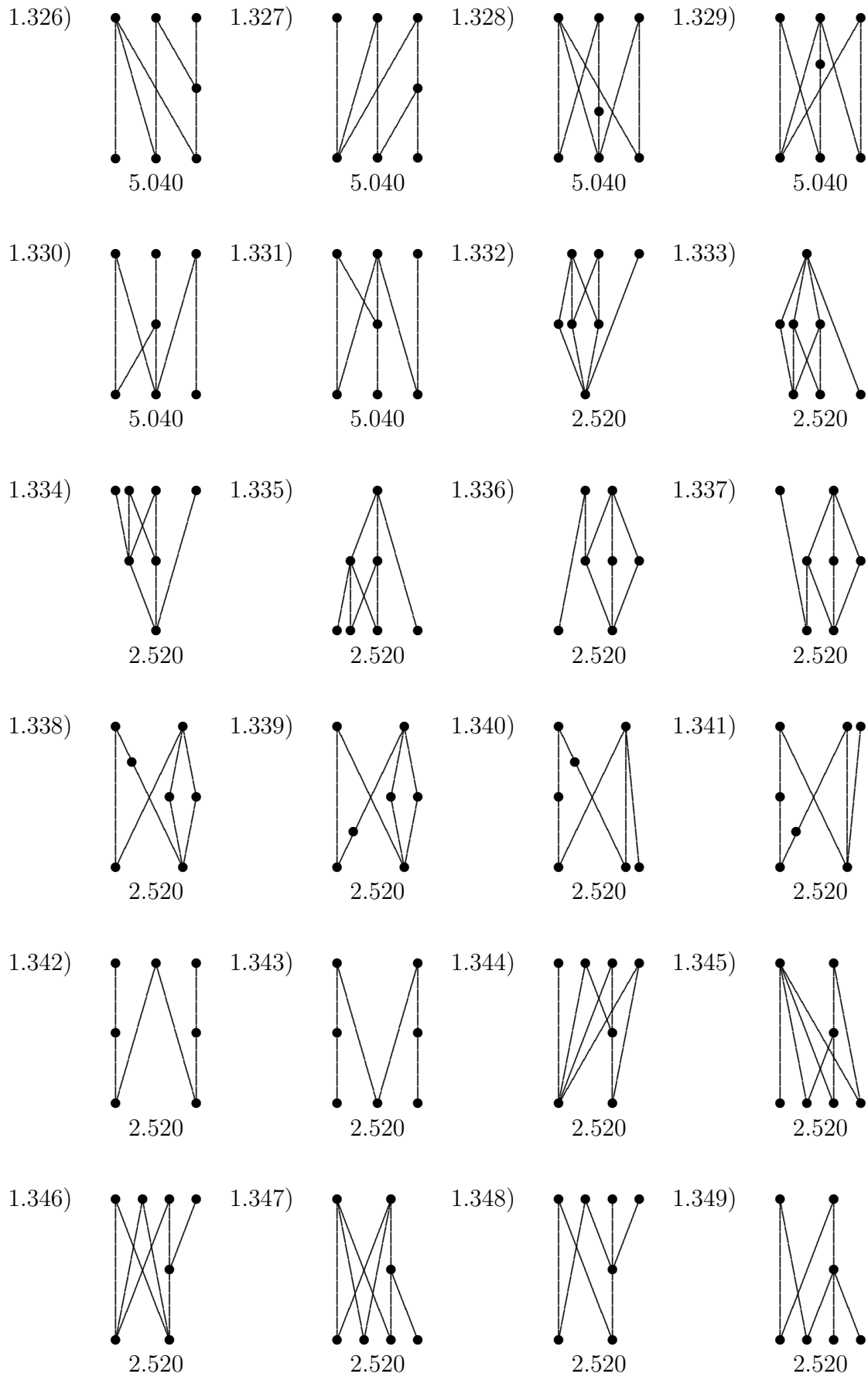




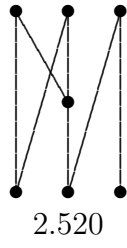


$|RB(\mathbf{7})| = 25$

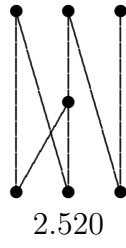




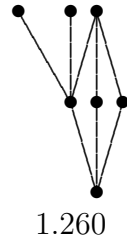
1.350)



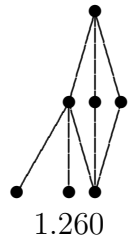
1.351)



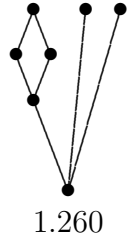
1.352)



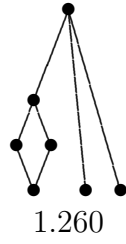
1.353)



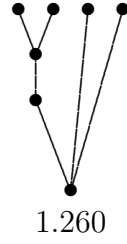
1.354)



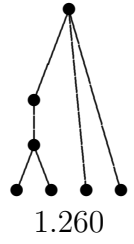
1.355)



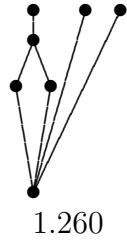
1.356)



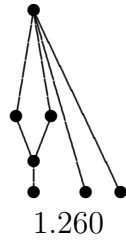
1.357)



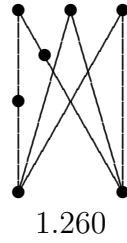
1.358)



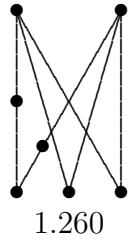
1.359)



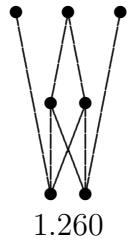
1.360)



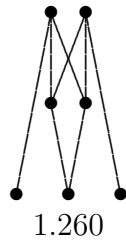
1.361)



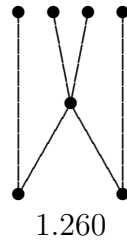
1.362)



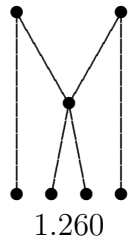
1.363)



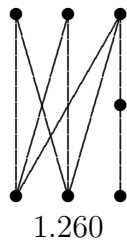
1.364)



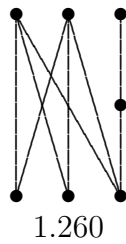
1.365)



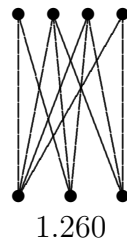
1.366)



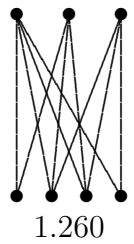
1.367)



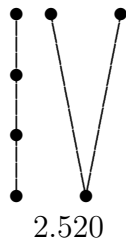
1.368)



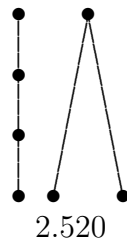
1.369)



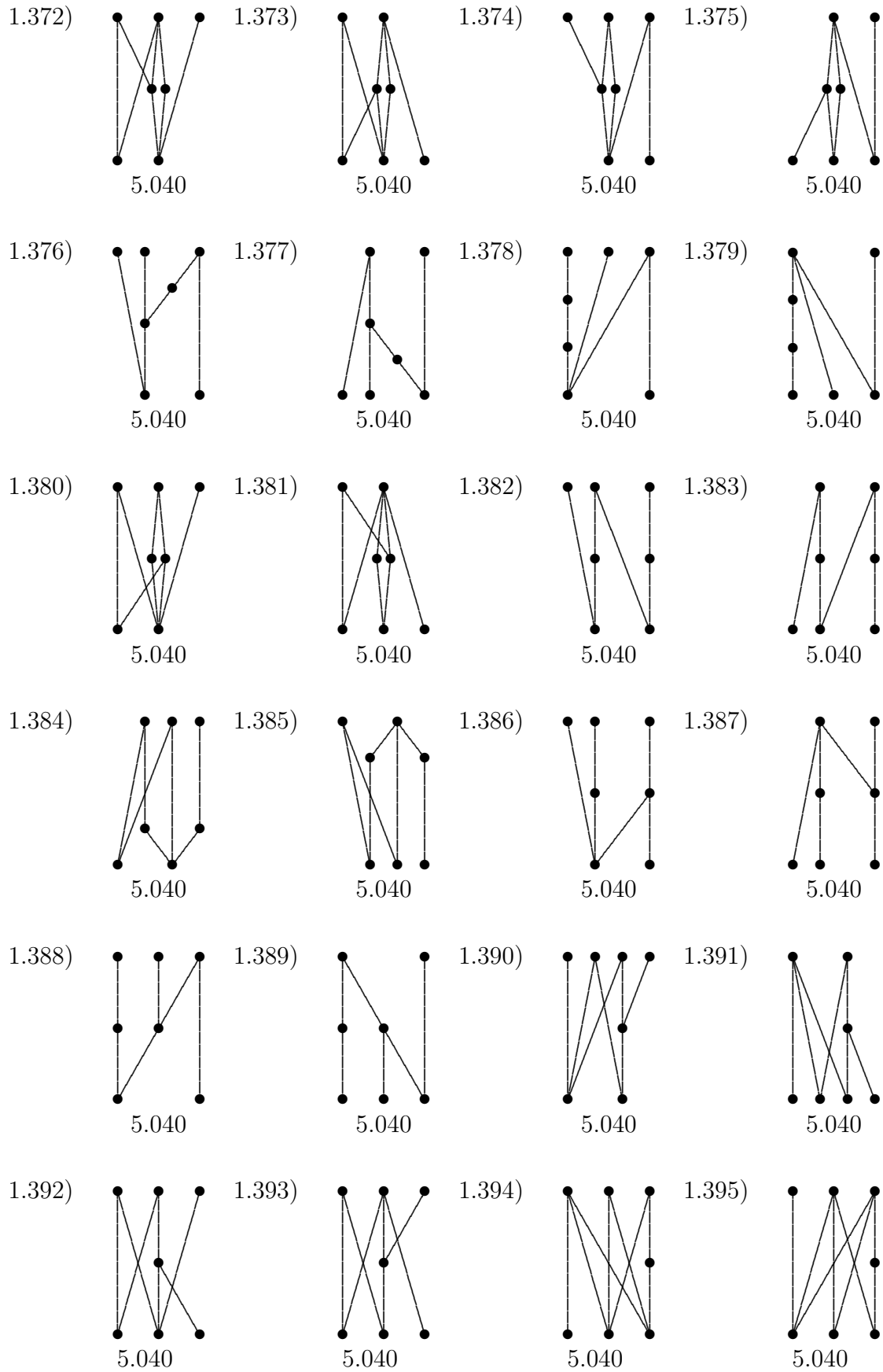
1.370)



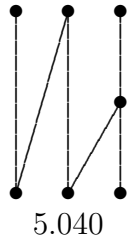
1.371)



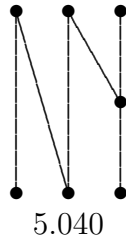
$$|RB(7)| = 26$$



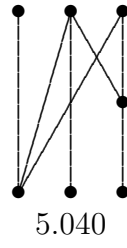
1.396)



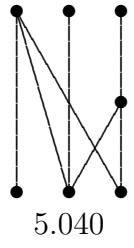
1.397)



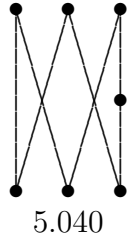
1.398)



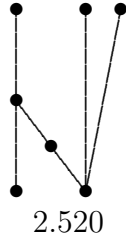
1.399)



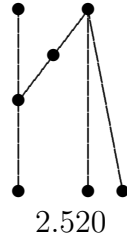
1.400)



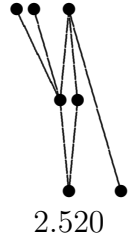
1.401)



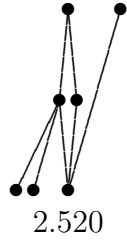
1.402)



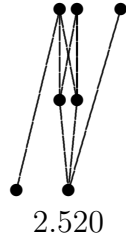
1.403)



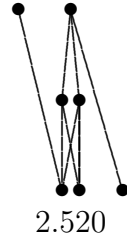
1.404)



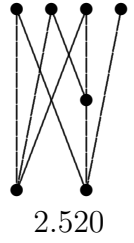
1.405)



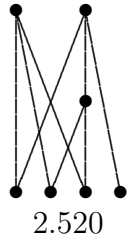
1.406)



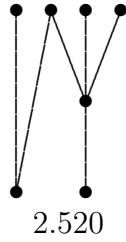
1.407)



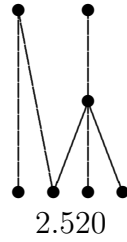
1.408)



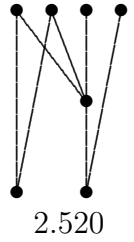
1.409)



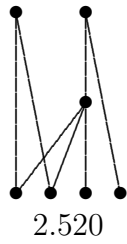
1.410)



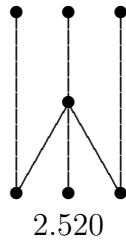
1.411)



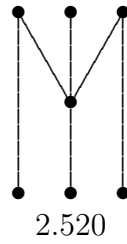
1.412)



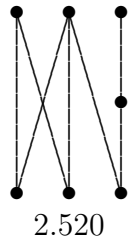
1.413)



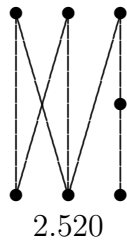
1.414)



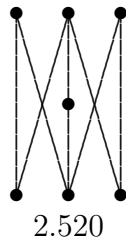
1.415)



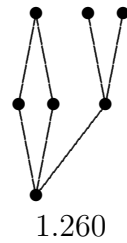
1.416)



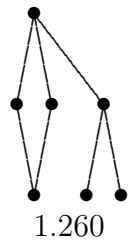
1.417)

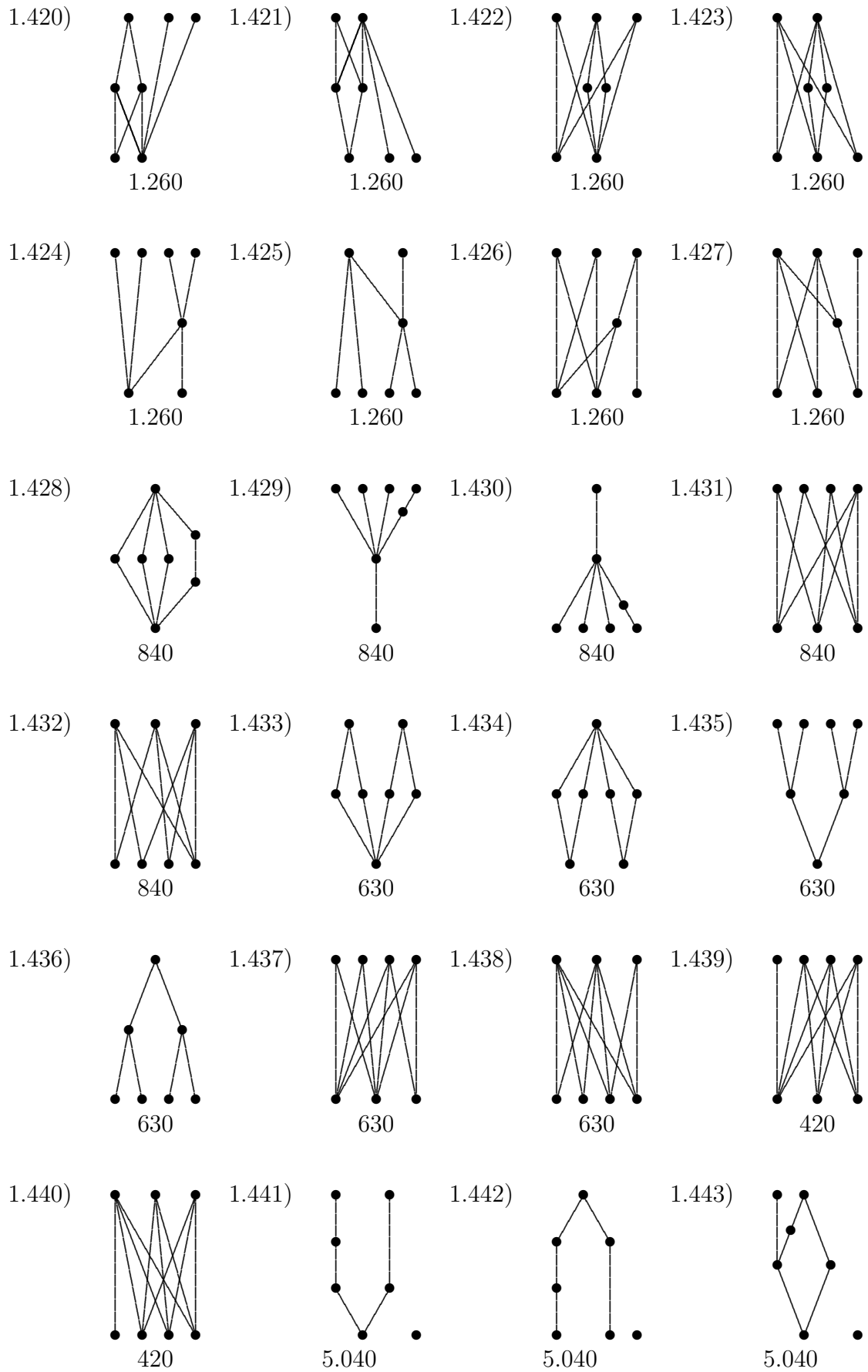


1.418)

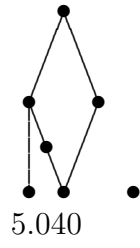


1.419)

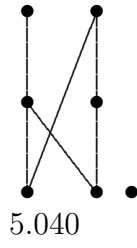




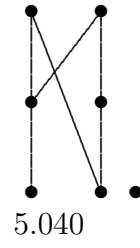
1.444)



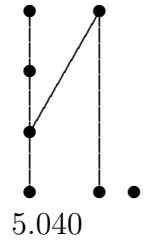
1.445)



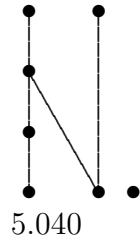
1.446)



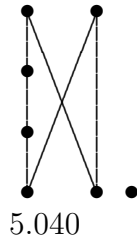
1.447)



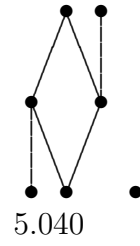
1.448)



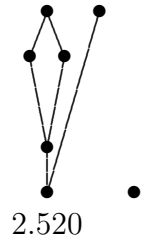
1.449)



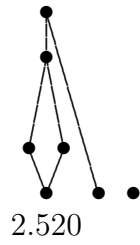
1.450)



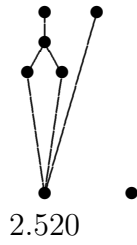
1.451)



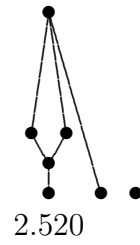
1.452)



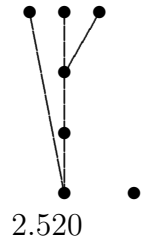
1.453)



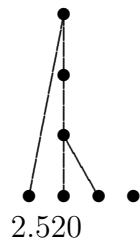
1.454)



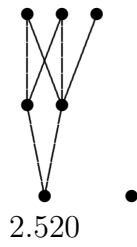
1.455)



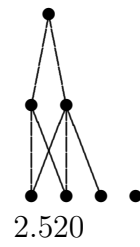
1.456)



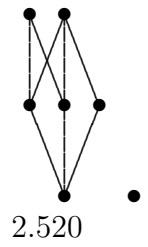
1.457)



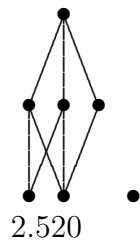
1.458)



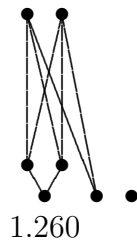
1.459)



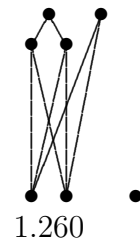
1.460)



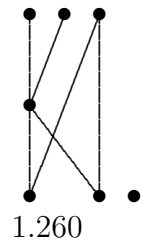
1.461)



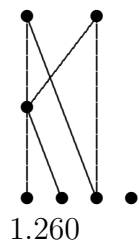
1.462)



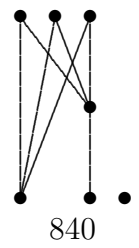
1.463)



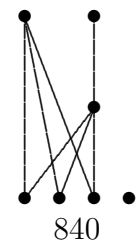
1.464)



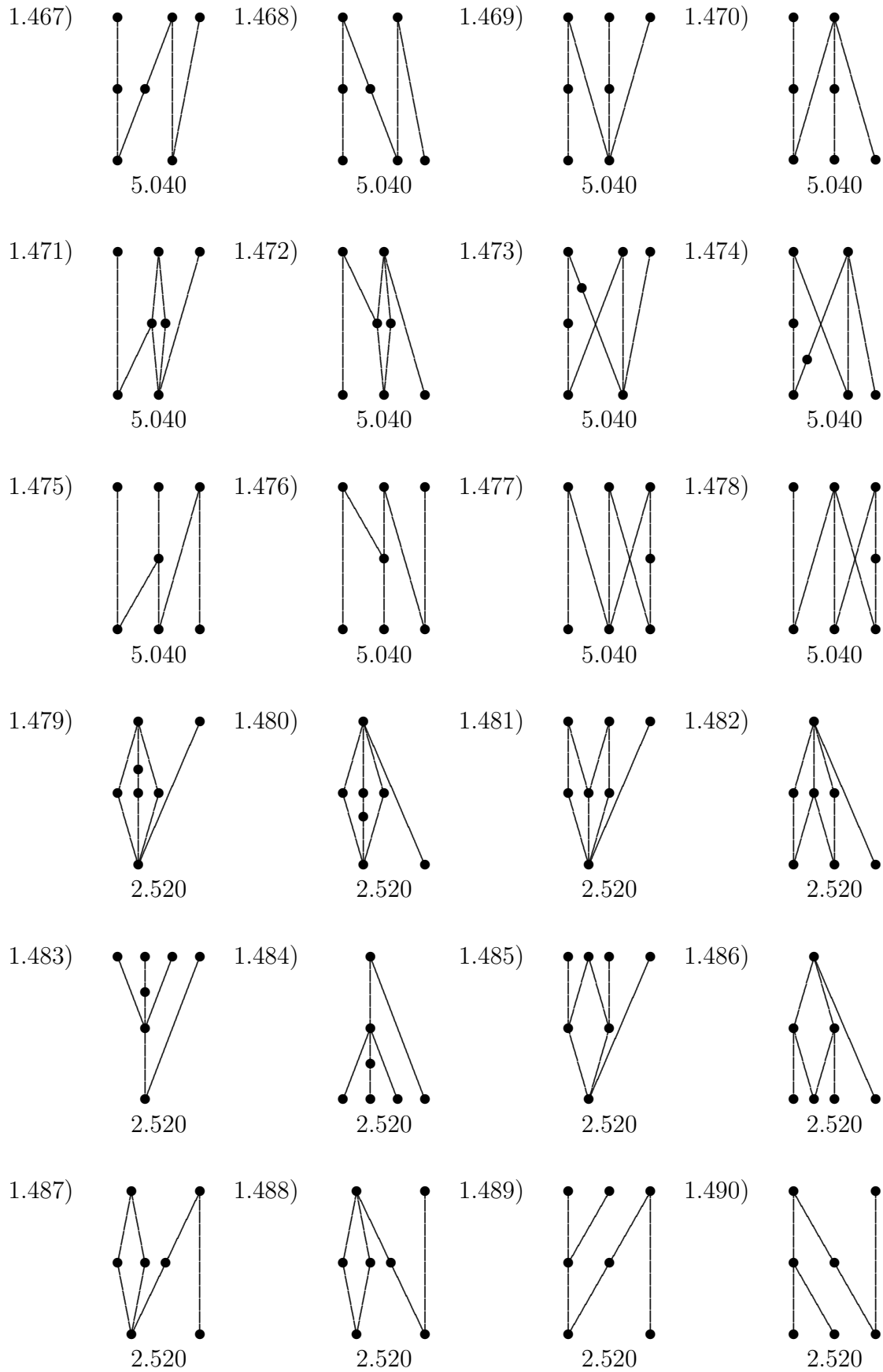
1.465)



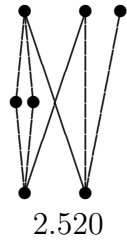
1.466)



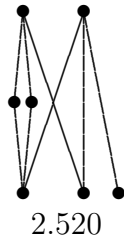
$$|RB(7)| = 27$$



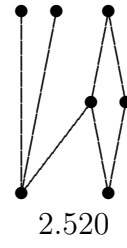
1.491)



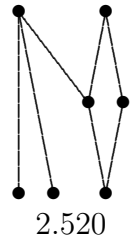
1.492)



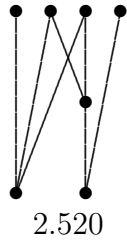
1.493)



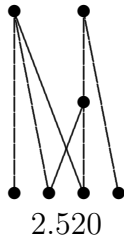
1.494)



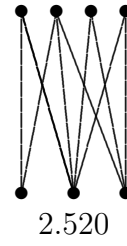
1.495)



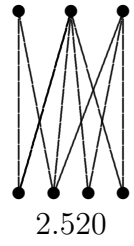
1.496)



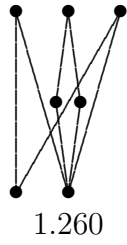
1.497)



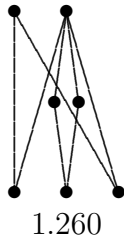
1.498)



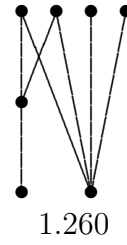
1.499)



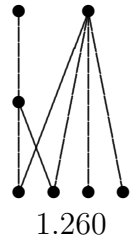
1.500)



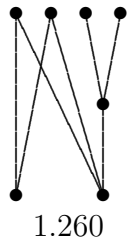
1.501)



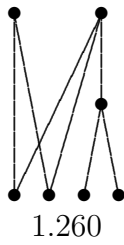
1.502)



1.503)



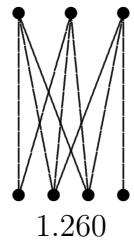
1.504)



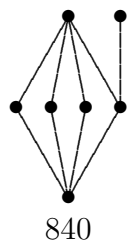
1.505)



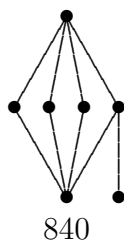
1.506)



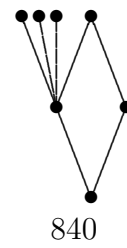
1.507)



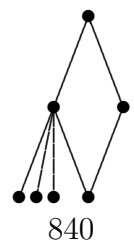
1.508)



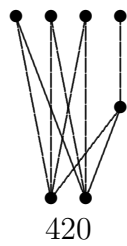
1.509)



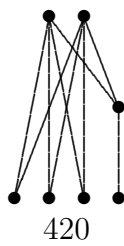
1.510)



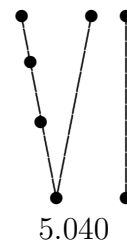
1.511)



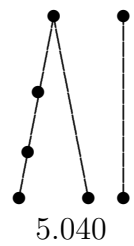
1.512)

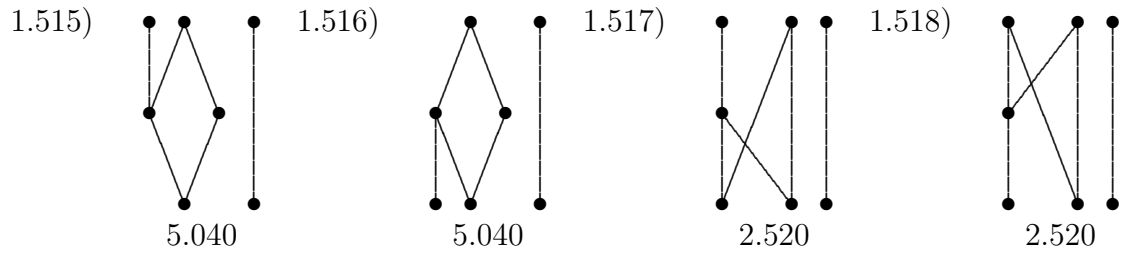
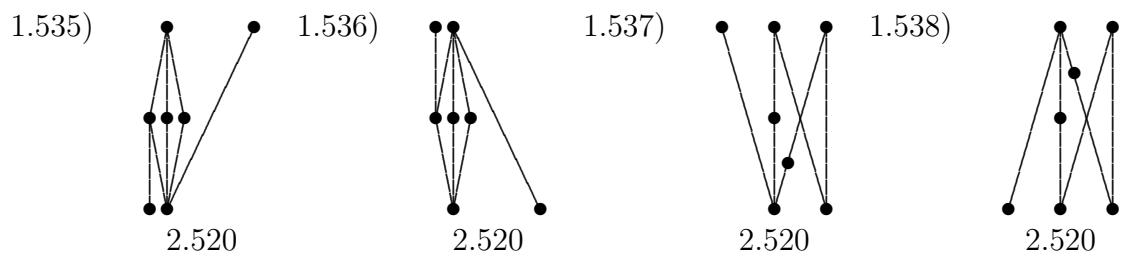
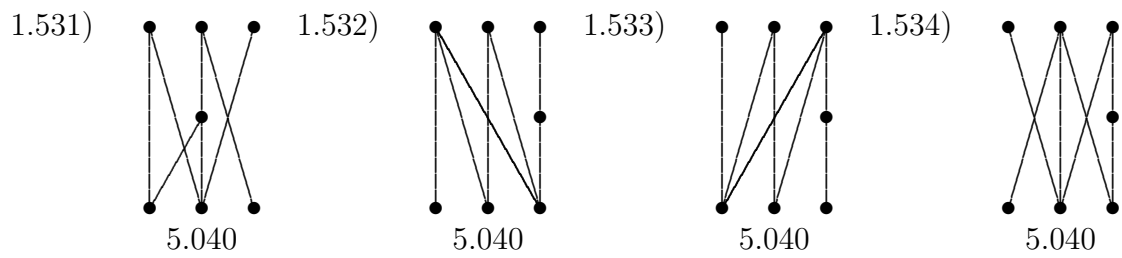
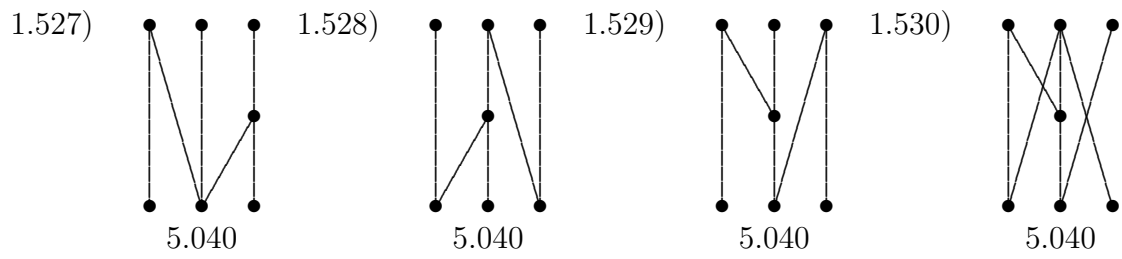
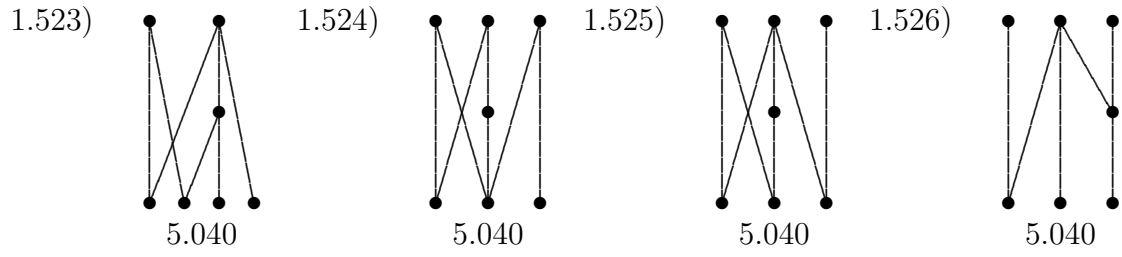
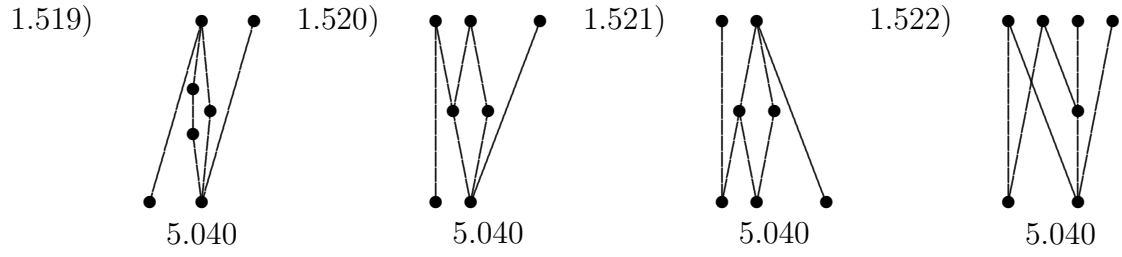


1.513)

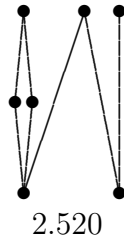


1.514)

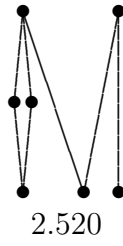



 $|RB(7)| = 28$


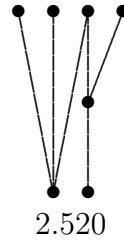
1.539)



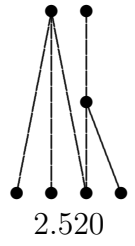
1.540)



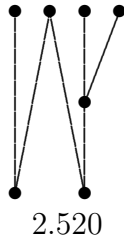
1.541)



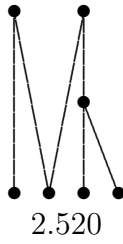
1.542)



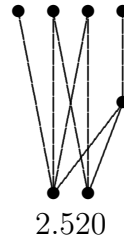
1.543)



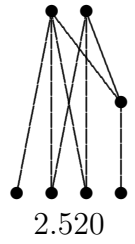
1.544)



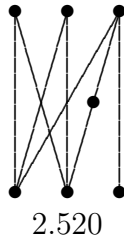
1.545)



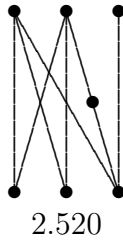
1.546)



1.547)



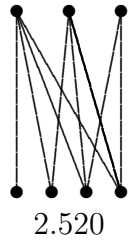
1.548)



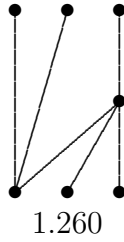
1.549)



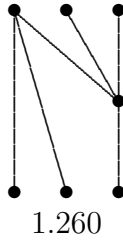
1.550)



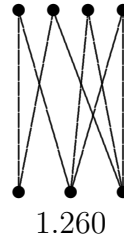
1.551)



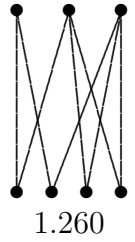
1.552)



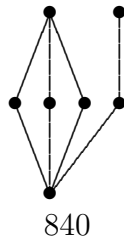
1.553)



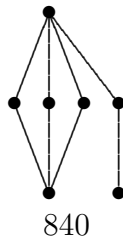
1.554)



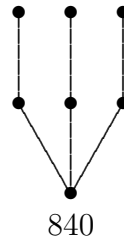
1.555)



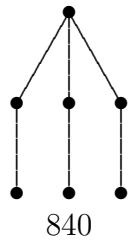
1.556)



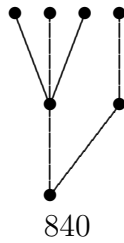
1.557)



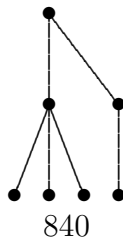
1.558)



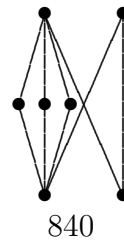
1.559)



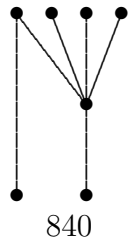
1.560)

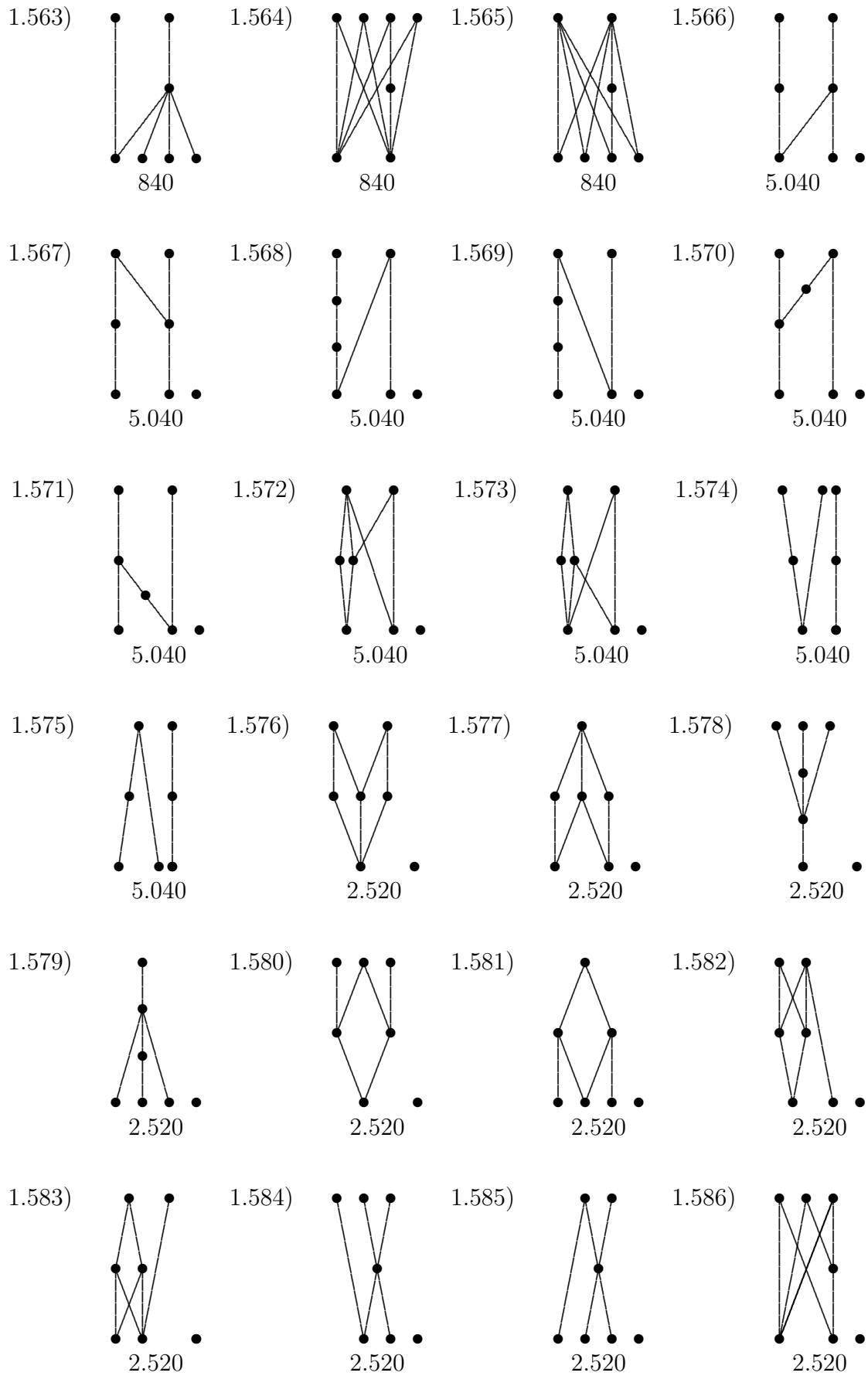


1.561)

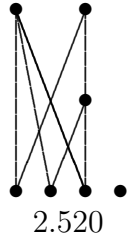


1.562)

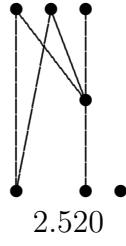




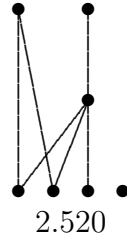
1.587)



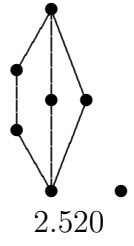
1.588)



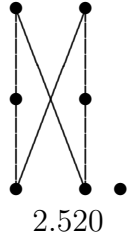
1.589)



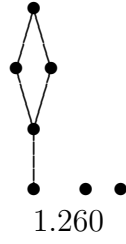
1.590)



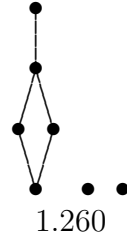
1.591)



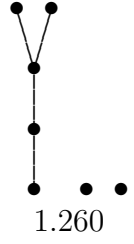
1.592)



1.593)



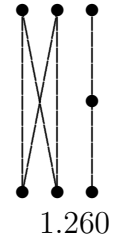
1.594)



1.595)

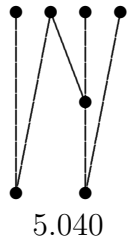


1.596)

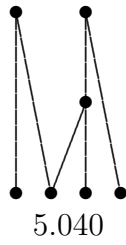


$|RB(7)| = 29$

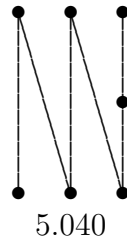
1.597)



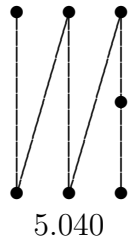
1.598)



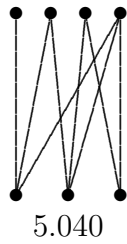
1.599)



1.600)



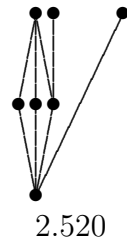
1.601)



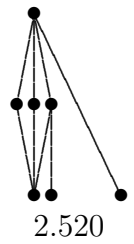
1.602)



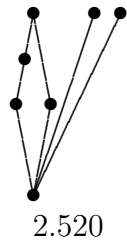
1.603)



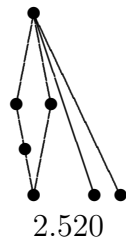
1.604)



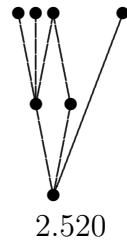
1.605)



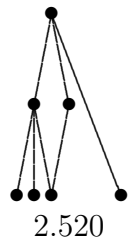
1.606)

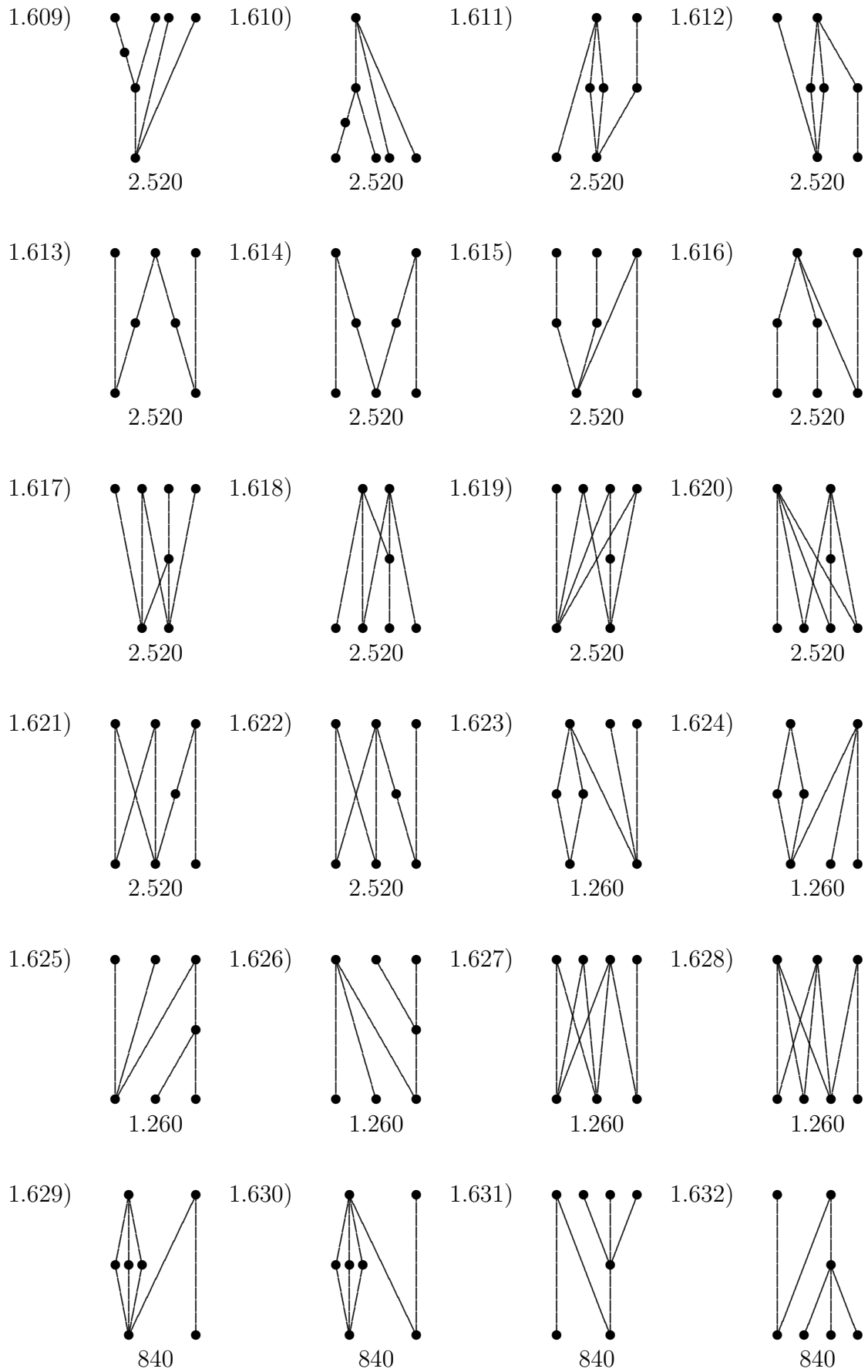


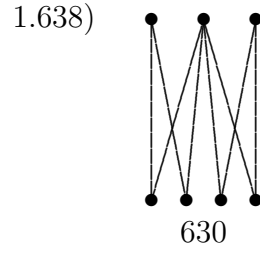
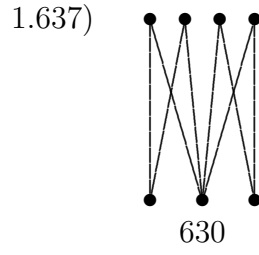
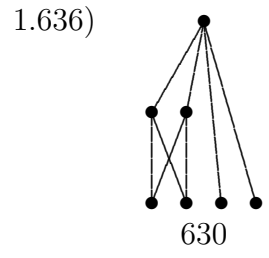
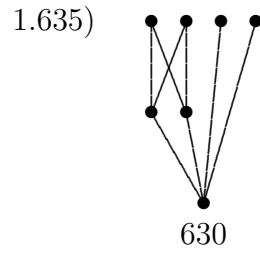
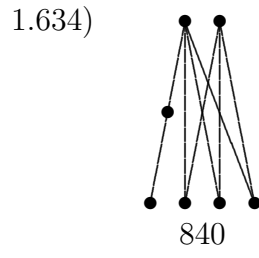
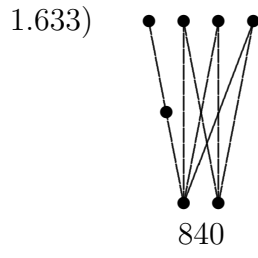
1.607)



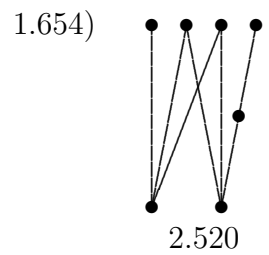
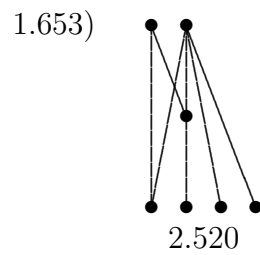
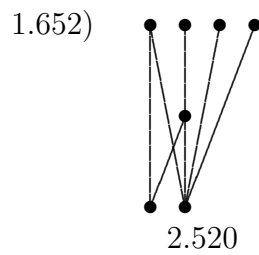
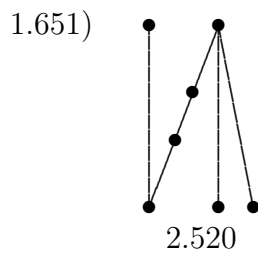
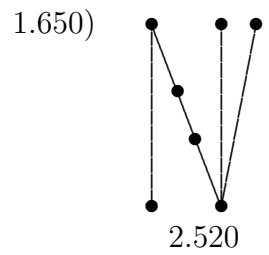
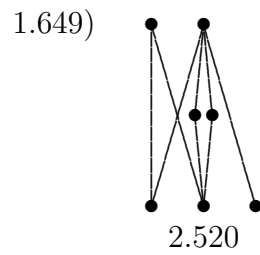
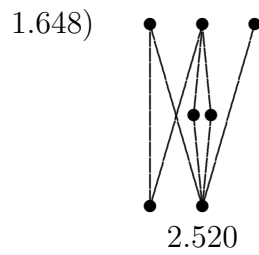
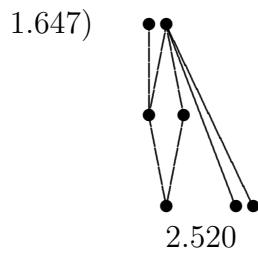
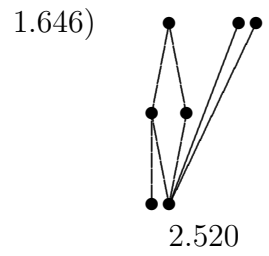
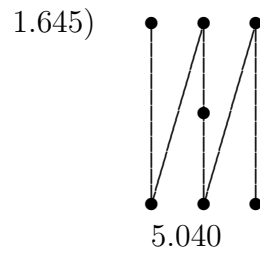
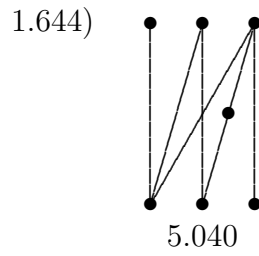
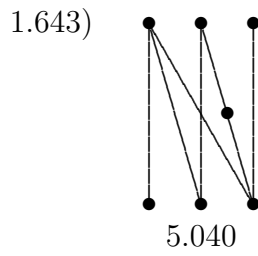
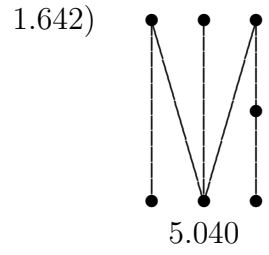
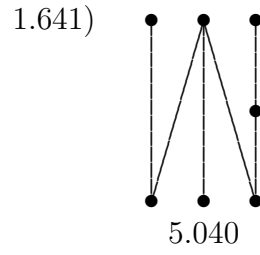
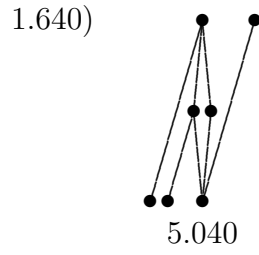
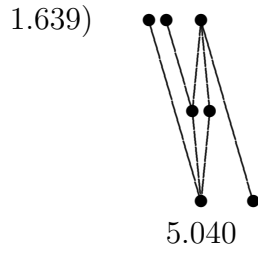
1.608)

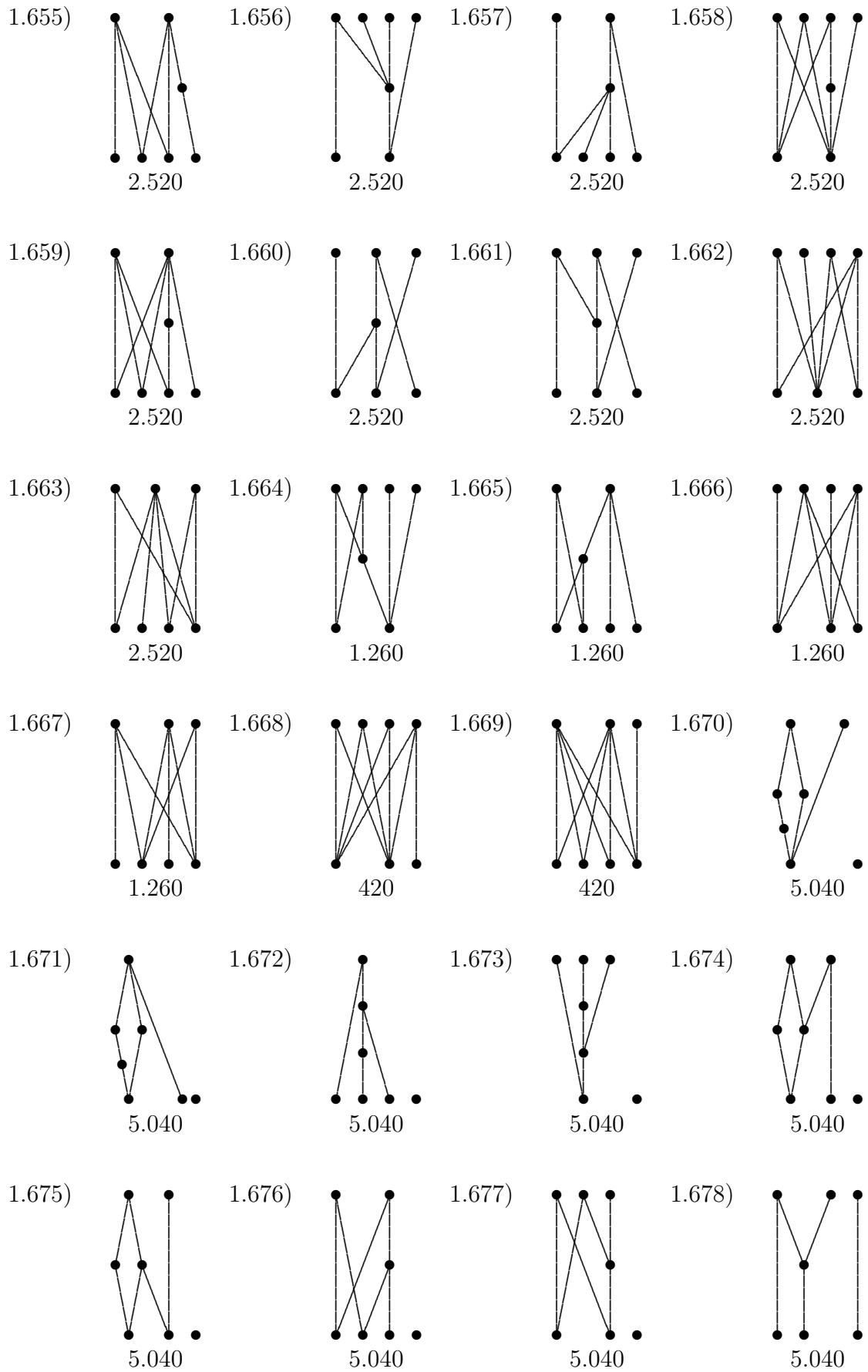




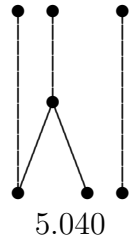


$|RB(\mathbf{7})| = 30$

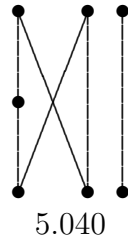




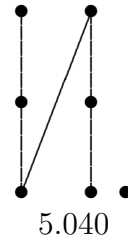
1.679)



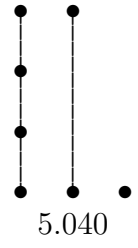
1.680)



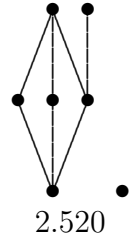
1.681)



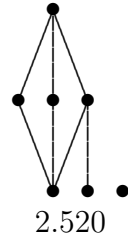
1.682)



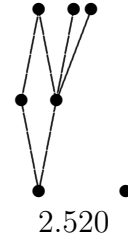
1.683)



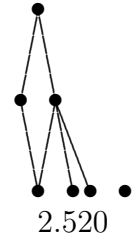
1.684)



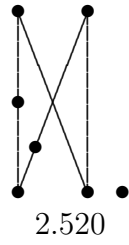
1.685)



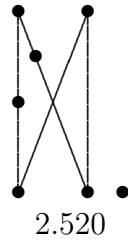
1.686)



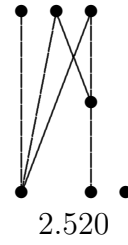
1.687)



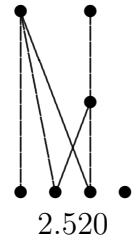
1.688)



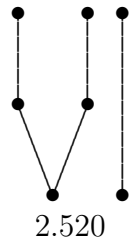
1.689)



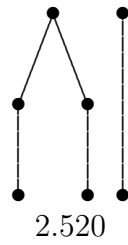
1.690)



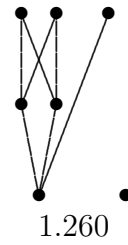
1.691)



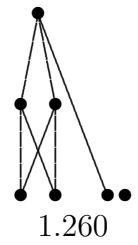
1.692)



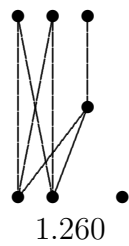
1.693)



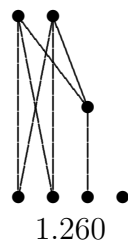
1.694)



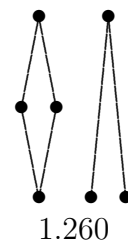
1.695)



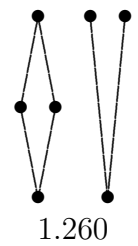
1.696)



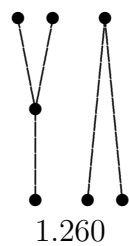
1.697)



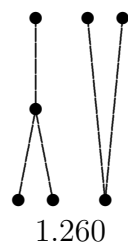
1.698)



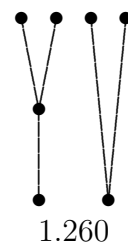
1.699)



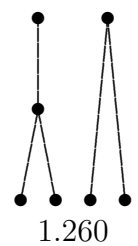
1.700)

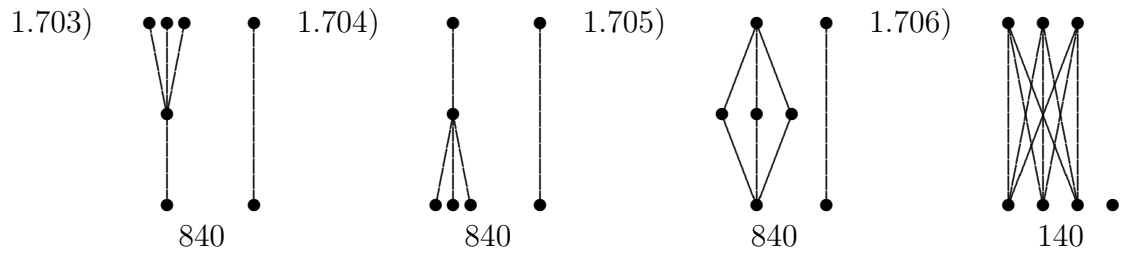


1.701)

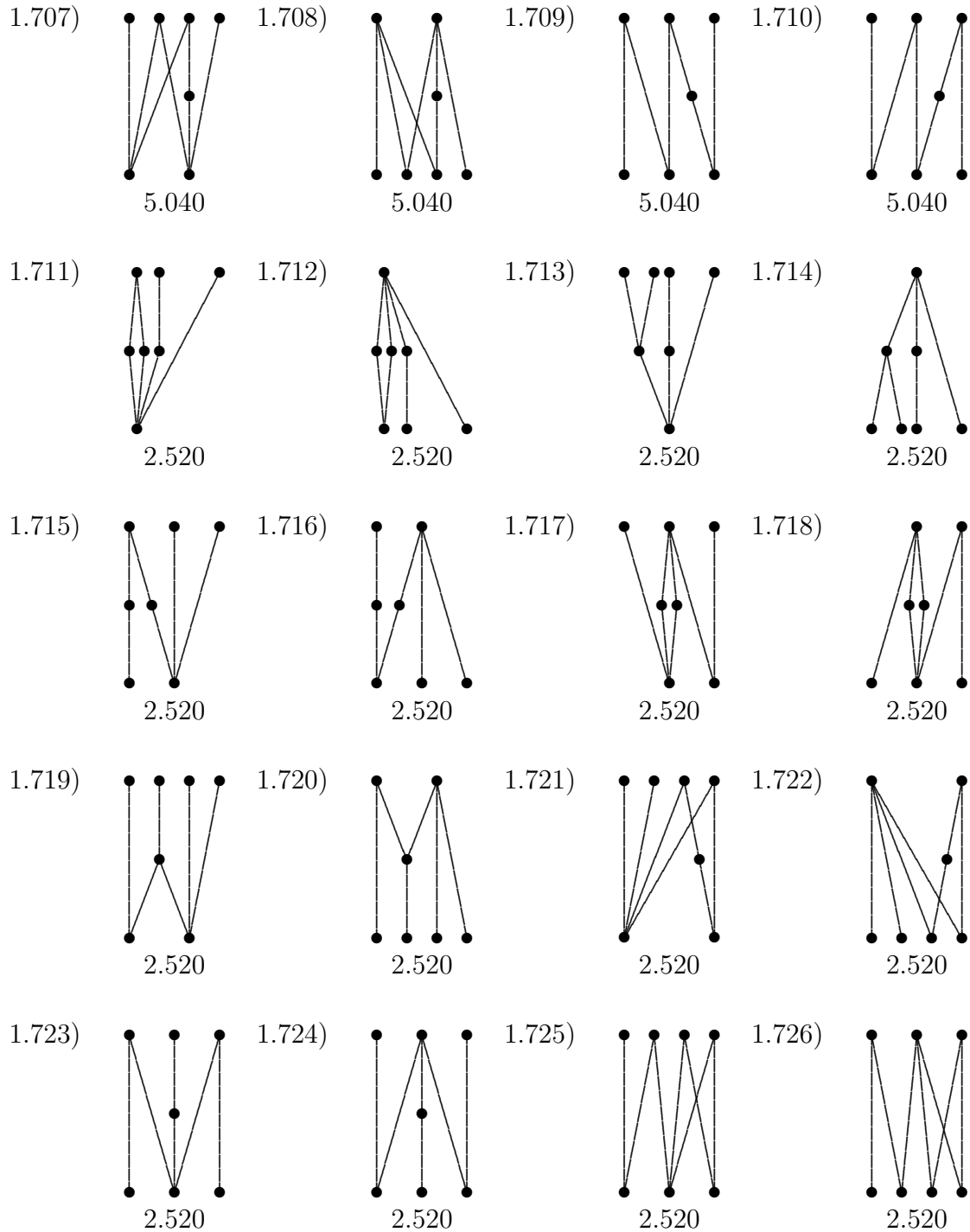


1.702)

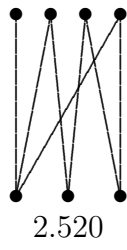




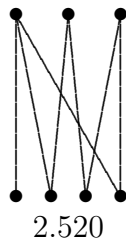
$$|RB(\mathbf{7})| = 31$$



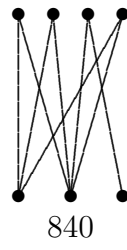
1.727)



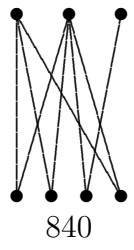
1.728)



1.729)

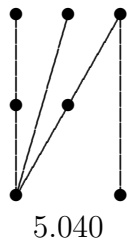


1.730)

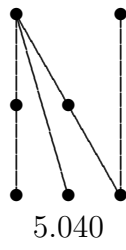


$|RB(\mathbf{7})| = 32$

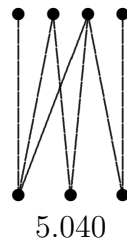
1.731)



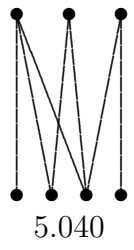
1.732)



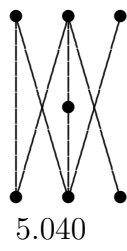
1.733)



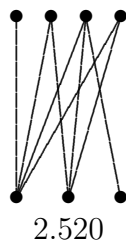
1.734)



1.735)



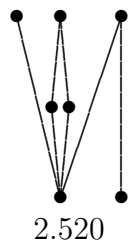
1.736)



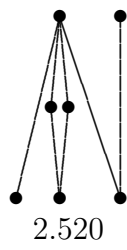
1.737)



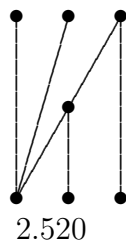
1.738)



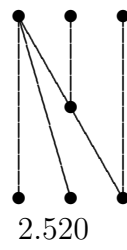
1.739)



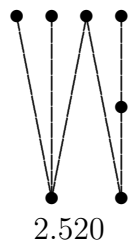
1.740)



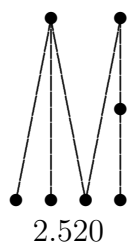
1.741)



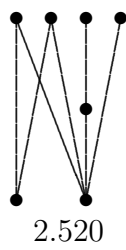
1.742)



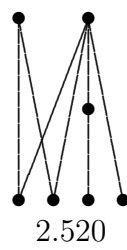
1.743)



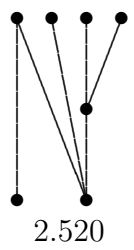
1.744)



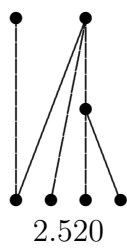
1.745)



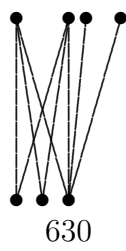
1.746)



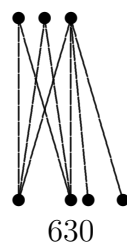
1.747)



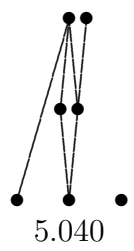
1.748)

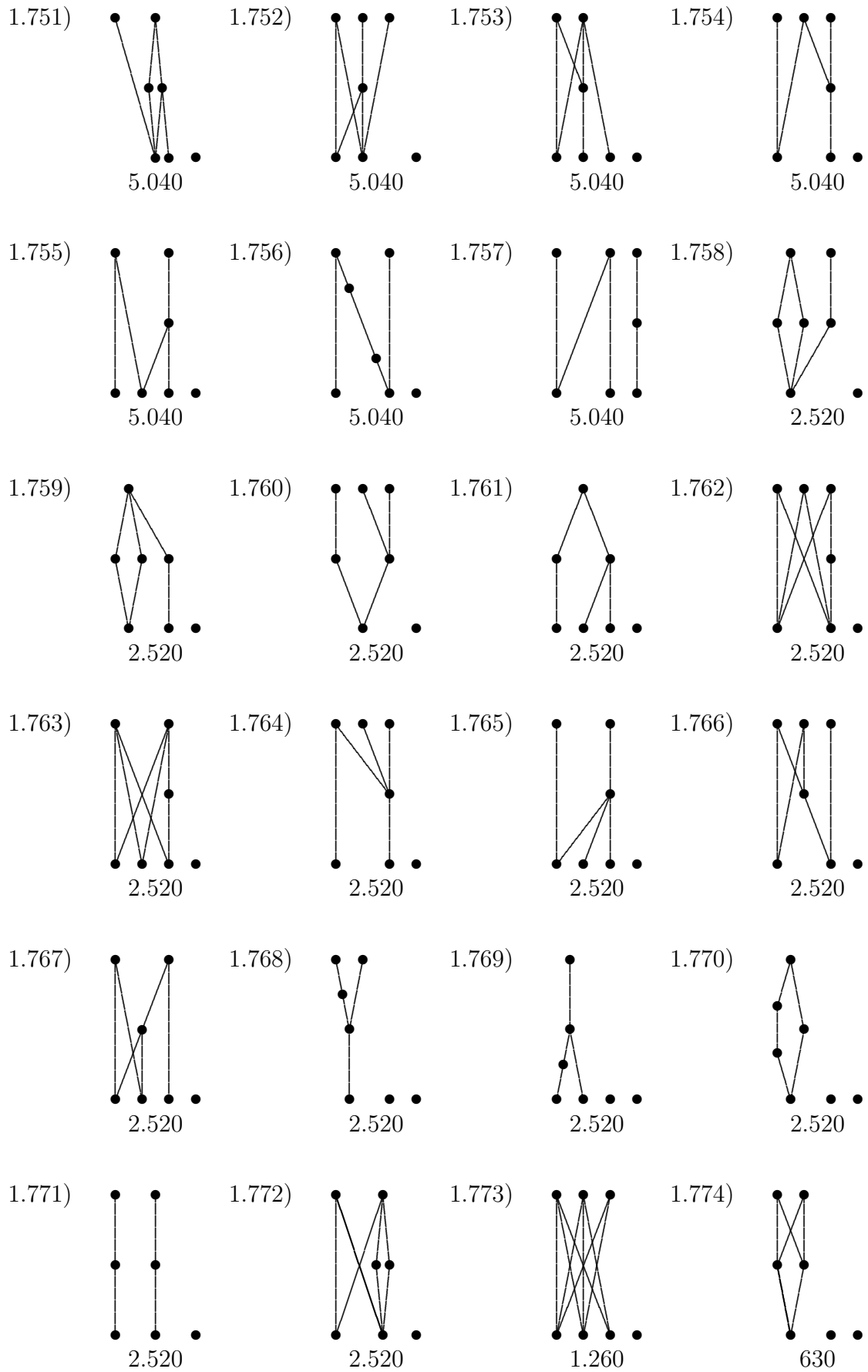


1.749)



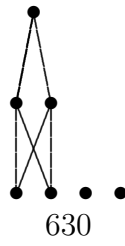
1.750)



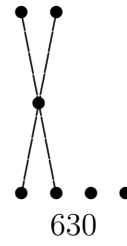


$$|RB(\mathbf{7})| = 33$$

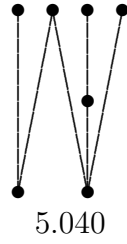
1.775)



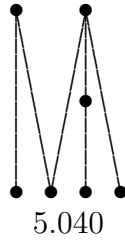
1.776)



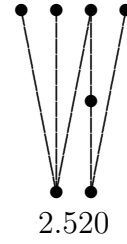
1.777)



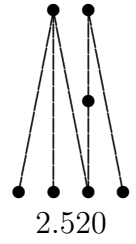
1.778)



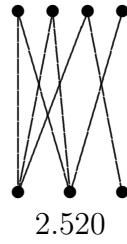
1.779)



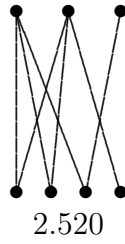
1.780)



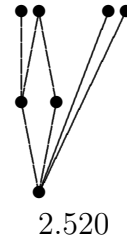
1.781)



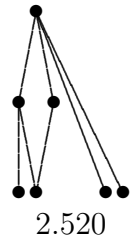
1.782)



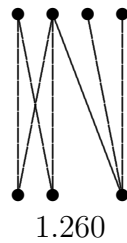
1.783)



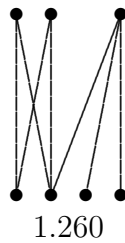
1.784)



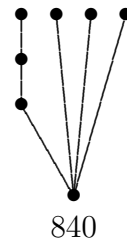
1.785)



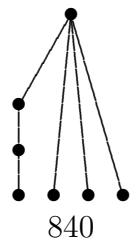
1.786)



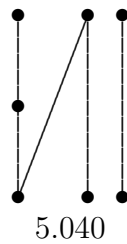
1.787)



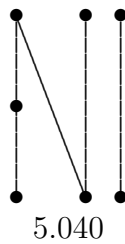
1.788)



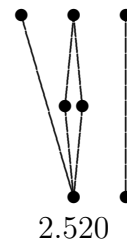
1.789)



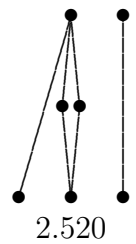
1.790)



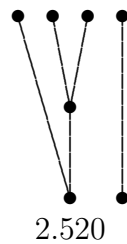
1.791)



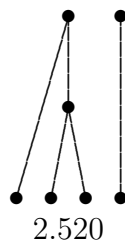
1.792)



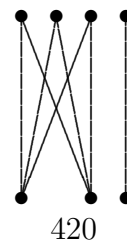
1.793)



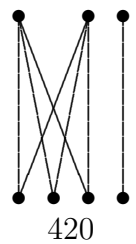
1.794)



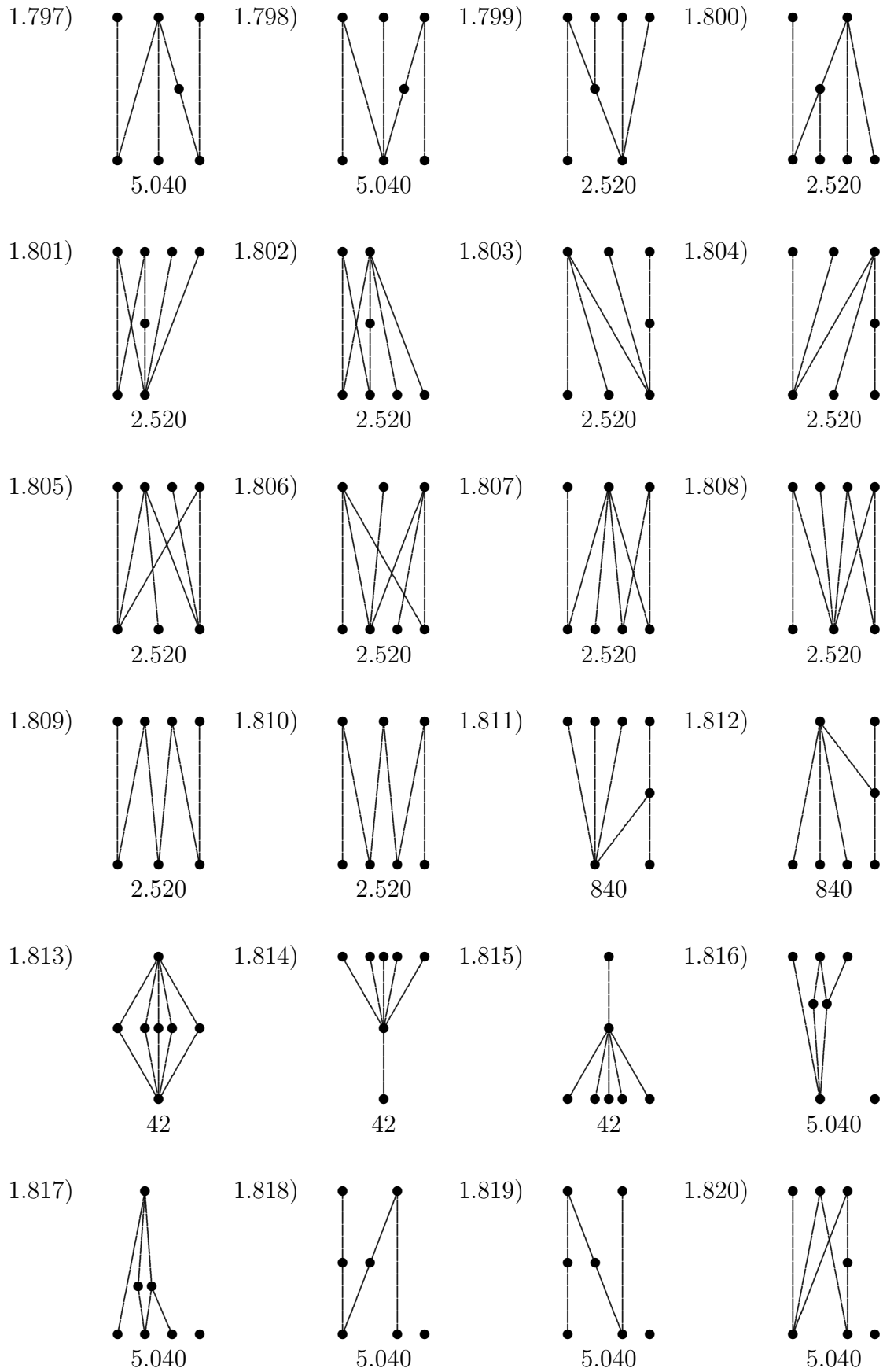
1.795)

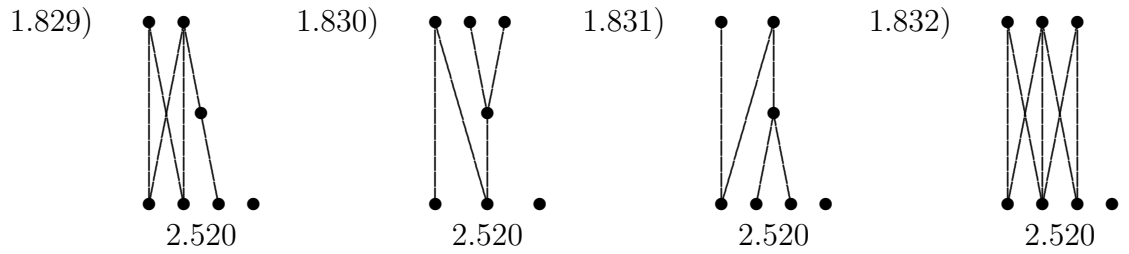
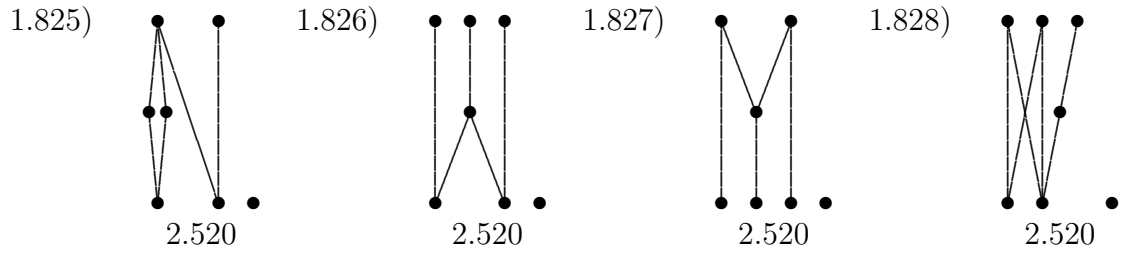
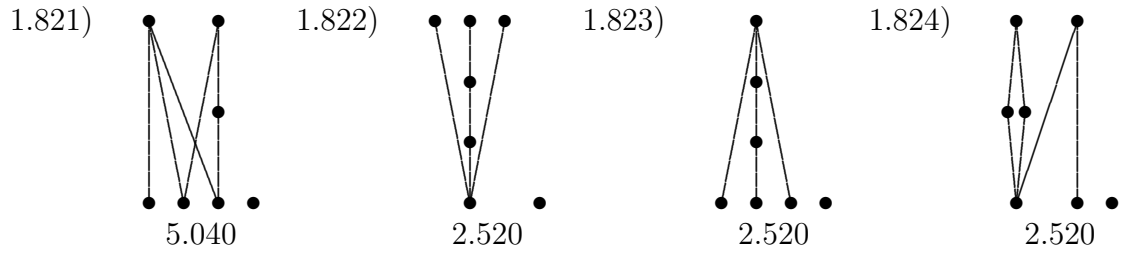


1.796)

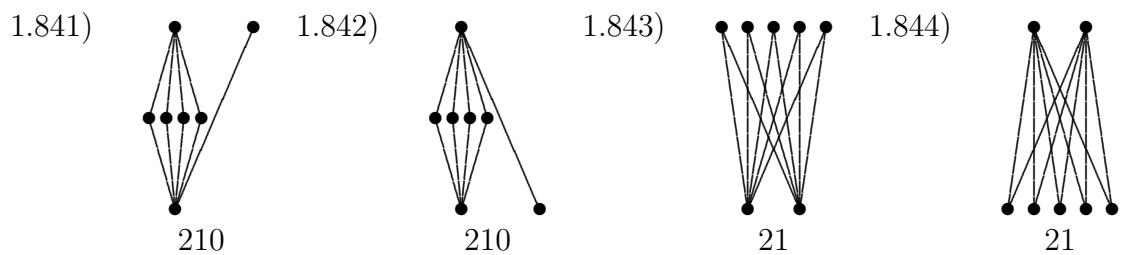
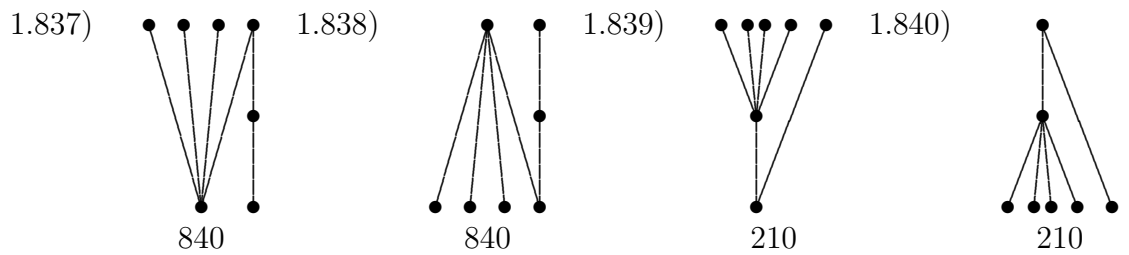
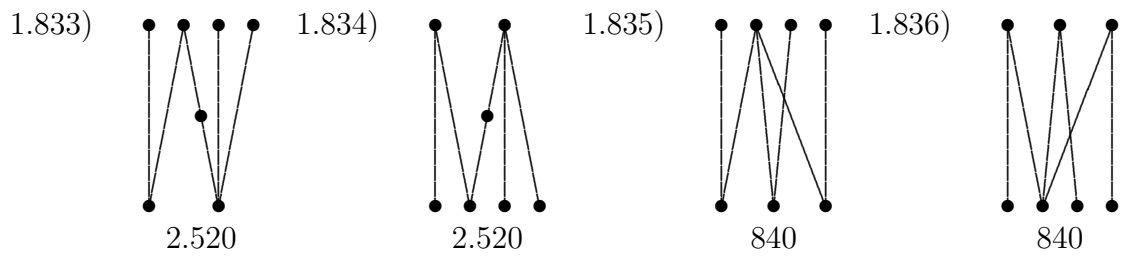


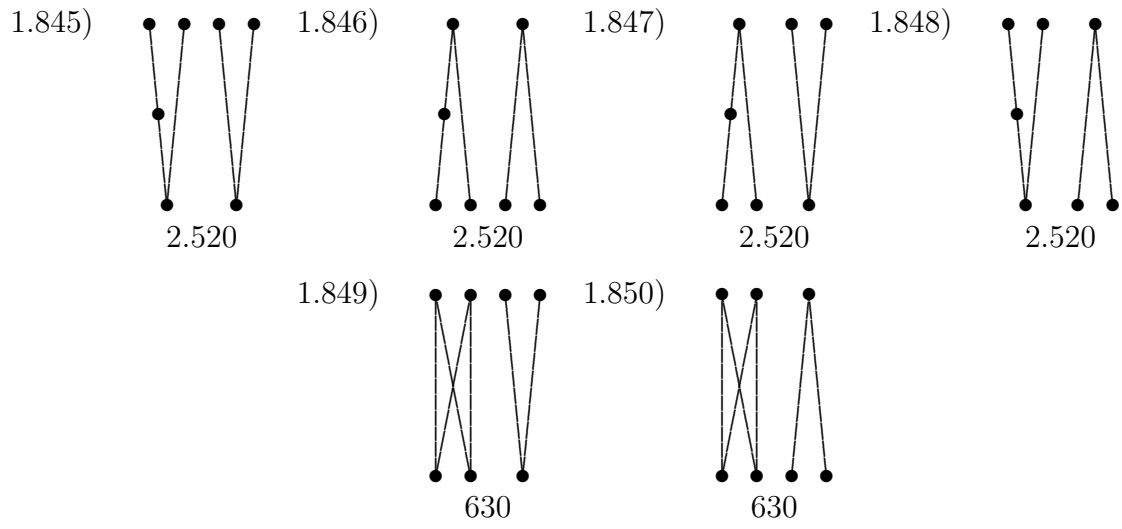
$$|RB(7)| = 34$$



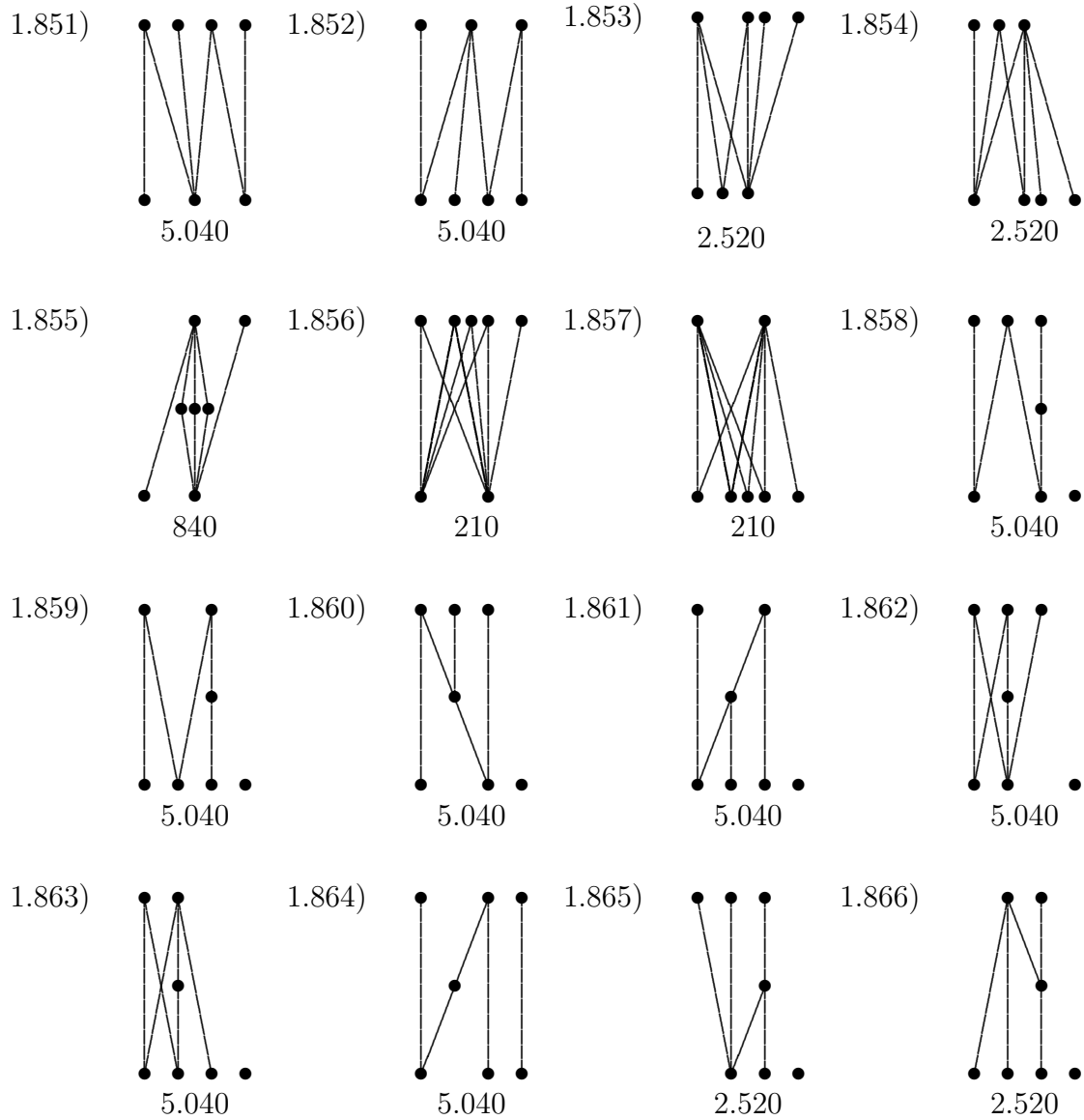


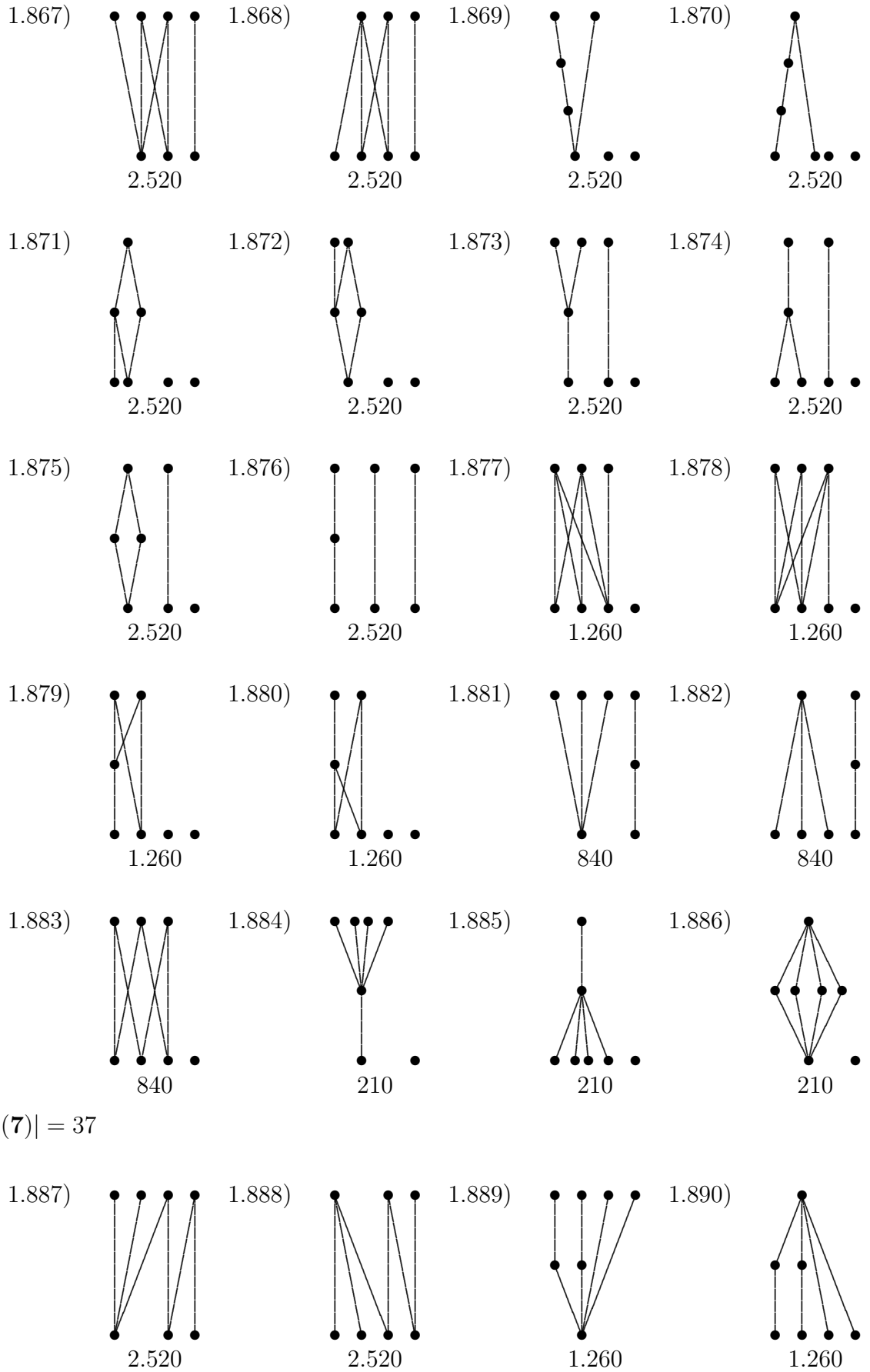
$|RB(7)| = 35$

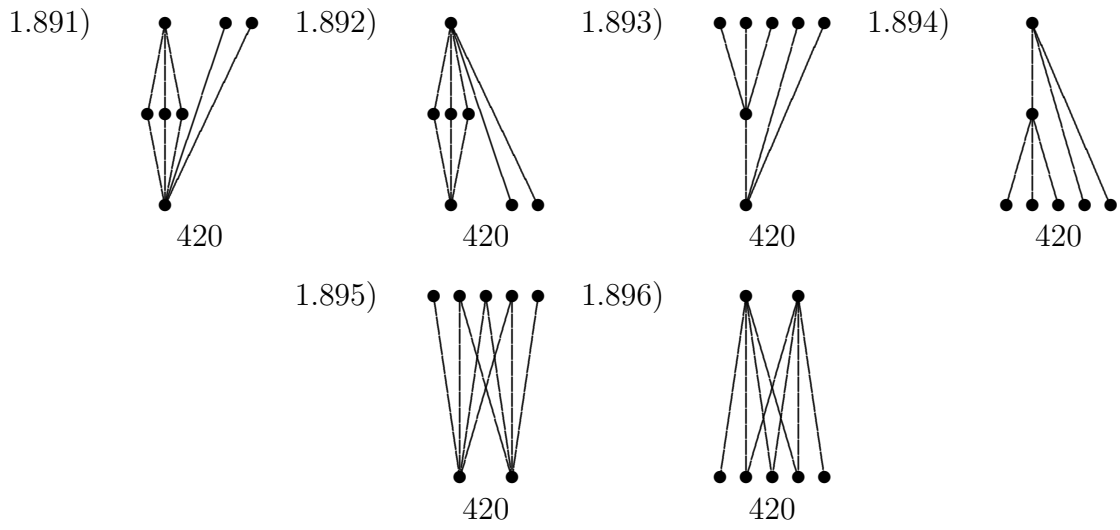
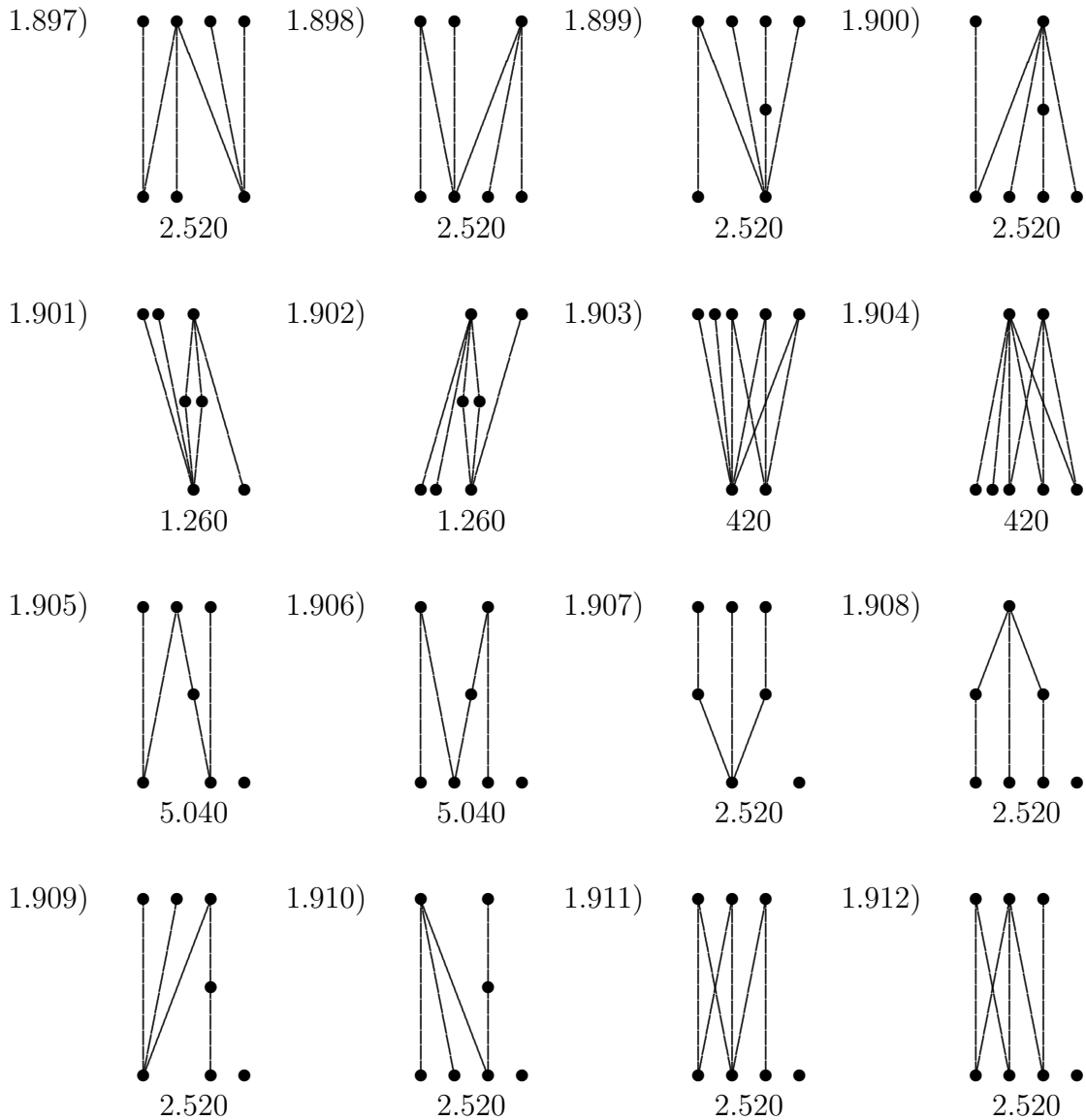




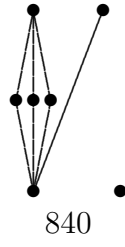
$$|RB(\mathbf{7})| = 36$$



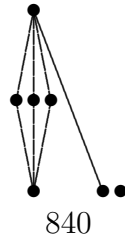



 $|RB(7)| = 38$


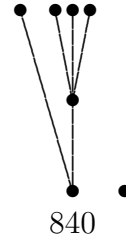
1.913)



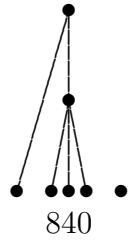
1.914)



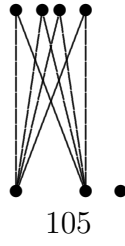
1.915)



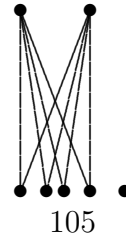
1.916)



1.917)



1.918)

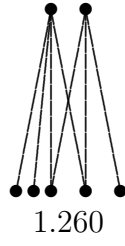


$|RB(\mathbf{7})| = 39$

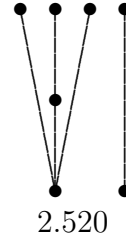
1.919)



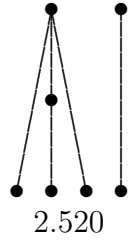
1.920)



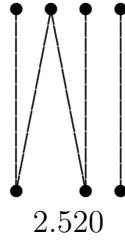
1.921)



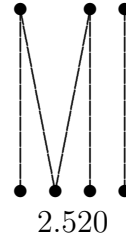
1.922)



1.923)

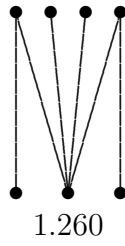


1.924)

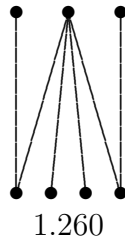


$|RB(\mathbf{7})| = 40$

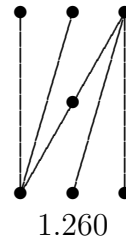
1.925)



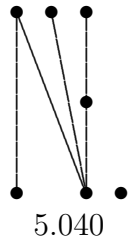
1.926)



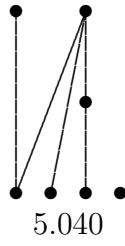
1.927)



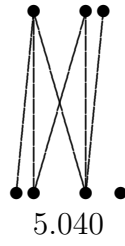
1.928)



1.929)



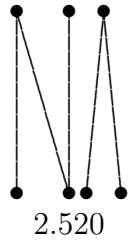
1.930)

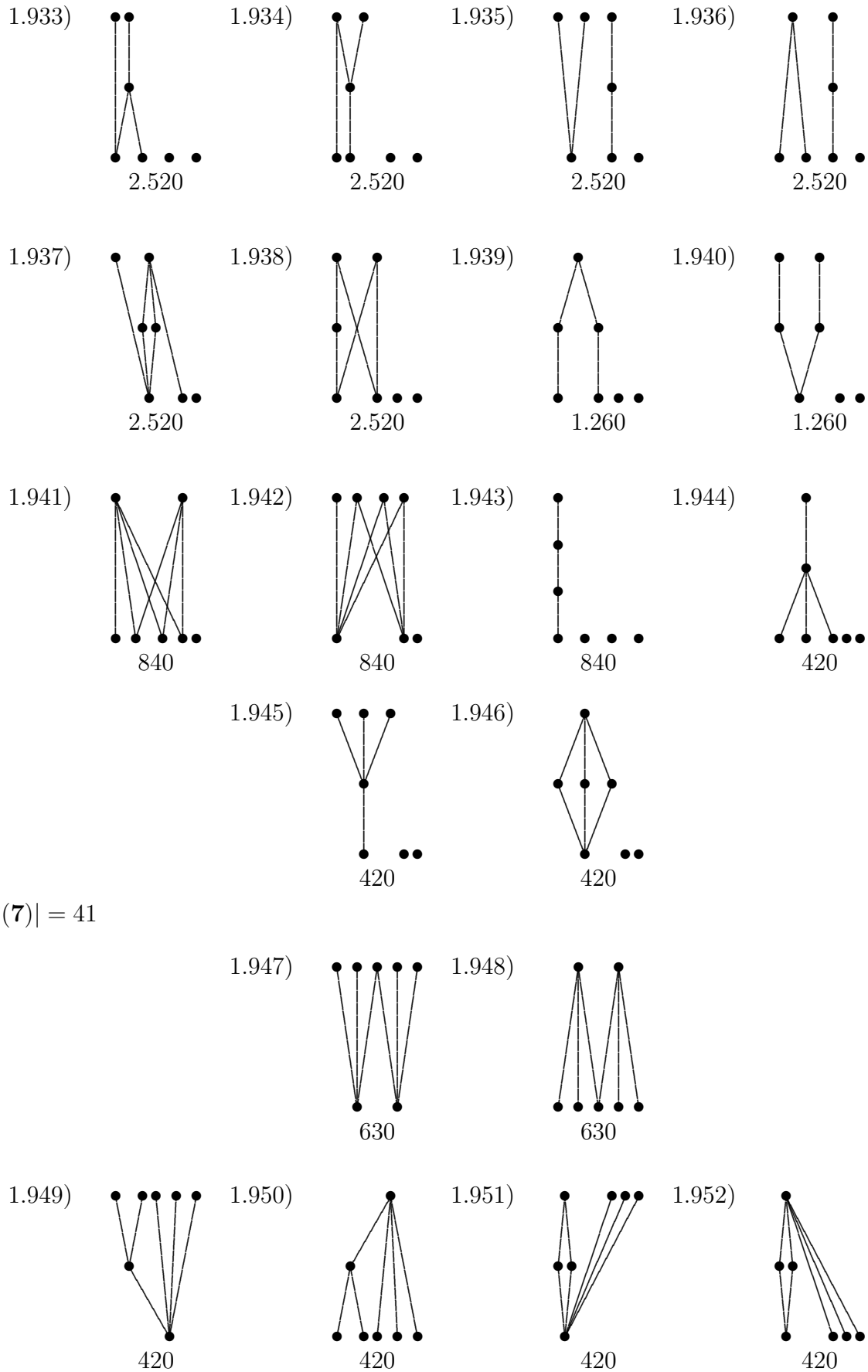


1.931)



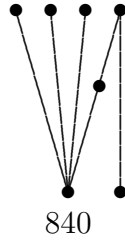
1.932)





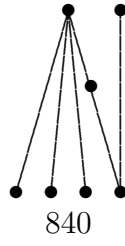
$$|RB(\mathbf{7})| = 42$$

1.953)



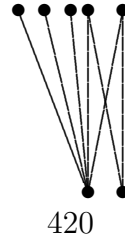
840

1.954)



840

1.955)



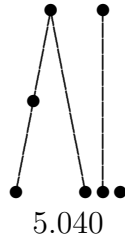
420

1.956)



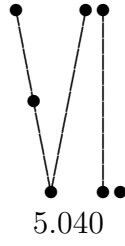
420

1.957)



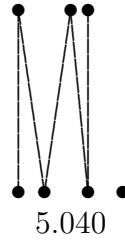
5.040

1.958)



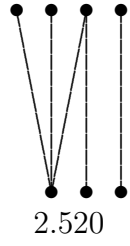
5.040

1.959)



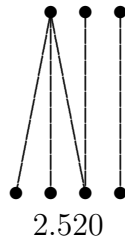
5.040

1.960)



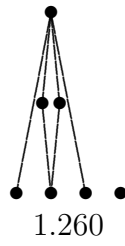
2.520

1.961)



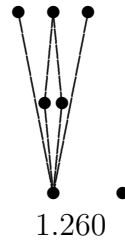
2.520

1.962)



1.260

1.963)



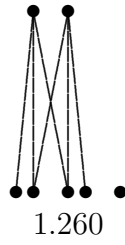
1.260

1.964)



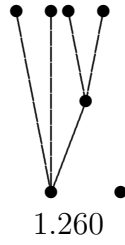
1.260

1.965)



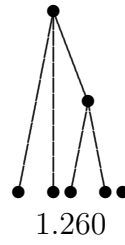
1.260

1.966)



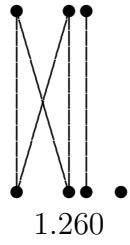
1.260

1.967)



1.260

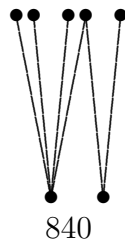
1.968)



1.260

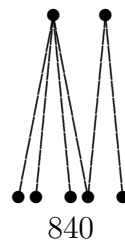
$$|RB(\mathbf{7})| = 43$$

1.969)



840

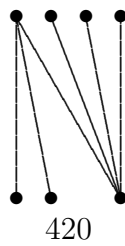
1.970)



840

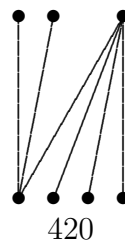
$$|RB(\mathbf{7})| = 44$$

1.971)

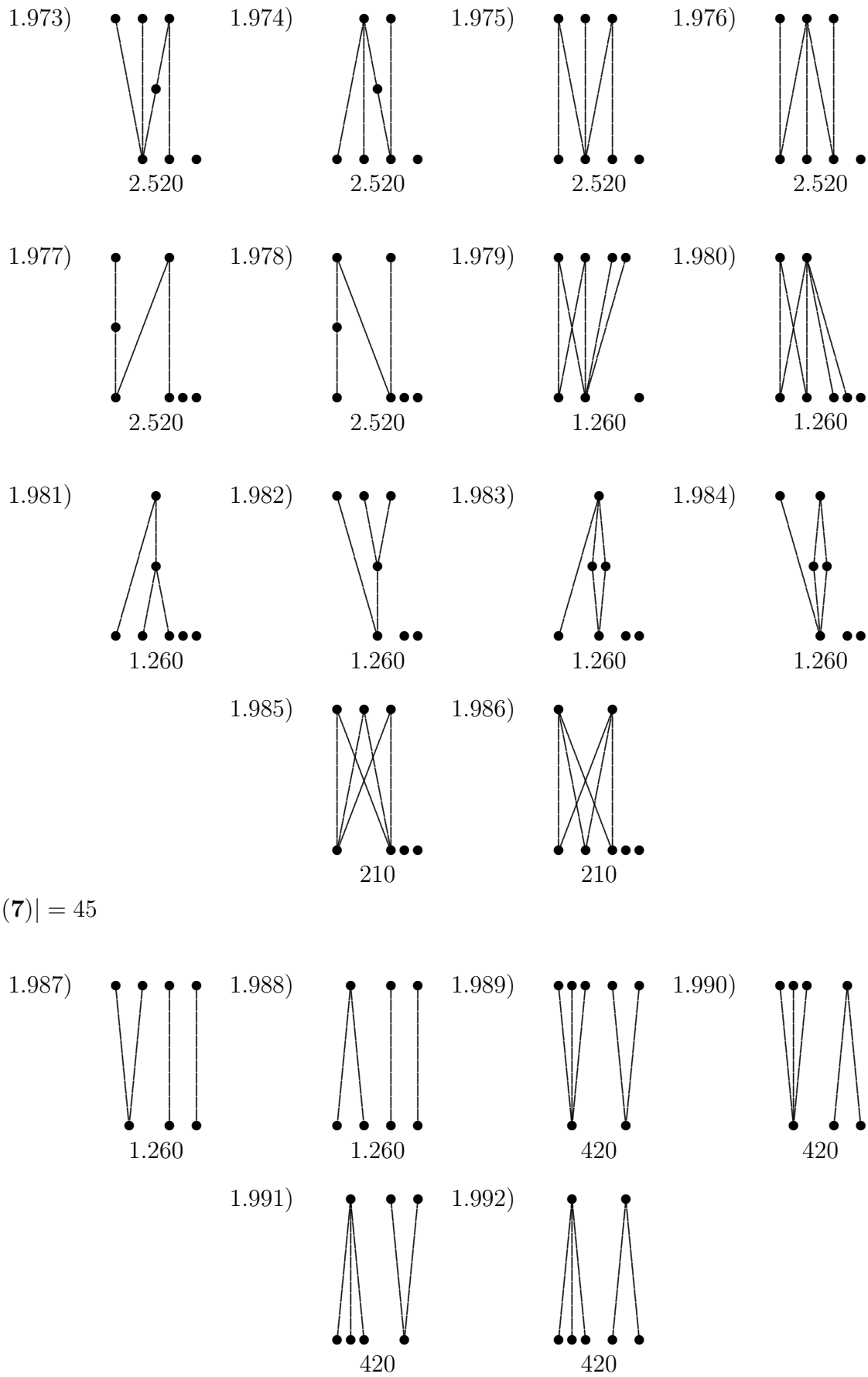


420

1.972)

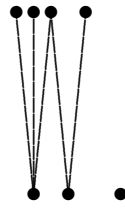


420



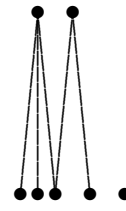
$$|RB(\mathbf{7})| = 46$$

1.993)



2.520

1.994)



2.520

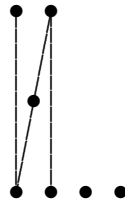
$$|RB(\mathbf{7})| = 48$$

1.995)



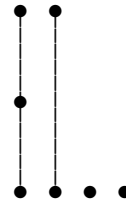
5.040

1.996)



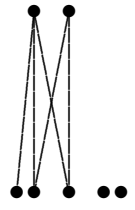
2.520

1.997)



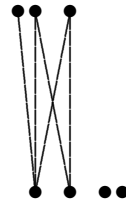
2.520

1.998)



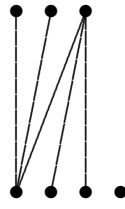
1.260

1.999)



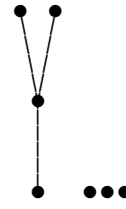
1.260

2.000)



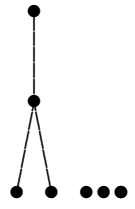
1.260

2.001)



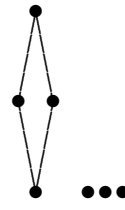
420

2.002)



420

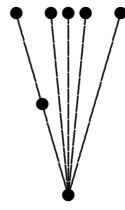
2.003)



420

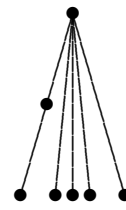
$$|RB(\mathbf{7})| = 49$$

2.004)



210

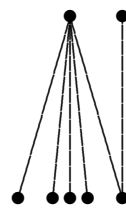
2.005)



210

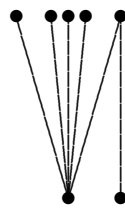
$$|RB(\mathbf{7})| = 50$$

2.006)



210

2.007)



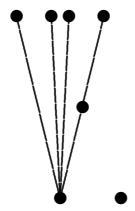
210

2.008)



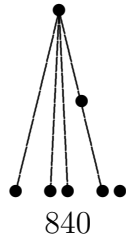
1.260

2.009)



840

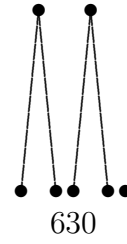
2.010)



2.011)

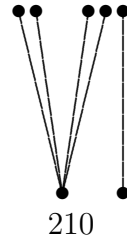


2.012)

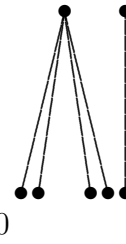


$$|RB(\mathbf{7})| = 51$$

2.013)

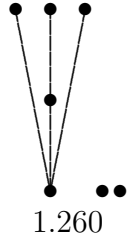


2.014)

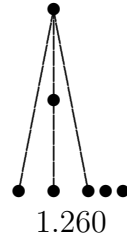


$$|RB(\mathbf{7})| = 52$$

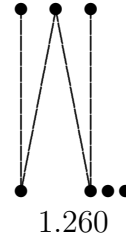
2.015)



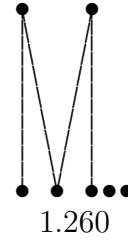
2.016)



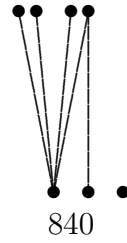
2.017)



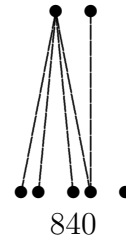
2.018)



2.019)

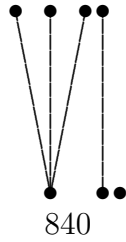


2.020)

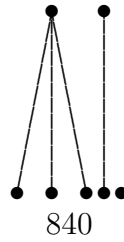


$$|RB(\mathbf{7})| = 54$$

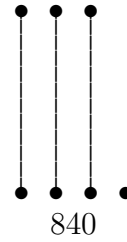
2.021)



2.022)

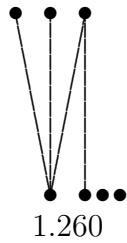


2.023)

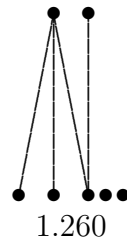


$$|RB(\mathbf{7})| = 56$$

2.024)



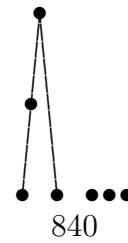
2.025)

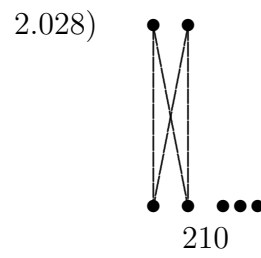


2.026)

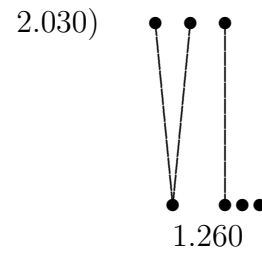
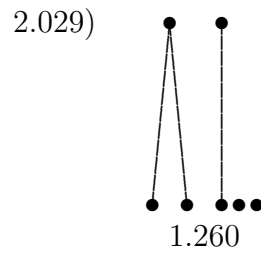


2.027)

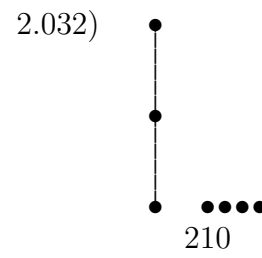
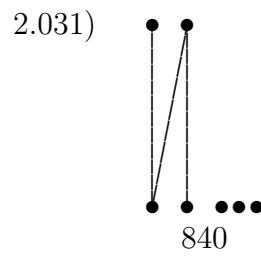




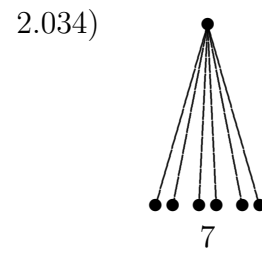
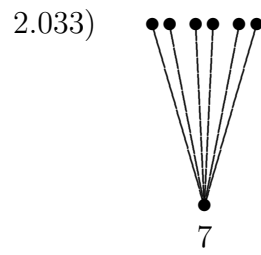
$$|RB(\mathbf{7})| = 60$$



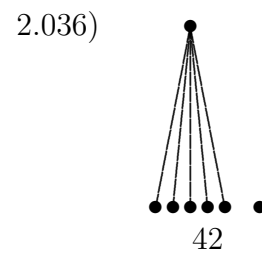
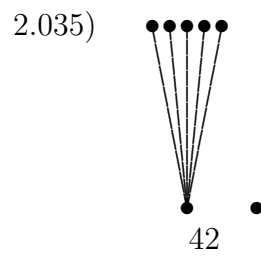
$$|RB(\mathbf{7})| = 64$$



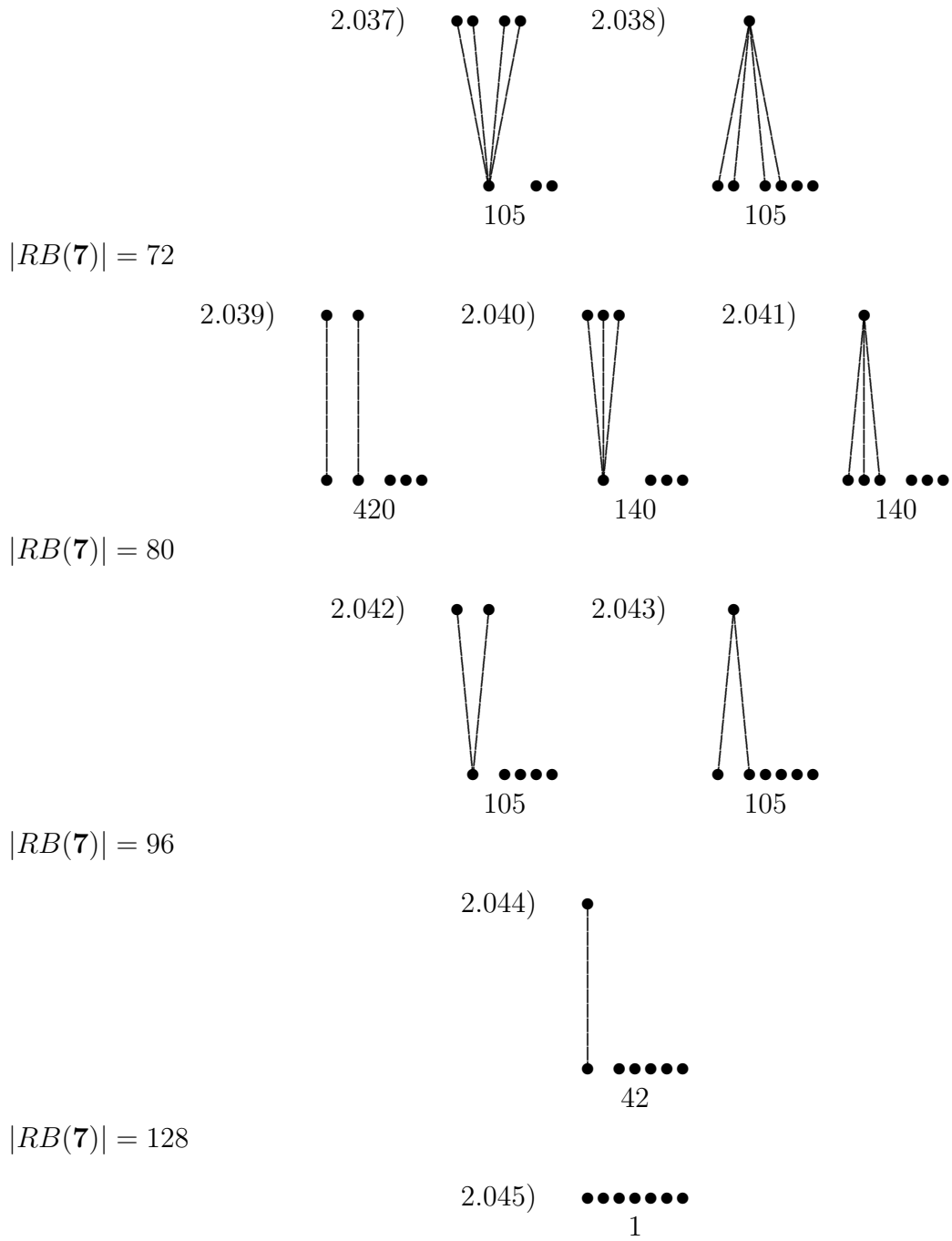
$$|RB(\mathbf{7})| = 65$$



$$|RB(\mathbf{7})| = 66$$



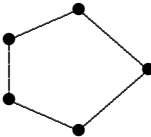
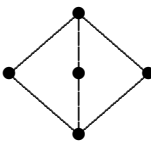
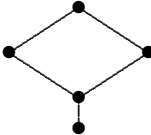
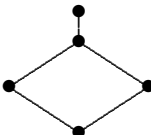
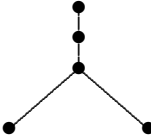
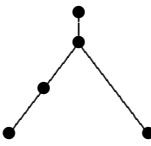
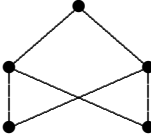
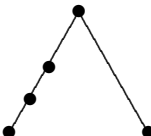
$$|RB(\mathbf{7})| = 68$$



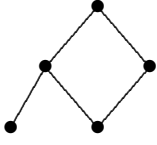
10. Los resultados de S. Savini e I. Viglizzo

10.1. $PC(n, n-5)$, $n \geq 6$.

Si $X \in PC(n, n-5)$ entonces $|m(X)| = n-5$ y $|X \setminus m(X)| = 5$. En los diagramas de la izquierda indicamos los posibles posets, no isomorfos, con 5 elementos

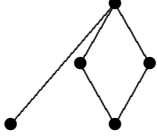
- 1)  $\binom{n}{n-5} 120 (5^{n-5} - 4^{n-5})$
- 2)  $\binom{n}{n-5} 120 (7^{n-5} - 6^{n-5})$
- 3)  $\binom{n}{n-5} 20 (9^{n-5} - 8^{n-5})$
- 4)  $\binom{n}{n-5} 60 (6^{n-5} - 5^{n-5})$
- 5)  $\binom{n}{n-5} 60 (6^{n-5} - 5^{n-5})$
- 6)  $\binom{n}{n-5} 60 (6^{n-5} - 2 \cdot 4^{n-5} + 3^{n-5})$
- 7)  $\binom{n}{n-5} 120 (7^{n-5} - 5^{n-5} - 4^{n-5} + 3^{n-5})$
- 8)  $\binom{n}{n-5} 30 (7^{n-5} - 2 \cdot 5^{n-5} + 4^{n-5})$
- 9)  $\binom{n}{n-5} 120 (8^{n-5} - 6^{n-5} - 4^{n-5} + 3^{n-5})$

10)



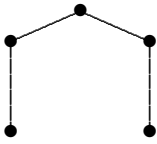
$$\binom{n}{n-5} 120 (8^{n-5} - 6^{n-5} - 5^{n-5} + 4^{n-5})$$

11)



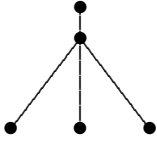
$$\binom{n}{n-5} 60 (10^{n-5} - 8^{n-5} - 5^{n-5} + 4^{n-5})$$

12)



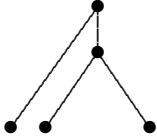
$$\binom{n}{n-5} 60 (9^{n-5} - 2 \cdot 6^{n-5} + 4^{n-5})$$

13)



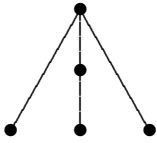
$$\binom{n}{n-5} 20 (9^{n-5} - 3 \cdot 5^{n-5} + 3 \cdot 3^{n-5} - 2^{n-5})$$

14)



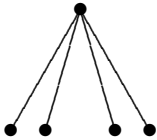
$$\binom{n}{n-5} 60 (10^{n-5} - 2 \cdot 6^{n-5} - 5^{n-5} + 4^{n-5} + 2 \cdot 3^{n-5} - 2^{n-5})$$

15)



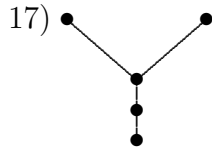
$$\binom{n}{n-5} 60 (12^{n-5} - 8^{n-5} - 2 \cdot 6^{n-5} + 2 \cdot 4^{n-5} + 3^{n-5} - 2^{n-5})$$

16)

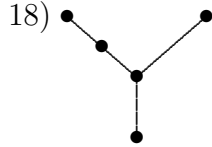


$$\binom{n}{n-5} 5 (16^{n-5} - 4 \cdot 8^{n-5} + 6 \cdot 4^{n-5} - 4 \cdot 2^{n-5} + 1)$$

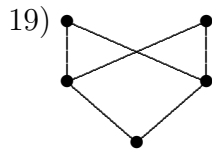
$$|PC(n, n-5, 1)| = \binom{n}{n-5} (5 \cdot 16^{n-5} + 60 \cdot 12^{n-5} + 120 \cdot 10^{n-5} + 100 \cdot 9^{n-5} + 80 \cdot 8^{n-5} + 270 \cdot 7^{n-5} - 540 \cdot 6^{n-5} - 480 \cdot 5^{n-5} + 540 \cdot 3^{n-5} - 160 \cdot 2^{n-5} + 5)$$



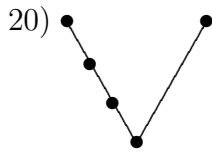
$$\binom{n}{n-5} 60 (6^{n-5} - 5^{n-5})$$



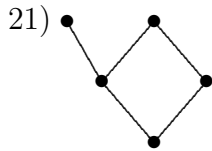
$$\binom{n}{n-5} 120 (7^{n-5} - 6^{n-5})$$



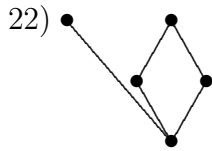
$$\binom{n}{n-5} 30 (7^{n-5} - 6^{n-5})$$



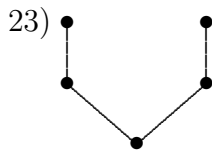
$$\binom{n}{n-5} 120 (8^{n-5} - 7^{n-5})$$



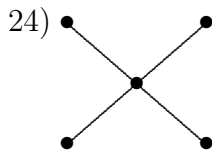
$$\binom{n}{n-5} 120 (8^{n-5} - 7^{n-5})$$



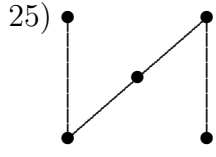
$$\binom{n}{n-5} 60 (10^{n-5} - 9^{n-5})$$



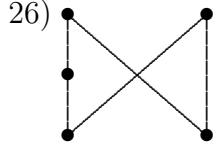
$$\binom{n}{n-5} 60 (9^{n-5} - 8^{n-5})$$



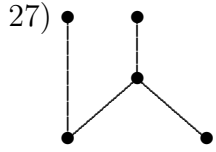
$$\binom{n}{n-5} 30 (7^{n-5} - 2 \cdot 5^{n-5} + 4^{n-5})$$



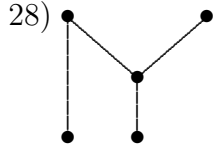
$$\binom{n}{n-5} 120 (11^{n-5} - 9^{n-5} - 6^{n-5} + 5^{n-5})$$



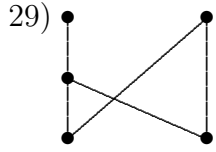
$$\binom{n}{n-5} 120 (9^{n-5} - 7^{n-5} - 6^{n-5} + 5^{n-5})$$



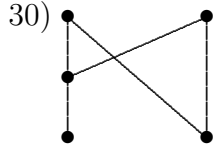
$$\binom{n}{n-5} 120 (9^{n-5} - 7^{n-5} - 6^{n-5} + 5^{n-5})$$



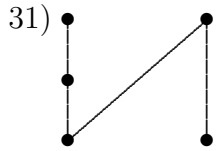
$$\binom{n}{n-5} 120 (9^{n-5} - 7^{n-5} - 5^{n-5} + 4^{n-5})$$



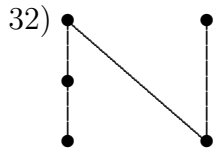
$$\binom{n}{n-5} 60 (8^{n-5} - 2 \cdot 6^{n-5} + 5^{n-5})$$



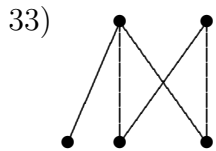
$$\binom{n}{n-5} 60 (8^{n-5} - 6^{n-5} - 5^{n-5} + 4^{n-5})$$



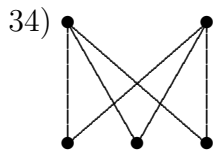
$$\binom{n}{n-5} 120 (10^{n-5} - 8^{n-5} - 6^{n-5} + 5^{n-5})$$



$$\binom{n}{n-5} 120 (10^{n-5} - 2 \cdot 7^{n-5} + 5^{n-5})$$

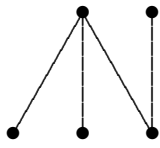


$$\binom{n}{n-5} 60 (11^{n-5} - 2 \cdot 7^{n-5} - 6^{n-5} + 5^{n-5} + 2 \cdot 4^{n-5} - 3^{n-5})$$



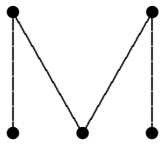
$$\binom{n}{n-5} 10 (10^{n-5} - 3 \cdot 6^{n-5} + 3 \cdot 4^{n-5} - 3^{n-5})$$

35)



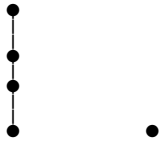
$$\binom{n}{n-5} 60 (13^{n-5} - 9^{n-5} - 2 \cdot 7^{n-5} + 2 \cdot 5^{n-5} + 4^{n-5} - 3^{n-5})$$

36)



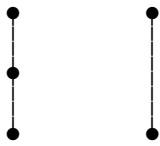
$$\binom{n}{n-5} 60 (12^{n-5} - 8^{n-5} - 2 \cdot 7^{n-5} + 2 \cdot 5^{n-5} + 4^{n-5} - 3^{n-5})$$

37)



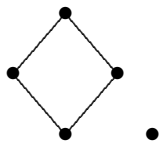
$$\binom{n}{n-5} 120 (9^{n-5} - 7^{n-5} - 5^{n-5} + 4^{n-5})$$

38)



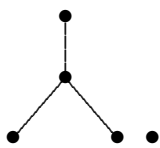
$$\binom{n}{n-5} 120 (11^{n-5} - 8^{n-5} - 7^{n-5} + 2 \cdot 4^{n-5} - 3^{n-5})$$

39)



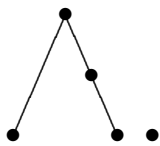
$$\binom{n}{n-5} 60 (11^{n-5} - 9^{n-5} - 6^{n-5} + 5^{n-5})$$

40)



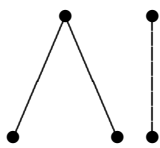
$$\binom{n}{n-5} 60 (11^{n-5} - 2 \cdot 7^{n-5} - 6^{n-5} + 5^{n-5} + 2 \cdot 4^{n-5} - 3^{n-5})$$

41)



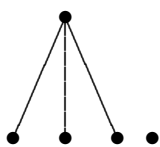
$$\binom{n}{n-5} 120 (13^{n-5} - 9^{n-5} - 2 \cdot 7^{n-5} + 2 \cdot 5^{n-5} + 4^{n-5} - 3^{n-5})$$

42)



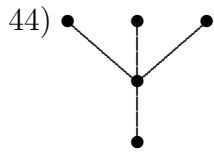
$$\binom{n}{n-5} 60 (14^{n-5} - 9^{n-5} - 2 \cdot 8^{n-5} - 6^{n-5} + 4 \cdot 5^{n-5} + 2 \cdot 4^{n-5} - 4 \cdot 3^{n-5} + 2^{n-5})$$

43)

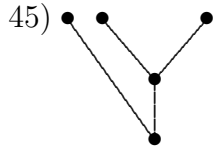


$$\binom{n}{n-5} 20 (17^{n-5} - 4 \cdot 9^{n-5} + 6 \cdot 5^{n-5} - 4 \cdot 3^{n-5} + 2^{n-5})$$

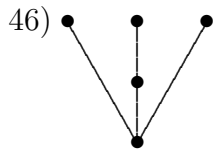
$$|PC(n, n-5, 2)| = \binom{n}{n-5} (20 \cdot 17^{n-5} + 60 \cdot 14^{n-5} + 180 \cdot 13^{n-5} + 60 \cdot 12^{n-5} + 420 \cdot 11^{n-5} + 310 \cdot 10^{n-5} - 20 \cdot 9^{n-5} - 120 \cdot 8^{n-5} - 1.620 \cdot 7^{n-5} - 1.020 \cdot 6^{n-5} + 1.260 \cdot 5^{n-5} + 1.200 \cdot 4^{n-5} - 810 \cdot 3^{n-5} + 80 \cdot 2^{n-5})$$



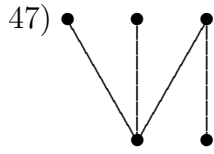
$$\binom{n}{n-5} 20 (9^{n-5} - 8^{n-5})$$



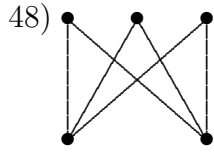
$$\binom{n}{n-5} 60 (10^{n-5} - 9^{n-5})$$



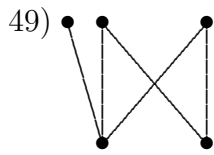
$$\binom{n}{n-5} 60 (12^{n-5} - 11^{n-5})$$



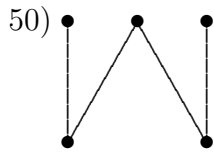
$$\binom{n}{n-5} 60 (13^{n-5} - 11^{n-5} - 8^{n-5} + 7^{n-5})$$



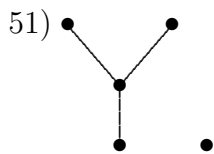
$$\binom{n}{n-5} 10 (10^{n-5} - 2 \cdot 8^{n-5} + 7^{n-5})$$



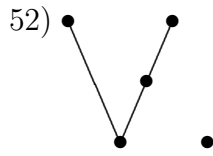
$$\binom{n}{n-5} 60 (11^{n-5} - 9^{n-5} - 8^{n-5} + 7^{n-5})$$



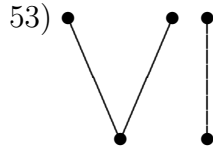
$$\binom{n}{n-5} 60 (12^{n-5} - 2 \cdot 9^{n-5} + 7^{n-5})$$



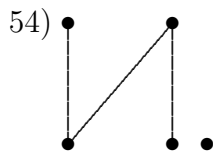
$$\binom{n}{n-5} 60 (11^{n-5} - 9^{n-5} - 6^{n-5} + 5^{n-5})$$



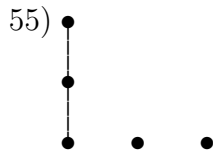
$$\binom{n}{n-5} 120 (13^{n-5} - 11^{n-5} - 7^{n-5} + 6^{n-5})$$



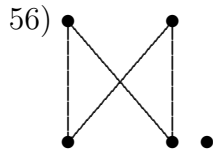
$$\binom{n}{n-5} 60 (14^{n-5} - 11^{n-5} - 9^{n-5} + 7^{n-5} - 6^{n-5} + 2 \cdot 5^{n-5} - 4^{n-5})$$



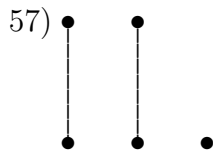
$$\binom{n}{n-5} 120 (15^{n-5} - 11^{n-5} - 9^{n-5} - 8^{n-5} + 7^{n-5} + 6^{n-5} + 5^{n-5} - 4^{n-5})$$



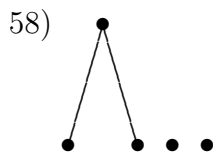
$$\binom{n}{n-5} 60 (15^{n-5} - 11^{n-5} - 2 \cdot 8^{n-5} + 6^{n-5} + 3 \cdot 5^{n-5} - 2 \cdot 4^{n-5})$$



$$\binom{n}{n-5} 30 (13^{n-5} - 2 \cdot 9^{n-5} + 2 \cdot 5^{n-5} - 4^{n-5})$$

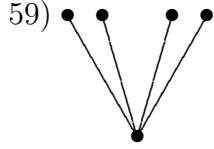


$$\binom{n}{n-5} 60 (17^{n-5} - 2 \cdot 11^{n-5} - 9^{n-5} - 7^{n-5} + 4 \cdot 6^{n-5} + 4 \cdot 5^{n-5} - 7 \cdot 4^{n-5} + 2 \cdot 3^{n-5})$$

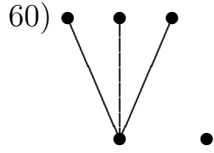


$$\binom{n}{n-5} 30 (19^{n-5} - 2 \cdot 11^{n-5} - 2 \cdot 10^{n-5} + 6 \cdot 6^{n-5} + 2 \cdot 5^{n-5} - 7 \cdot 4^{n-5} + 2 \cdot 3^{n-5})$$

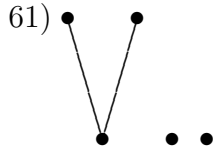
$$|PC(n, n-5, 3)| = \binom{n}{n-5} (30 \cdot 19^{n-5} + 60 \cdot 17^{n-5} + 180 \cdot 15^{n-5} + 60 \cdot 14^{n-5} + 210 \cdot 13^{n-5} + 120 \cdot 12^{n-5} - 540 \cdot 11^{n-5} + 10 \cdot 10^{n-5} - 580 \cdot 9^{n-5} - 400 \cdot 8^{n-5} + 190 \cdot 7^{n-5} + 600 \cdot 6^{n-5} + 840 \cdot 5^{n-5} - 960 \cdot 4^{n-5} + 180 \cdot 3^{n-5})$$



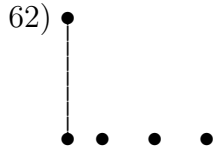
$$\binom{n}{n-5} 5 (16^{n-5} - 15^{n-5})$$



$$\binom{n}{n-5} 20 (17^{n-5} - 15^{n-5} - 9^{n-5} + 8^{n-5})$$



$$\binom{n}{n-5} 30 (19^{n-5} - 15^{n-5} - 2 \cdot 10^{n-5} + 2 \cdot 8^{n-5} - 7^{n-5} + 3 \cdot 6^{n-5} - 2 \cdot 5^{n-5})$$



$$\binom{n}{n-5} 20 (23^{n-5} - 15^{n-5} - 3 \cdot 12^{n-5} - 9^{n-5} + 8^{n-5} + 6 \cdot 7^{n-5} + 9 \cdot 6^{n-5} - 18 \cdot 5^{n-5} + 6 \cdot 4^{n-5})$$

$$|PC(n, n-5, 4)| = \binom{n}{n-5} (20 \cdot 23^{n-5} + 30 \cdot 19^{n-5} + 20 \cdot 17^{n-5} + 5 \cdot 16^{n-5} - 75 \cdot 15^{n-5} - 60 \cdot 12^{n-5} - 60 \cdot 10^{n-5} - 40 \cdot 9^{n-5} + 100 \cdot 8^{n-5} + 90 \cdot 7^{n-5} + 270 \cdot 6^{n-5} - 420 \cdot 5^{n-5} + 120 \cdot 4^{n-5})$$



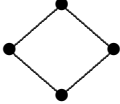
$$|PC(n, n-5, 5)| = \binom{n}{n-5} (31^{n-5} - 5 \cdot 16^{n-5} - 10 \cdot 10^{n-5} + 20 \cdot 9^{n-5} + 30 \cdot 7^{n-5} - 60 \cdot 6^{n-5} + 24 \cdot 5^{n-5})$$

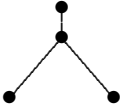
$$|PC(n, n-5)| = \binom{n}{n-5} (31^{n-5} + 20 \cdot 23^{n-5} + 60 \cdot 19^{n-5} + 100 \cdot 17^{n-5} + 5 \cdot 16^{n-5} + 105 \cdot 15^{n-5} + 120 \cdot 14^{n-5} + 390 \cdot 13^{n-5} - 120 \cdot 11^{n-5} + 250 \cdot 10^{n-5} - 620 \cdot 9^{n-5} - 424 \cdot 8^{n-5} - 1.310 \cdot 7^{n-5} - 210 \cdot 6^{n-5} + 2.804 \cdot 5^{n-5} + 366 \cdot 4^{n-5} - 630 \cdot 3^{n-5} + 76 \cdot 2^{n-5} + 1)$$

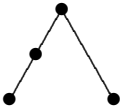
10.2. $PC(n, n-4)$, $n \geq 5$.

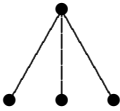
Si $X \in PC(n, n-4)$ entonces $|m(X)| = n-4$ y $|X \setminus m(X)| = 4$. En los diagramas de la izquierda indicamos los posibles posets, no isomorfos, con 4 elementos

1)  $\binom{n}{n-4} 24 (4^{n-4} - 3^{n-4})$

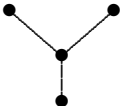
2)  $\binom{n}{n-4} 12 (5^{n-4} - 4^{n-4})$

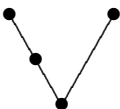
3)  $\binom{n}{n-4} 12 (5^{n-4} - 2 \cdot 3^{n-4} + 2^{n-4})$

4)  $\binom{n}{n-4} 24 (6^{n-4} - 4^{n-4} - 3^{n-4} + 2^{n-4})$

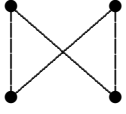
5)  $\binom{n}{n-4} 4 (8^{n-4} - 3 \cdot 4^{n-4} + 3 \cdot 2^{n-4} - 1)$

$$|PC(n, n-4, 1)| = \binom{n}{n-4} (4 \cdot 8^{n-4} + 24 \cdot 6^{n-4} + 24 \cdot 5^{n-4} - 24 \cdot 4^{n-4} - 72 \cdot 3^{n-4} + 48 \cdot 2^{n-4} - 4)$$

6)  $\binom{n}{n-4} 12 (5^{n-4} - 4^{n-4})$

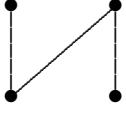
7)  $\binom{n}{n-4} 24 (6^{n-4} - 5^{n-4})$

8)



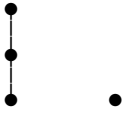
$$\binom{n}{n-4} 6 (6^{n-4} - 2 \cdot 4^{n-4} + 3^{n-4})$$

9)



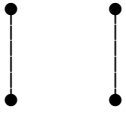
$$\binom{n}{n-4} 24 (7^{n-4} - 5^{n-4} - 4^{n-4} + 3^{n-4})$$

10)



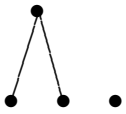
$$\binom{n}{n-4} 24 (7^{n-4} - 5^{n-4} - 4^{n-4} + 3^{n-4})$$

11)



$$\binom{n}{n-4} 12 (8^{n-4} - 2 \cdot 5^{n-4} - 4^{n-4} + 3 \cdot 3^{n-4} - 2^{n-4})$$

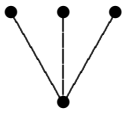
12)



$$\binom{n}{n-4} 12 (9^{n-4} - 3 \cdot 5^{n-4} + 3 \cdot 3^{n-4} - 2^{n-4})$$

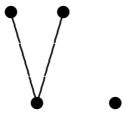
$$|PC(n, n-4, 2)| = \binom{n}{n-4} (12 \cdot 9^{n-4} + 12 \cdot 8^{n-4} + 48 \cdot 7^{n-4} + 30 \cdot 6^{n-4} - 120 \cdot 5^{n-4} - 84 \cdot 4^{n-4} + 126 \cdot 3^{n-4} - 24 \cdot 2^{n-4})$$

13)



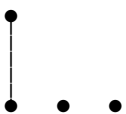
$$\binom{n}{n-4} 4 (8^{n-4} - 7^{n-4})$$

14)



$$\binom{n}{n-4} 12 (9^{n-4} - 7^{n-4} - 5^{n-4} + 4^{n-4})$$

15)



$$\binom{n}{n-4} 12 (11^{n-4} - 7^{n-4} - 2 \cdot 6^{n-4} - 5^{n-4} + 5 \cdot 4^{n-4} - 2 \cdot 3^{n-4})$$

$$|PC(n, n-4, 3)| = \binom{n}{n-4} (12 \cdot 11^{n-4} + 12 \cdot 9^{n-4} + 4 \cdot 8^{n-4} - 28 \cdot 7^{n-4} - 24 \cdot 6^{n-4} - 24 \cdot 5^{n-4} + 72 \cdot 4^{n-4} - 24 \cdot 3^{n-4})$$

16) • • • •

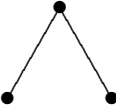
$$|PC(n, n-4, 4)| = \binom{n}{n-4} (15^{n-4} - 4 \cdot 8^{n-4} - 3 \cdot 6^{n-4} + 12 \cdot 5^{n-4} - 6 \cdot 4^{n-4})$$

$$|PC(n, n-4)| = \binom{n}{n-4} (15^{n-4} + 12 \cdot 11^{n-4} + 24 \cdot 9^{n-4} + 16 \cdot 8^{n-4} + 20 \cdot 7^{n-4} + 27 \cdot 6^{n-4} - 108 \cdot 5^{n-4} - 42 \cdot 4^{n-4} + 30 \cdot 3^{n-4} + 24 \cdot 2^{n-4} - 4)$$

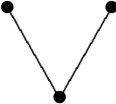
10.3. $PC(n, n-3)$, $n \geq 4$.

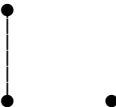
Si $X \in PC(n, n-3)$ entonces $|m(X)| = n-3$ y $|X \setminus m(X)| = 3$. En los diagramas de la izquierda indicamos los posibles posets, no isomorfos, con 3 elementos

1)  $\binom{n}{n-3} 6(3^{n-3} - 2^{n-3})$

2)  $\binom{n}{n-3} 3(4^{n-3} - 2 \cdot 2^{n-3} + 1)$

$$|PC(n, n-3, 1)| = \binom{n}{n-3} (3 \cdot 4^{n-3} + 6 \cdot 3^{n-3} - 12 \cdot 2^{n-3} + 3)$$

3)  $\binom{n}{n-3} 3(4^{n-3} - 3^{n-3})$

4)  $\binom{n}{n-3} 6(5^{n-3} - 2 \cdot 3^{n-3} + 2^{n-3})$

$$|PC(n, n-3, 2)| = \binom{n}{n-3} (6 \cdot 5^{n-3} + 3 \cdot 4^{n-3} - 15 \cdot 3^{n-3} + 6 \cdot 2^{n-3})$$

5) • • •

$$|PC(n, n-3, 3)| = \binom{n}{n-3} (7^{n-3} - 3 \cdot 4^{n-3} + 2 \cdot 3^{n-3}).$$

$$|PC(n, n-3)| = \binom{n}{n-3} (7^{n-3} + 6 \cdot 5^{n-3} + 3 \cdot 4^{n-3} - 7 \cdot 3^{n-3} - 6 \cdot 2^{n-3} + 3)$$

$|PC(n, n-3, 3)|$ se determina del siguiente modo. Sea $M(X) = \{1, 2, 3\}$ y consideremos los conjuntos (ver definición de P_i en la sección 2):

$$A_1 = P_1 \cap \overline{P_2} \cap \overline{P_3}, \quad A_2 = P_1 \cap P_2 \cap \overline{P_3}, \quad A_3 = P_1 \cap P_2 \cap P_3,$$

$$A_4 = P_1 \cap \overline{P_2} \cap P_3, \quad A_5 = \overline{P_1} \cap P_2 \cap \overline{P_3}, \quad A_6 = \overline{P_1} \cap P_2 \cap P_3,$$

$$A_7 = \overline{P_1} \cap \overline{P_2} \cap P_3.$$

y $A = \{A_1, A_2, A_3, A_4, A_5, A_6, A_7\}$. Luego $\bigcup_{i=1}^7 A_i = \bigcup_{i=1}^3 P_i$.

Sea $G = \{A_1, \dots, A_7\}^{m(X)}$, luego $|G| = 7^{n-3}$.

Como el poset debe ser conexo entonces buscamos el conjunto F de las funciones que satisfagan:

$$B_1 = A_2 \cup A_3 \cup A_4 \neq \emptyset, \quad B_2 = A_2 \cup A_3 \cup A_6 \neq \emptyset, \quad B_3 = A_3 \cup A_4 \cup A_6 \neq \emptyset.$$

Cabe aclarar que si decimos $A_i \neq \emptyset$ esto debe entenderse como una abreviatura de “el conjunto de las funciones $g \in G$ tales que para algún $x \in m(X)$, $g(x) \in A_i$ ”. Observemos que estas condiciones implican:

$$P_1 = A_1 \cup A_2 \cup A_3 \cup A_4 \neq \emptyset, \quad P_2 = A_2 \cup A_3 \cup A_5 \cup A_6 \neq \emptyset,$$

$$P_3 = A_3 \cup A_4 \cup A_6 \cup A_7 \neq \emptyset.$$

Tenemos entonces que

$$F = \{g \in G : \text{existen } x, y, z \in m(X) \text{ tal que } g(x) \in B_1, g(y) \in B_2, g(z) \in B_3\}$$

Nos interesa determinar $|F|$. Observemos que

$$|F| = |G| - |\overline{F}| = 7^{n-3} - |\overline{F}|$$

y que

$$\overline{F} = \bigcup_{i=1}^3 F_i$$

donde

$$F_1 = \{g \in G : g(x) \in A_1 \cup A_5 \cup A_6 \cup A_7, \forall x \in m(X)\}$$

$$F_2 = \{g \in G : g(x) \in A_1 \cup A_4 \cup A_5 \cup A_7, \forall x \in m(X)\}$$

$$F_3 = \{g \in G : g(x) \in A_1 \cup A_2 \cup A_5 \cup A_7, \forall x \in m(X)\}$$

Observemos que:

$$F_1 \cap F_2 = \{g \in G : g(x) \in A_1 \cup A_5 \cup A_7, \forall x \in m(X)\}$$

$$F_1 \cap F_3 = \{g \in G : g(x) \in A_1 \cup A_5 \cup A_7, \forall x \in m(X)\}$$

$$F_2 \cap F_3 = \{g \in G : g(x) \in A_1 \cup A_5 \cup A_7, \forall x \in m(X)\}$$

$$F_1 \cap F_2 \cap F_3 = \{g \in G : g(x) \in A_1 \cup A_5 \cup A_7, \forall x \in m(X)\}$$

luego

$$|\overline{F}| = \sum_{i=1}^3 |F_i| - |F_1 \cap F_2| - |F_1 \cap F_3| - |F_2 \cap F_3| + |F_1 \cap F_2 \cap F_3|$$

y como

$$|F_1| = |F_2| = |F_3| = 4^{n-3}$$

y

$$|F_1 \cap F_2| = |F_1 \cap F_3| = |F_2 \cap F_3| = |F_1 \cap F_2 \cap F_3| = 3^{n-3}$$

entonces

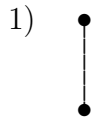
$$|\overline{F}| = 3 \cdot 4^{n-3} - 2 \cdot 3^{n-3}$$

luego

$$|F| = 7^{n-3} - (3 \cdot 4^{n-3} - 2 \cdot 3^{n-3}) = 7^{n-3} - 3 \cdot 4^{n-3} + 2 \cdot 3^{n-3}$$

10.4. $PC(n, n-2)$, $n \geq 3$.

Si $X \in PC(n, n-2)$ entonces $|m(X)| = n-2$ y $|X \setminus m(X)| = 2$. En los diagramas de la izquierda indicamos los posibles posets, no isomorfos, con 2 elementos



$$|PC(n, n-2, 1)| = \binom{n}{n-2} (2 \cdot 2^{n-2} - 2)$$

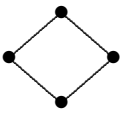
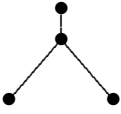
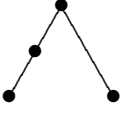
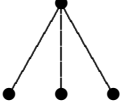


$$|PC(n, n-2, 2)| = \binom{n}{n-2} (3^{n-2} - 2^{n-2})$$

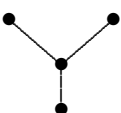
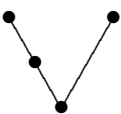
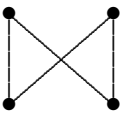
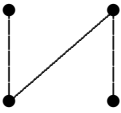
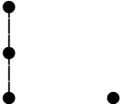
$$|PC(n, n-2)| = \binom{n}{n-2} (3^{n-2} + 2^{n-2} - 2).$$

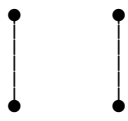
10.5. $PNC(n, n-4)$, $n \geq 5$.

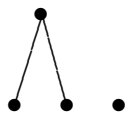
Si $X \in PNC(n, n-4)$ entonces $|m(X)| = n-4$ y $|X \setminus m(X)| = 4$. En los diagramas de la izquierda indicamos los posibles posets, no isomorfos, con 4 elementos

- | | | |
|----|--|--|
| 1) |  | $\binom{n}{n-4} 24 (5^{n-4} - 2 \cdot 4^{n-4} + 3^{n-4})$ |
| 2) |  | $\binom{n}{n-4} 12 (6^{n-4} - 2 \cdot 5^{n-4} + 4^{n-4})$ |
| 3) |  | $\binom{n}{n-4} 12 (6^{n-4} - 5^{n-4} - 2 \cdot 4^{n-4} + 3 \cdot 3^{n-4} - 2^{n-4})$ |
| 4) |  | $\binom{n}{n-4} 24 (7^{n-4} - 6^{n-4} - 5^{n-4} + 2 \cdot 3^{n-4} - 2^{n-4})$ |
| 5) |  | $\binom{n}{n-4} 4 (9^{n-4} - 8^{n-4} - 3 \cdot 5^{n-4} + 3 \cdot 4^{n-4} + 3 \cdot 3^{n-4} - 4 \cdot 2^{n-4} + 1)$ |

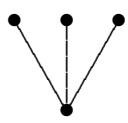
$$PNC^{(1)}(n, n-4, 1) = \binom{n}{n-4} (4 \cdot 9^{n-4} - 4 \cdot 8^{n-4} + 24 \cdot 7^{n-4} - 48 \cdot 5^{n-4} - 48 \cdot 4^{n-4} + 120 \cdot 3^{n-4} - 52 \cdot 2^{n-4} + 4)$$

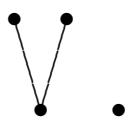
- | | | |
|-----|---|--|
| 6) |  | $\binom{n}{n-4} 12 (6^{n-4} - 2 \cdot 5^{n-4} + 4^{n-4})$ |
| 7) |  | $\binom{n}{n-4} 24 (7^{n-4} - 2 \cdot 6^{n-4} + 5^{n-4})$ |
| 8) |  | $\binom{n}{n-4} 6 (7^{n-4} - 6^{n-4} - 2 \cdot 5^{n-4} + 3 \cdot 4^{n-4} - 3^{n-4})$ |
| 9) |  | $\binom{n}{n-4} 24 (8^{n-4} - 7^{n-4} - 6^{n-4} + 2 \cdot 4^{n-4} - 3^{n-4})$ |
| 10) |  | $\binom{n}{n-4} 24 (8^{n-4} - 7^{n-4} - 6^{n-4} + 5^{n-4})$ |

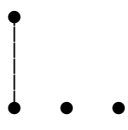
11)  $\binom{n}{n-4} 12 (9^{n-4} - 8^{n-4} - 2 \cdot 6^{n-4} + 2 \cdot 5^{n-4} + 2 \cdot 4^{n-4} - 3 \cdot 3^{n-4} + 2^{n-4})$

12)  $\binom{n}{n-4} 12 (10^{n-4} - 9^{n-4} - 2 \cdot 6^{n-4} + 2 \cdot 5^{n-4} + 4^{n-4} - 3^{n-4})$

$$PNC^{(\cdot)}(n, n-4, 2) = \binom{n}{n-4} (12 \cdot 10^{n-4} + 36 \cdot 8^{n-4} - 18 \cdot 7^{n-4} - 138 \cdot 6^{n-4} + 60 \cdot 5^{n-4} + 114 \cdot 4^{n-4} - 78 \cdot 3^{n-4} + 12 \cdot 2^{n-4}).$$

13)  $\binom{n}{n-4} 4 (9^{n-4} - 2 \cdot 8^{n-4} + 7^{n-4})$

14)  $\binom{n}{n-4} 12 (10^{n-4} - 9^{n-4} - 8^{n-4} + 7^{n-4})$

15)  $\binom{n}{n-4} 12 (12^{n-4} - 11^{n-4} - 8^{n-4} + 7^{n-4} + 5^{n-4} - 3 \cdot 4^{n-4} + 3 \cdot 3^{n-4} - 2^{n-4})$

$$PNC^{(\cdot)}(n, n-4, 3) = \binom{n}{n-4} (12 \cdot 12^{n-4} - 12 \cdot 11^{n-4} + 12 \cdot 10^{n-4} - 8 \cdot 9^{n-4} - 32 \cdot 8^{n-4} + 28 \cdot 7^{n-4} + 12 \cdot 5^{n-4} - 36 \cdot 4^{n-4} + 36 \cdot 3^{n-4} - 12 \cdot 2^{n-4}).$$

16) 

$$PNC^{(\cdot)}(n, n-4, 4) = \binom{n}{n-4} (16^{n-4} - 15^{n-4} + 3 \cdot 6^{n-4} - 12 \cdot 5^{n-4} + 12 \cdot 4^{n-4} - 4 \cdot 2^{n-4} + 1).$$

$$PNC(n, n-4) = \binom{n}{n-4} (16^{n-4} - 15^{n-4} + 12 \cdot 12^{n-4} - 12 \cdot 11^{n-4} + 24 \cdot 10^{n-4} - 4 \cdot 9^{n-4} + 34 \cdot 7^{n-4} - 135 \cdot 6^{n-4} + 12 \cdot 5^{n-4} + 42 \cdot 4^{n-4} + 78 \cdot 3^{n-4} - 56 \cdot 2^{n-4} + 5).$$

10.6. $|PNC(n, n-3)|$, $n \geq 4$.

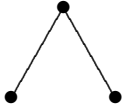
Si $X \in PNC(n, n-3)$ entonces $|m(X)| = n-3$ y $|X \setminus m(X)| = 3$. En los diagramas de la izquierda indicamos los posibles posets, no isomorfos, con 3 elementos

1)



$$\binom{n}{n-3} (6 \cdot 4^{n-3} - 2 \cdot 3^{n-3} + 2^{n-3})$$

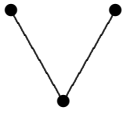
2)



$$\binom{n}{n-3} (3 \cdot 5^{n-3} - 4^{n-3} - 2 \cdot 3^{n-3} + 3 \cdot 2^{n-3} - 1)$$

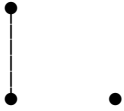
$$|PNC^{(')} (n, n-3, 1)| = \binom{n}{n-3} (3 \cdot 5^{n-3} + 3 \cdot 4^{n-3} - 18 \cdot 3^{n-3} + 15 \cdot 2^{n-3} - 3).$$

3)



$$\binom{n}{n-3} (3 \cdot 5^{n-3} - 2 \cdot 4^{n-3} + 3^{n-3})$$

4)



$$\binom{n}{n-3} (6 \cdot 6^{n-3} - 5^{n-3} - 4^{n-3} + 3^{n-3})$$

$$|PNC^{(')} (n, n-3, 2)| = \binom{n}{n-3} (6 \cdot 6^{n-3} - 3 \cdot 5^{n-3} - 12 \cdot 4^{n-3} + 9 \cdot 3^{n-3}).$$

5)



$$|PNC^{(')} (n, n-3, 3)| = \binom{n}{n-3} (8^{n-3} - 7^{n-3} - 2 \cdot 3^{n-3} + 3 \cdot 2^{n-3} - 1).$$

$$|PNC(n, n-3)| = \binom{n}{n-3} (8^{n-3} - 7^{n-3} + 6 \cdot 6^{n-3} - 9 \cdot 4^{n-3} - 11 \cdot 3^{n-3} + 18 \cdot 2^{n-3} - 4).$$

10.7. $|PNC(n, n-2)|$, $n \geq 3$.

Si $X \in PNC(n, n-2)$ entonces $|m(X)| = n-2$ y $|X \setminus m(X)| = 2$. En los diagramas de la izquierda indicamos los posibles posets, no isomorfos, con 2 elementos

1)



$$|PNC^{(')} (n, n-2, 1)| = \binom{n}{n-2} (2 \cdot 3^{n-2} - 4 \cdot 2^{n-2} + 2)$$

$$2) \quad \bullet \quad \bullet \quad |PNC^{(')} (n, n-2, 2)| = \binom{n}{n-2} (4^{n-2} - 3^{n-2} - 2^{n-2} + 1)$$

Luego

$$|PNC(n, n-2)| = \binom{n}{n-2} (4^{n-2} + 3^{n-2} - 5 \cdot 2^{n-2} + 3)$$

Referencias

- [1] Alexandrov P.S., *Diskrete Räume*, Mat. Sb. (N.S.) 2 (1937), 501-518.
- [2] Birkhoff G., *Lattice Theory*, American Mathematical Society, Colloq. Publ. Vol. 25, revised edition, Providence, 1948.
- [3] Borevič Z. I. *On the question of the enumeration of finite topologies* (Russian) Zap. Naučn Sem. Leningrad. Otdel. Mat.Inst. Stelov (LOMI) 71 (1977), 47-65, 283.
- [4] Borevič Z. I, Venslav V., Dobrovolskiĭ E. and Rodionov V.I., *The number of labeled topologies on nine points* (Russian), Zap. Naučn Sem. Leningrad. Otdel. Mat.Inst. Stelov (LOMI) 75 (1978), 35-42, 195.
- [5] Borevič Z. I, Bumagin and Rodionov V.I., *The number of labeled topologies on ten points* (Russian), Zap. Naučn Sem. Leningrad. Otdel. Mat.Inst. Stelov (LOMI) 86 (1979), 5-10, 188.
- [6] Brualdi R.A., Chan Jung H. and Trotter Jr. W.T., *On the poset of all posets on n elements*, Discrete Applied Mathematics, 50 (1994), 111-123.
- [7] Brinkmann G. and McKay B.D., *Posets on up to 16 Points*, Order 19 (2002), 147-179.
- [8] Butler K. K. H., *The number of partially ordered sets*, Journal of Combinatorial Theory (B) ,13, (1972), 276-289.
- [9] Chatterji S.D., *The number of topologies on n points*, Kent State University, NASA Technical Report (1966)
- [10] Chaunier C. and Lygeros N., *Progrés dans l'énumération des posets*, C.R. Acad. Sci. Paris série I, 314 (1991), 691-694.
- [11] Chaunier C. and Lygeros N., *The number of orders with thirteen elements*, Order 9 (1992), 203-204.
- [12] Comtet L., *Recouvrement, bases de filtre et topologies d'un ensemble fini*, Compt. Rend. Acad. Sci. Paris, Ser. A-B, 262 (1966), 1091-1094.
- [13] Culberson J.C. and Rawlins G.J.E., *New results from an algorithm for counting posets*, Order 7(4) (1991), 361-374.
- [14] Chatterji, S. D., *The number of topologies on n points*, Kent State University, NASA Thecnical Reports. (1966).

- [15] Das S.K., *A machine representation of finite T_0 topologies*, J. ACM, 24 (1977), 676-692.
- [16] Dwinger Ph., *A classification of the partial orderings on a finite set*, Proceedings A 89, (1986), 271-287.
- [17] Ern , M., *Endliche Topologies*. Diploma thesis. Universit t M nchen. (1970)
- [18] Ern , M., *Struktur und Anzahlformeln f r Topologien auf endlichen Mengen*, Ph. Dissertation, Universit t M nchen. (1972)
- [19] Ern , M., *Struktur und Anzahlformeln f r Topologien auf endlichen Mengen*, Manuscripts Math. 11 (1974), 221-259.
- [20] Ern , M., *On the cardinalities of finite topologies and the number of antichains in partially ordered sets*, Discrete Mathematics 35 (1981) 119-133.
- [21] Ern  M. and Stege K., *Counting finite posets and topologies*, Order 8, (1991), 247-265.
- [22] Ern  M. and Stege K., *The number of topologies and orders on 15 points*, Preprint, Universit t Hannover, (1993).
- [23] Evans J. W., Harary F. and Lynn M. S., *On the computer enumeration of finite topologies*, Comm. ACM 10 (1967), 295-297, 313.
- [24] Heitzig J. and Reinhold J., *The number of unlabeled orders on fourteen elements*, Order 17 (2000), 333-341.
- [25] Krishnamurty V., *On the number of topologies on a finite set*, Amer. Math. Monthly 73, (1966), 154-157.
- [26] Lygeros N. and Zimmermann P., *Computation of $p(14)$; the number of posets with 14 elements 1.338.193.159.771*, <http://www.lygeros.org/Math/poset.html>
- [27] Mielants W., *On the number of ordering-structures*, Riv. Mat. Univ. Parma (3) 1, (1972), 83-89.
- [28] M hring R.H., *Algorithmic aspects of comparability graphs and interval graphs*, Graphs and Order: The Role of Graphs in the Theory of Ordered Sets and Its Applications, volume 17 of NATO ADv. Stud. Ser. C: Math. Phys. Sci, (1984), 41-102.
- [29] Monteiro L., *Free bounded distributive lattices on n generators*, Multiple valued logic 6, (2001), 175-192,
- [30] Neggers J., *Counting finite posets*, Acta Math. Acad. Sci. Hungaricae, Tomus 31, 3-4 (1978), 233-258.
- [31] Rodionov V.I., *A relation in finite topologies* (Russian), Zap. Nau n Sem. Leningrad. Otdel. Mat.Inst. Stelov (LOMI) 103 (1979), 114-116, 159.
- [32] Rodionov V.I., *Some recursive relations in finite topologies* (Russian), Zap. Nau n Sem. Leningrad. Otdel. Mat.Inst. Stelov (LOMI) 114 (1982), 174-179, 221.

- [33] Rozenfel'd R.A., *The number of partial orderings on a 6-set* (Russian), Rostock Math. Kollq. 28 (1985), 46-48
- [34] Shafaat A., *On the number of topologies definable for a finite set*, J. Austral. Math. Soc. 8 (1968), 194-198
- [35] Sharp Jr. H., *Quasi-orderings and topologies on a finite set*, Proc. Amer. Math. Soc. 17 (1966), 1344-1349
- [36] Sharp Jr. H., *Cardinality of finite topologies*, J. Combinatorial Theory 5 (1968), 82-86
- [37] Sloane, N.J. A., *The On-Line Encyclopedia of Integer Sequences*, <http://oeis.org>
- [38] Stanley, R.P., *On the number of open sets of finite topologies*, J. Combinatorial Theory (A), 10, (1971), 74-79
- [39] Stephen D., *Topology on a finite sets*, Amer. Math. Monthly 75 (1968), 739-741