ANÁLISIS

Una generalización de los espacios H_μ y H'_μ y del espacio de multiplicadores .

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La Transformación de Hankel definida por:

$$\hbar_\mu f = \int_0^\infty f(x)\sqrt{xy}J_\mu(xy)\,dx$$

donde $0 < y < \infty$, $\mu \in I\!R$, $\mu \geq -\frac{1}{2}$ y J_μ la función de Bessel de primera clase y de orden μ, ha sido estudiada por A. H. Zemanian en [1]. Allí se define un espacio de prueba denotado por $\mathcal{H}_\mu$ que es numerablemente multinormado y además un espacio de Fréchet y se demuestra que la transformación de Hankel es un automorfismo de $\mathcal{H}_\mu$ lo cual permite extender la transformación de Hankel al dual $\mathcal{H}'_\mu$ vía la transformación adjunta. Koh [2], construye una generalización n-dimensional de estos espacios, obteniendo propiedades análogas.

Se va ha exponer aquí otra generalización n-dimensional de los espacios $\mathcal{H}_\mu$ y $\mathcal{H}'_\mu$ que posee propiedades análogas a las estudiadas por Zemanian y Koh. También se han estudiado propiedades del espacio $\mathcal{O}$ de los multiplicadores de $\mathcal{H}_\mu$ y $\mathcal{H}'_\mu$.

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Remarks on $\sum_{n\leqslant N}\frac{(-1)^{n-1}}{n^{\frac{1}{2}+it}}$ and $\zeta(\frac{1}{2}+it)$.

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On the distribution $\left[\delta^{(\ell)}\left(P_+^s\right)\right]^m$

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The purpose of this Note is the proof of the formula

$$\left[\delta^{(\ell)}\left(P_+^s\right)\right]^m = \left(\frac{1}{(-1)^{s-1}}\right)^m (-1)^{m-1} \cdot A_{s,p,q,n,\ell,m} \cdot \delta^{(m(\ell+s)-1)}(P_+) , \qquad (4,13)$$

where

$$A_{s,p,q,n,\ell,m} = \left(\frac{1}{2}\frac{2}{(s-1)!}\right)^m \frac{(\ell+s-1)!}{(m(\ell+s)-1)!} \cdot \left(\frac{1}{2\left[\psi\left(\frac{p}{2}\right) - \psi\left(\frac{n}{2}\right)\right]}\right)^{m-1} , \qquad (4,16)$$

$$\psi(k) = -C + 1 + \frac{1}{2} + \cdots + \frac{1}{k-1} , \qquad (1,35)$$

and $C = \lim_{n\to\infty}\left(\sum_{m=1}^{n} \frac{1}{n} - \log m\right) = 0,577215664$ is the Euler's constant. Here δ is the Dirac-delta function and P_+^s is defined by

$$(P_+^\lambda, \varphi) = \int_{P>0} P^\lambda(x)\varphi(x)dx \; ; \qquad (1,2)$$

with

$$P = P(x) = x_1^2 + \cdots + x_p^2 - x_{p+1}^2 - \cdots - x_{p+q}^2 \; . \qquad (1,1)$$

We note that the formula (4.13) is a generalization of the one-dimensional formula $\left[\delta^{(k)}(x^n)\right]^m$ due to A.P. Khapalyuk [6]. To arrive at our desired formula (4.13), we obtain several others interesting results. These conclusions are, under some conditions, the following formulas,
$\delta(P^2)=2P^{-1}\cdot\delta(P)$, (2,3), $\delta^{(k)}(G)\cdot G^{-k-1}=-\frac{1}{2}\frac{k!}{(-1)^k(2k+1)!}\delta^{(2k+1)}(G)$, (2,5),
$\delta(G)\cdot G^{-1}=-\frac{1}{2}\delta'(G)$, (2,6), $\delta(P^s)=s\left(P^{-1}\right)^{s-1}\delta(P)$, (2,7), $\delta\left(P^2\right)=-\delta'(P)$, (2,8),
$\delta(P^s)=\frac{(-1)^k s}{k!}P^{k+1-s}\delta^{(k)}(P)$, (2,14), $P^{k+1-s}\cdot\delta^{(k)}(P)=\frac{(-1)^{k+1-s}k!}{(s-1)!}\delta^{(s-1)}(P)$, (2,15),
$P_+^{k+1-s}\cdot\delta^{(k)}(P)=\frac{1}{2}\frac{(-1)^{k+1-s}}{(s-1)!}k!\delta^{(s-1)}(P_+)$, (2,16),
$\delta(P^s)=\frac{s}{(-1)^{s-1}(s-1)!}\delta^{(s-1)}(P)$, (2,17), $\delta\left(P_+^s\right)=\frac{1}{2}\frac{s}{(-1)^{s-1}(s-1)!}\delta^{(s-1)}(P_+)$, (2,18),
$\delta^{(\ell)}(P^s)=\frac{s}{(-1)^s(s-1)!}\delta^{(\ell+s-1)}(P)$, (3,4), $\delta^{(\ell)}(P_+^s)=\frac{1}{2}\frac{s}{(-1)^{s-1}(s-1)!}\delta^{(\ell+s-1)}(P_+)$, (3,5),

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Some remarks on orbits of disk authomorphisms

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The nature of composition operators is connected with theoretical aspects of the underlying space. Moreover, strikingly kinds of their cyclical behavior are intrinsically tied with the dynamical of the inducing maps. Their knowledge provide new tools in the resolution of problems of complex polynomial approximation, analytic functional equations and functional analysis (cf. [1]). We consider the open unit disk D endowed with the Poincaré or the (equivalent) pseudohyperbolic metric. We prove that the set of orbits is a non dense subspace of the product space $D^{\mathbb{N}_0}$, that the set $Aut\,(D)$ of analytic authomorphims on D is a topological group and we classify their stabilizers.

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Descripción de las cápsulas convexas de los conjuntos fraccionarios en las bases $-n+i$ y puntos dobles en el contorno para la base -2+i.

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On spaces associated with primitives of distributions in one-sided Hardy spaces

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The purpose of this work is to present the spaces $\mathcal{H}^{p,+}_{q,\alpha}(w)$ on the real line, where $0 < p \leq 1$, $1 < q < \infty$, $\alpha > 0$, and with weights w belonging to the class A_s^+ introduced by E. Sawyer. Given $f \in L^q_{loc}(R)$, we consider its class F in E^q_N, the quotient space between $L^q_{loc}(R)$ and the linear subspace formed by all the polynomials of degree at most N. We define the maximal function

$$N^+_{q,\alpha}(F,x) = \inf\{n^+_{q,\alpha}(f,x) : f \in F\},$$

where

$$n^+_{q,\alpha}(f,x) = \sup_{\rho>0} \frac{1}{\rho^\alpha}(\frac{1}{\rho}\int_x^{x+\rho} |f(y)|\, dy)^{1/q},$$

and $\alpha = N + \beta$, for $0 < \beta \leq 1$.

We shall say that a class F in E^q_N belongs to $\mathcal{H}^{p,+}_{q,\alpha}(w)$ if $N^+_{q,\alpha}(F,x) \in L^p(w)$. Furthermore, we shall say that a class A is a p-atom in $\mathcal{H}^{p,+}_{q,\alpha}(w)$, if there exist a representative $a(x)$ of A and an interval I such that supp(a) is contained in I and $N^+_{q,\alpha}(A,x) \leq w(I)^{-1/p}$.

In this work, we give a decomposition into atoms for elements of $\mathcal{H}^{p,+}_{q,\alpha}(w)$. Furthermore, if α is a natural number weprove that we can identify the spaces $\mathcal{H}^{p,+}_{q,\alpha}(w)$ with the one-sided Hardy spaces $H^{p,+}(w)$ introduced by L. de Rosa and C. Segovia. This identification is given by the operator D^α.

In the case n-dimensional, when α is a even and for the Lebesgue measure, i.e., $w \equiv 1$, this spaces were studied by A. Gatto, J. Jiménez and C. Segovia.