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WHEN THE IDENTITY IS A (σ,τ)-DERIVATION

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ABSTRACT. The concept of (σ,τ)-derivation on an algebra generalizes the usual notion of derivation. We focus our attention on conditions under which the identity operator on an associative algebra $\mathscr{U}$ on a field $\mathfrak{K}$ is a (σ,τ)-derivation.

Let $\mathscr{U}$, $\mathscr{B}$ be associative algebras on a fixed field $\mathfrak{K}$. Let us consider a $\mathscr{B}$-bimodule $\mathfrak{X}$ and linear mappings $\sigma,\tau:\mathscr{U}\to\mathscr{B}$ and $d:\mathscr{U}\to\mathfrak{X}$. Then d is called a (σ,τ)-derivation from $\mathscr{U}$ into $\mathfrak{X}$ if $d(ab)=\sigma(a)d(b)+d(a)\tau(b)$ for all $a,b\in\mathscr{U}$. If $\sigma=\tau$ we will simply say that d is a σ-derivation (see the Examples 2, 3, 4, 5 and 6 below). This notion generalizes the current concept of a derivation and actually it constitutes a matter of intensive research. For example, for studies concerning inner (σ,τ)-derivations of the type $d_x(a)=\sigma(a)x-x\tau(a)$, where $a\in\mathscr{U}$ and $x\in\mathfrak{X}$, the reader can see [7], [2]. For researches in the context of Banach bimodules see [3], [5]. The problematic about automatic continuity of σ-derivations or (σ,τ)-amenability on C^*-algebras is treated in [4] and [6] res-pectively. However, the investigation of the issue is more limited in the context of general algebras. If d is a (σ,τ)-derivation it is straightforward to see that

$$(\sigma(ab)-\sigma(a)\sigma(b))\,d(c)=d(a)\,(\tau(bc)-\tau(b)\tau(c))$$

for all $a,b,c\in\mathscr{U}$. It is timely to point out that if $\mathscr{U}$ and $\mathscr{B}$ are Banach associative algebras, d is a σ-derivation whose separating space has null right annihilator and σ is continuous, then σ becomes a homomorphism. In this setting, d is bounded when the set $\{\sigma(ab)-\sigma(a)\sigma(b):a,b\in\mathscr{U}\}$ has null left annihilator (cf. [4], Remark 2.4).

Our aim in this article is to determine conditions under which the identity mapping $\mathrm{Id}_{\mathscr{U}}$ on an associative algebra $\mathscr{U}$ is a (σ,τ)-derivation (see Th. 10 below). When this is the case it will be seen that the mappings σ and τ have a very simple structure and are automatically continuous in the context of normed algebras. In Prop. 7 we will see that linear mappings σ and τ on a Banach algebra $\mathscr{U}$ become bounded if there is some bounded (σ,τ)-derivation with null annihilator ideals. A last example of (σ,τ)-derivations connected with the identity mapping in a Banach space sequence will be given in Ex. 12.

Notation 1. *We shall denote the usual unitization of an associative algebra $\mathscr{U}$ over a field $\mathfrak{K}$ as $\mathscr{U}^\sharp$. So, $\mathscr{U}^\sharp=\mathscr{U}\times\mathfrak{K}$ is provided with the natural $\mathfrak{K}$-vector space structure together with the multiplication of elements (a_1,k_1) and (a_2,k_2) of $\mathscr{U}^\sharp$ defined as $(a_1,k_1)(a_2,k_2)=(a_1a_2+k_2a_1+k_1a_2,k_1k_2)$. Thus $\mathscr{U}^\sharp$ becomes an associative algebra on $\mathfrak{K}$ with unit element $(0,1)$. Let us denote by $j:\mathscr{U}\hookrightarrow\mathscr{U}^\sharp$ the injection of $\mathscr{U}$ into $\mathscr{U}^\sharp$, $p:\mathscr{U}^\sharp\to\mathscr{U}$ the projection of $\mathscr{U}^\sharp$ onto $\mathscr{U}$, $q:\mathscr{U}^\sharp\to\mathfrak{K}$ the projection of $\mathscr{U}^\sharp$ onto $\mathfrak{K}$. If $\sigma,\tau\in\mathscr{L}(\mathscr{U})$ then $\mathscr{D}(\sigma,\tau)$ (or simply $\mathscr{D}(\sigma)$ if $\sigma=\tau$) will denote the linear subspace of $\mathscr{L}(\mathscr{U})$ of (σ,τ)-derivations from $\mathscr{U}$ into $\mathscr{U}$. As usual, if $a\in\mathscr{U}$ we will denote by L_a,R_a the elements of $\mathscr{L}(\mathscr{U})$ such that $L_a(x)=ax$ and $R_a(x)=xa$ for all $x\in\mathscr{U}$.*

Palabras clave. Annihilator of an algebra; (σ,τ)-derivations; multipliers of an algebra; separating space of a linear map; annihilator ideals.

Example 2. *Given* $\mathfrak{h} \in \mathrm{Hom}(\mathscr{U}, \mathscr{B})$ *and a derivation* $D : \mathscr{B} \to \mathfrak{X}$*, the mapping* $d = D \circ \mathfrak{h}$ *is a* $\mathfrak{h}$*-derivation. In particular, if* $n \in \mathbb{N}$ *and* $n > 1$ *let* d *be a non-zero* $\mathfrak{h}$*-derivation on the matrix associative algebra* $\mathscr{U} = \mathrm{M}_n(\mathfrak{K})$*. If* $\mathfrak{h}$ *is a homomorphism its kernel is a bilateral ideal of* $\mathscr{U}$*. Further,* $\ker(\mathfrak{h}) \subseteq \ker(d)$ *and as* $\mathscr{U}$ *is simple it has no non trivial bilateral ideals. Consequently,* $\mathfrak{h}$ *must be injective. Indeed,* $\mathfrak{h}$ *becomes an isomorphism and we can define* $D : \mathscr{U} \to \mathscr{U}$ *so that* $D(x) = d\left(\mathfrak{h}^{-1}(x)\right)$ *if* $x \in \mathscr{U}$*. It is readily seen that* D *is a derivation and* $d = D \circ \mathfrak{h}$*.*

Example 3. *If* $\mathscr{U}$ *is an associative algebra, every* $\mathfrak{h} \in \mathrm{Hom}(\mathscr{U})$ *is a* $\mathfrak{h}/2$*-derivation.*

Example 4. *If* $\mathscr{U} = \mathrm{M}_2(\mathfrak{K})$ *and* $x \in \mathscr{U}$*, let*

$$\sigma(x) = \begin{bmatrix} x_{11} & 0 \\ 0 & x_{22} \end{bmatrix}, \quad d(x) = \begin{bmatrix} 0 & x_{12} \\ x_{21} & 0 \end{bmatrix}.$$

Then d *becomes a* σ*-derivation but* $\sigma \notin \mathrm{Hom}(\mathscr{U})$*. More generally, if* $n \in \mathbb{N}$ *and* $\mathscr{U} = \mathrm{M}_n(\mathfrak{K})$*, any linear mapping* $T \in \mathscr{L}(\mathscr{U})$ *uniquely defines* n^2*-matrices* $\left(T^{k,h}\right)_{1 \le k,h \le n} \in \mathscr{U}$ *so that* $T(x) = (\sum_{k,h=1}^{n} T_{i,j}^{k,h} x_{k,h})_{1 \le i,j \le n}$ *for all* $x \in \mathscr{U}$*. It is straightforward to see that if* d *is a* σ*-derivation the following system of matricial equations hold:*

$$\delta_{j,k} d^{i,h} = d^{i,j} \sigma^{k,h} + \sigma^{i,j} d^{k,h}, \quad 1 \le i,j,k,h \le n.$$

Example 5. *Let* $\mathscr{U} = \mathfrak{K}^n$*. It is an associative algebra if we consider the usual* $\mathfrak{K}$*-vector space structure and the product of* $x, y \in \mathscr{U}$ *is defined as*

$$x \cdot y = \left(\sum_{i=1}^{j} x_{j-i+1} y_i\right)_{1 \le j \le n}.$$

Let $d \in \mathscr{L}(\mathscr{U})$ *so that* $d(x_1, \ldots, x_n) = (x_2, \ldots, x_n, 0)$*. If* $n = 2$*,* $a, b \in \mathfrak{K}$ *and* $\sigma, \tau \in \mathscr{L}(\mathscr{U})$ *have the canonical representation*

$$[\sigma] = \begin{bmatrix} 1 & a \\ 0 & b \end{bmatrix}, \quad [\tau] = \begin{bmatrix} 1 & -a \\ 0 & -b \end{bmatrix},$$

then d *is a* (σ, τ)*-derivation. The same conclusion holds if* $n = 3$*,* $a, b, c \in \mathfrak{K}$ *and* $\sigma, \tau \in \mathscr{L}(\mathscr{U})$ *have the canonical representation*

$$[\sigma] = \begin{bmatrix} 1 & a & 0 \\ 0 & b & a \\ 0 & c & b-1 \end{bmatrix}, \quad [\tau] = \begin{bmatrix} 1 & -a & 0 \\ 0 & 1-b & -a \\ 0 & -c & -b \end{bmatrix}.$$

However, d *is not a* (σ, τ)*-derivation if* $n \ge 4$*.*

Example 6. *Let* $\mathscr{U}$ *be an abelian associative algebra over a field* $\mathfrak{K}$*,* $\mathrm{Id}_{\mathscr{U}} \in \mathscr{D}(\sigma, \tau)$*. Then* $a(\sigma(b) - \tau(b)) = (\sigma(a) - \tau(a))b$ *if* $a, b \in \mathscr{U}$*, i.e.* $\sigma - \tau$ *is a multiplier of* $\mathscr{U}$*.*

Proposition 7. *Let* $\mathscr{U}$ *be an associative Banach algebra and let* d *be a bounded* (σ, τ)*-derivation. If the left and right annihilator ideals of* $d(\mathscr{U})$ *are zero then* σ *and* τ *are bounded.*

Proof. Let $\{y\} \cup \{x_n\}_{n \in \mathbb{N}}$ in $\mathscr{U}$ so that $(x_n, \sigma(x_n)) \to (0, y)$ in $\mathscr{U} \times \mathscr{U}$. If $z \in \mathscr{U}$ then $d(x_n z) = \sigma(x_n) d(z) + d(x_n) \tau(z)$ for any $n \in \mathbb{N}$. Letting $n \to \infty$ we get $yd(z) = 0$, i.e. $y = 0$ since $d(\mathscr{U})$ has null left annihilator. Consequently, σ is bounded since its separating space is trivial (cf. [1], p. 39). Analogously, it is seen that τ is also bounded. □

Definition 8. *Given* $\sigma, \tau \in \mathscr{L}(\mathscr{U})$ *we will say that* $\mathrm{Id}_{\mathscr{U}}$ *is a* (σ, τ)*-extendible derivation if it is a* (σ, τ)*-derivation and there are natural extensions* $\sigma^\sharp, \tau^\sharp \in \mathscr{L}(\mathscr{U}^\sharp)$ *of* σ *and* τ *so that* $\mathrm{Id}_{\mathscr{U}^\sharp}$ *is also a* $\left(\sigma^\sharp, \tau^\sharp\right)$*-derivation.*

Proposition 9. *Let $\mathscr{U}$ be a unitary associative algebra on a field $\mathfrak{K}$, $\sigma,\tau\in\mathscr{L}(\mathscr{U})$. So, $\mathrm{Id}_{\mathscr{U}}\in\mathscr{D}(\sigma,\tau)$ if and only if there are unique elements $k\in\mathfrak{K}$ and $a_0\in\mathscr{U}$ such that $\sigma=k\mathrm{Id}_{\mathscr{U}}+R_{a_0}$ and $\tau=(1-k)\,\mathrm{Id}_{\mathscr{U}}-L_{a_0}$.*

Proof. ($\Rightarrow$) If e denotes the unit of $\mathscr{U}$ and $x\in\mathscr{U}$ then

$$x=\sigma(x)+x\tau(e)=\sigma(e)x+\tau(x).$$

As $e=\sigma(e)+\tau(e)$ the claim follows making $k=0$ and $a_0=\sigma(e)$.

($\Leftarrow$) Let $\sigma=k\mathrm{Id}_{\mathscr{U}}+R_{a_0}$ and $\tau=(1-k)\,\mathrm{Id}_{\mathscr{U}}-L_{a_0}$, where $k\in\mathfrak{K}$ and $a_0\in\mathscr{U}$. If $x,y\in\mathscr{U}$ we have

$$\sigma(x)\,y+x\tau(y)=(kx+xa_0)+x(y-ky-a_0y)=xy$$

and the condition is sufficient.

□

Theorem 10. *Let $\mathscr{U}$ be an associative algebra over a field $\mathfrak{K}$.*

(i) *Let $\sigma,\tau\in\mathscr{L}(\mathscr{U})$. So, $\mathrm{Id}_{\mathscr{U}}$ is a (σ,τ)-extendible derivation if and only if there exist $k\in\mathfrak{K}$ and $a_0\in\mathscr{U}$ so that $\sigma=k\mathrm{Id}_{\mathscr{U}}+R_{a_0}$ and $\tau=(1-k)\,\mathrm{Id}_{\mathscr{U}}-L_{a_0}$.*

(ii) *If $\sigma^\sharp,\tau^\sharp\in\mathscr{L}(\mathscr{U}^\sharp)$ and $\mathrm{Id}_{\mathscr{U}^\sharp}\in\mathscr{D}(\sigma^\sharp,\tau^\sharp)$ then*

$$(p\circ\sigma^\sharp\circ j+q\circ\sigma^\sharp\circ j)(a)\,b+a(p\circ\tau^\sharp\circ j+q\circ\tau^\sharp\circ j)(b)=ab$$

for all $a,b\in\mathscr{U}$.

Proof. **(i)($\Rightarrow$)** Let us look for the structure of extensions $\sigma^\sharp$ of σ and $\tau^\sharp$ of τ in $\mathscr{L}\left(\mathscr{U}^\sharp\right)$ so that $\mathrm{Id}_{\mathscr{U}^\sharp}$ is still a $\left(\sigma^\sharp,\tau^\sharp\right)$-derivation. Let

$$\widetilde{\sigma}=q\circ\sigma^\sharp\circ j,\quad \widetilde{\tau}=q\circ\tau^\sharp\circ j$$

in $\mathscr{L}(\mathscr{U},\mathfrak{K})$, $(a_\sigma,k_\sigma)=\sigma^\sharp(0,1)$ and $(a_\tau,k_\tau)=\tau^\sharp(0,1)$ in $\mathscr{U}^\sharp$. Then $\sigma^\sharp$ and $\tau^\sharp$ should act on $(a,k)\in\mathscr{U}^\sharp$ as

$$\sigma^\sharp(a,k)=(\sigma(a)+ka_\sigma,\widetilde{\sigma}(a)+kk_\sigma),$$
$$\tau^\sharp(a,k)=(\tau(a)+ka_\tau,\widetilde{\tau}(a)+kk_\tau).$$

Now, if $\sigma^\sharp$ extends σ we have

$$(\sigma(a),\widetilde{\sigma}(a))=\left(\sigma^\sharp\circ j\right)(a)=(j\circ\sigma)(a)=(\sigma(a),0)$$

for all $a\in\mathscr{U}$, i.e. $\widetilde{\sigma}=0$. Analogously, $\widetilde{\tau}=0$. Now, let $m=(a,k)$ and $n=(b,h)$ in $\mathscr{U}^\sharp$. Since $\mathscr{U}^\sharp$ is an associative algebra over $\mathfrak{K}$ and $\mathrm{Id}_{\mathscr{U}^\sharp}$ is assumed to be a $\left(\sigma^\sharp,\tau^\sharp\right)$-derivation we see that

$$\begin{aligned}p(mn)&=p\left(\sigma^\sharp(m)\,n+m\tau^\sharp(n)\right)\\&=\sigma(a)\,b+a\tau(b)+k\left[a_\sigma b+k_\sigma b+\tau(b)\right]\\&\quad+h\left[\sigma(a)+aa_\tau+k_\tau a\right]+kh\left(a_\sigma+a_\tau\right)\\&=ab+kb+ha,\\q(mn)&=q\left(\sigma^\sharp(m)\,n+m\tau^\sharp(n)\right)\\&=kh(k_\sigma+k_\tau)\\&=kh.\end{aligned}$$

Consequently, as $\mathrm{Id}_{\mathscr{U}}$ is actually a (σ,τ)-derivation the equation

$$k\left[a_\sigma b+k_\sigma b+\tau(b)\right]+h\left[\sigma(a)+aa_\tau+k_\tau a\right]+kh\left(a_\sigma+a_\tau\right)=kb+ha$$

should hold for all $a,b\in\mathscr{U}$, $k,h\in\mathfrak{K}$. In particular, by choosing $a=b=0$ and $k=h=1$ we see that $a_\sigma+a_\tau=0$. Hence

$$\begin{aligned}k_\sigma+k_\tau&=q\left(a_\sigma+a_\tau,k_\sigma+k_\tau\right)\\&=q\left[\sigma^\sharp(0,1)(0,1)+(0,1)\tau^\sharp(0,1)\right]\\&=q\left[\mathrm{Id}_{\mathscr{U}^\sharp}((0,1)(0,1))\right]\\&=1\end{aligned}$$

and

$$k\left[a_\sigma b+k_\sigma b+\tau(b)\right]+h\left[\sigma(a)-aa_\sigma+(1-k_\sigma)a\right]=kb+ha.$$

So, if $k=0$ and $h=1$ we see that $\sigma=k_\sigma\mathrm{Id}_{\mathscr{U}}+R_{a_\sigma}$, while if $k=1$ and $h=0$ then $\tau=(1-k_\sigma)\mathrm{Id}_{\mathscr{U}}-L_{a_\sigma}$. Now (i) follows immediately.

(i)($\Leftarrow$) If $k_0\in\mathfrak{K}$ and $a_0\in\mathscr{U}$ we will prove that $\mathrm{Id}_{\mathscr{U}}$ is a (σ,τ)-extendible derivation if $\sigma=k_0\mathrm{Id}_{\mathscr{U}}+R_{a_0}$ and $\tau=(1-k_0)\mathrm{Id}_{\mathscr{U}}-L_{a_0}$. For, as $\mathscr{U}$ is an associative algebra on $\mathfrak{K}$, it is easy to see that $\mathrm{Id}_{\mathscr{U}}\in\mathscr{D}(\sigma,\tau)$. Further, let $\sigma_k=(k_0-k)\mathrm{Id}_{\mathscr{U}^\sharp}+R_{(a_0,k)}$ and $\tau_k=(1-k_0+k)\mathrm{Id}_{\mathscr{U}^\sharp}-L_{(a_0,k)}$. Then $\mathrm{Id}_{\mathscr{U}^\sharp}\in\mathscr{D}(\sigma_k,\tau_k)$ and given $x\in\mathscr{U}$ we have

$$\begin{aligned}\sigma_k(x,0)&=((k_0-k)x+xa_0+kx,0)=(\sigma(x),0),\\\tau_k(x,0)&=((1-k_0+k)x-a_0x-kx,0)=(\tau(x),0),\end{aligned}$$

i.e. σ_k and τ_k are natural extensions of σ and τ respectively. Finally, by Prop. 9 we have $\mathrm{Id}_{\mathscr{U}^\sharp}\in\mathscr{D}(\sigma_k,\tau_k)$ and the assertion holds.

(ii) It is straightforward.

□

Corollary 11. *If $\mathrm{Id}_{\mathscr{U}}$ is a $\mathfrak{h}$-extendible derivation then $\mathfrak{h}(ab)=a\mathfrak{h}(b)=\mathfrak{h}(a)b$ for all $a,b\in\mathscr{U}$, i.e. $\mathfrak{h}$ becomes a multiplier of $\mathscr{U}$.*

Example 12. *Let $\mathscr{U}=c_0(\mathbb{N})$ be the usual Banach associative algebra of convergent (complex) sequences on $\mathbb{N}$ to zero. Thus $\mathscr{U}^\sharp$ can be identified with the Banach subspace of $l^\infty(\mathbb{N})$ of convergent sequences. Let $\sigma^\sharp,\tau^\sharp\in\mathscr{L}(\mathscr{U}^\sharp)$ so that $\mathrm{Id}_{\mathscr{U}^\sharp}\in\mathscr{D}(\sigma^\sharp,\tau^\sharp)$. By Th. 10, (ii), if we write*

$$\sigma=p\circ\sigma^\sharp\circ j,\quad\widetilde{\sigma}=q\circ\sigma^\sharp\circ j,\quad\tau=p\circ\tau^\sharp\circ j,\quad\widetilde{\tau}=q\circ\tau^\sharp\circ j,$$

then

$$(\sigma+\widetilde{\sigma})(a)b+a(\tau+\widetilde{\tau})(b)=ab \tag{1}$$

for all $a,b\in c_0(\mathbb{N})$. As $\sigma,\tau\in\mathscr{B}(c_0(\mathbb{N}))$, if $x\in c_0(\mathbb{N})$ then

$$\sigma(x)=\left\{\sum_{n=1}^\infty\sigma_{m,n}x_n\right\}_{m\in\mathbb{N}}\qquad\text{and}\qquad\tau(x)=\left\{\sum_{n=1}^\infty\tau_{m,n}x_n\right\}_{m\in\mathbb{N}}.$$

Indeed, the complex matrices $\{\sigma_{m,n}\}_{m,n\in\mathbb{N}}$ and $\{\tau_{m,n}\}_{m,n\in\mathbb{N}}$ are unique, their columns belong to $c_0(\mathbb{N})$ and their rows are uniformly bounded in $l^1(\mathbb{N})$. Besides, as $\widetilde{\sigma},\widetilde{\tau}\in c_0(\mathbb{N})^*$ we can write $\widetilde{\sigma}\equiv\{\sigma_m\}_{m\in\mathbb{N}}$ and $\widetilde{\tau}\equiv\{\tau_m\}_{m=1}^\infty$ in $l^1(\mathbb{N})$. Letting $a=\{\delta_{i,k}\}_{k\in\mathbb{N}}$ and $b=\{\delta_{j,k}\}_{k\in\mathbb{N}}$ in (1), where $i,j\in\mathbb{N}$, we obtain the following equations:

$$\delta_{i,j}a=a\left(\{\tau_{m,j}\}_{m=1}^\infty+\tau_j\right)+\left(\{\sigma_{m,i}\}_{m=1}^\infty+\sigma_i\right)b. \tag{2}$$

From (2) we conclude that

$$\sigma_i+\sigma_{i,i}+\tau_i+\tau_{i,i}=1 \quad \text{for all } i\in\mathbb{N},$$
$$\sigma_{j,i}+\sigma_i=\tau_{i,j}+\tau_j=0 \quad \text{for all } i,j\in\mathbb{N}, i\neq j.$$

So, $\sigma_i=-\lim_{j\to\infty}\sigma_{j,i}=0$ *and* $\tau_j=-\lim_{i\to\infty}\tau_{i,j}=0$ *for all* $i,j\in\mathbb{N}$*, i.e.* $\widetilde{\sigma}=\widetilde{\tau}=0$*. Since* $\sigma_{j,i}=\tau_{i,j}=0$ *if* $j\neq i$ *then* σ *and* τ *become diagonal operators, and as* $\sigma_{i,i}+\tau_{i,i}=1$ *for all* i *then* $\sigma+\tau=\mathrm{Id}_{c_0(\mathbb{N})}$ *and* $\mathrm{Id}_{c_0(\mathbb{N})}\in\mathscr{D}(\sigma,\tau)$*. We observe that, as diagonal operators,* σ *and* τ *are multipliers on* $c_0(\mathbb{N})$*. However, given a diagonal operator* $\mathfrak{h}\in\mathscr{B}(c_0(\mathbb{N}))$ *then* $\mathrm{Id}_{\mathscr{U}}$ *would be a not* $\mathfrak{h}/2$*-extendible derivation. For instance, consider the diagonal operator* $\mathfrak{h}$ *defined by a bounded complex sequence without limit point* $\{\mathfrak{h}_n\}_{n\in\mathbb{N}}$*. Then* $\mathrm{Id}_{c_0(\mathbb{N})}\in\mathscr{D}(\mathfrak{h}/2)$*. If it were* $\mathfrak{h}/2$*-extendible, by Th. 10 we could write*

$$\mathfrak{h}/2=k\mathrm{Id}_{c_0(\mathbb{N})}+a_0\mathrm{Id}_{c_0(\mathbb{N})}=(1-k)\mathrm{Id}_{c_0(\mathbb{N})}-a_0\mathrm{Id}_{c_0(\mathbb{N})} \tag{3}$$

for some $k\in\mathbb{C}$ *and* $a_0\in c_0(\mathbb{N})$*. Thus* $x=2\left\{kx_n+a_0^n x_n\right\}_{n=1}^{\infty}$ *if* $x\in c_0(\mathbb{N})$*. Hence* $1=2(k+a_0^n)$ *for all* $n\in\mathbb{N}$ *and necessarily* $k=1/2$*. By (3) we have* $\mathfrak{h}_n x_n/2=(1/2+a_0^n)x_n$ *if* $x\in c_0(\mathbb{N})$ *and* $n\in\mathbb{N}$*. Thus* $\mathfrak{h}_n/2=1/2+a_0^n$ *for all* $n\in\mathbb{N}$*, which contradicts our election of* $\mathfrak{h}$*.*

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